ORBIS LONGITVDINES REPERTÆ È MAGNETIS À POLO DECLINATIONE.

Magnete paulum vtrinque sæpe deuia Dat inuenire portum vbique Plancius.

Navigator, taking a bearing of the sun (Stradanus, about 1600, see note 42)

THE PRINCIPAL WORKS
OF
SIMON STEVIN

EDITED BY

ERNST CRONE, E. J. DIJKSTERHUIS, R. J. FORBES
M. G. J. MINNAERT, A. PANNEKOEK

AMSTERDAM
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SIMON STEVIN

VOLUME III

ASTRONOMY

EDITED BY

A. PANNEKOEK

NAVIGATION

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SIMON STEVIN

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THE ASTRONOMICAL WORKS

OF

SIMON STEVIN

DE HEMELLOOP

THE HEAVENLY MOTIONS

From the *Wisconstighe Ghedachtenissen* (Work XI; i, 3)

INTRODUCTION TO THE WORK

§ 1

In order to understand the place of Stevin's work on astronomical theory it is necessary first to give a short survey of the opinions prevalent among scholars in the second half of the sixteenth century.

Copernicus' great work *De revolutionibus* appeared in 1543 under the more extensive title, due to Osiander: *De revolutionibus orbium coelestium.* Immediately after its publication it became the object of assiduous study, at the main Protestant university of Wittenberg as well as among scholars at other seats of learning. This interest concerned not so much the heliocentric theory, but rather the numerical elements of the orbits. From the first the heliocentric theory was sharply attacked by the Protestant theologians. Melanchthon (the *"praeceptor Germaniae"*), the foremost among the Wittenberg professors, in a series of lectures and in his *Initia doctrinae physicae,* dismissed it as absurd [1]. This qualification determined the opinion of contemporary authors. At the same time, however, Copernicus, because of the new basis he afforded for the computation of the celestial motions, was praised as the renovator of astronomy, the first and most famous of astronomers, the new Ptolemy. Several students of the book computed in advance the positions of the planets or the moon with the aid of the new data, and they were able to show that these were in better agreement with the observations than the Alphosine Tables. Foremost among them was Erasmus Reinhold [2], professor of mathematics at Wittenberg. First he had to correct several errors in the computations of Copernicus, and in many cases he derived new elements himself. Thus he was able to construct new and better tables, called the Prussian Tables, published in 1551 and reprinted several times afterwards. The new heliocentric world-system, however, is not even alluded to in this work. Reinhold's tables were used by Johannes Stadius [3] for the computation of his "Ephemerides" (daily tables) of the celestial bodies. These tables, destined chiefly for use in astrological prognostics, were published in 1556 for the first time, and in later years in five new editions.

In his valuable work on the origin and the extension of the Copernican doctrine, Ernst Zinner [4] enumerates a number of authors of widely used textbooks on astronomy. Besides Melanchthon's book, mentioned above, with 17 impressions, and Clavius' explanation of the astronomical work of Sacrobosco, which appeared in 1570 and up to 1618 passed through 19 impressions [5], he mentions Kaspar Peucer, Hartmann Beyer, Michael Neander, Victor Strigel, Heinrich Brucaeus, Georg Bachmann, Alb. Leoninus, Paolo Donati, G. A. Magini, Jean Bodin and others [6]. They all reject the earth's motion or are silent on it. And he remarks:

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[1] Ph. Melanchthon, *Initia Doctrinae Physicae,* 1549 (Ed. Bretschneider in *Corpus Reformatorum,* Vol. 13, p. 179). *Cf.* p. 216 Liber I.

[2] Erasmus Reinhold, *Prutenicae Tabulae coelestium motuum* (Tübingen, 1551).

[3] Joh. Stadius, *Ephemerides novae et exactae, ab Anno 1554* (Köln, 1556).

[4] Ernst Zinner, *Entstehung und Ausbreitung der coppernicanischen Lehre* (Erlangen, 1943).

[5] Chr. Clavius, *Opera mathematica V tomis distributa* (Mainz, 1612).

[6] Kaspar Peucer, *Hypotheses astronomicae* (1571).—Hartmann Beyer, *Quaestiones novae* (1549—1573, 6 impressions).—Michael Neander, *Elementa Sphaericae doctrinae* (1561).—

"Were there any adherents of the new doctrine? We might have doubts, if we consider that until 1590 the work of Copernicus was reprinted only once [7]), whereas the textbooks mentioned above together saw 62 impressions. In these, the new doctrine either was not mentioned at all or was termed absurd; seldom was its special character set forth" [8]). Only in England in 1576 an enthusiastic description of the new doctrine was added by Thomas Digges [9]) to a new edition of a book on prognostics by his father [10]).

Yet the number of adherents slowly increased. Christopher Rothmann, astronomer at the Cassel observatory, in his correspondence with Tycho Brahe, in 1589-90, with very well-chosen arguments defended the motion of the earth [11]). Tycho Brahe himself tried to combine the simplicity of the Copernican system with the central position of the earth at rest by a system specially devised — in 1583, as he said —: the planets in describing circles about the sun are carried along with it in its yearly orbit. Though here the motions were only nominally different from Ptolemy's, the Tychonic system found adherence as a symptom of an incipient critical attitude towards the old doctrine. In Italy in 1585 Benedetti spoke of the earth as a subordinate body [12]). Giordano Bruno, extending the Copernican system into a fantastic conception of a world of innumerable suns and inhabited planets, expounded it during his travels all over Europe. Kepler [13]) in his *"Mysterium Cosmographicum"*, his first work, published in 1596, endeavoured to give the heliocentric system a deeper philosophical basis by explaining the number of the planets (six) and their distances by connecting them with the five regular polyhedra. The English physician William Gilbert [14]) in 1600, in his book on magnetism, introduced the daily rotation of the earth as Copernicus had done, but he did not speak of its yearly revolution.

This enumeration shows that when Simon Stevin, in explaining to his illustrious pupil the motion of the celestial bodies, expounded the Copernican as the true beside the Ptolemaic as the untrue system, he sided with an extremely small group of renovators. Whereas the other adherents had expressed their opinion occasionally, in short remarks or in connection with other subjects, Stevin's book was the first textbook destined to give a simple and easy exposition of the heliocentric theory. Soon after its publication (in Dutch in 1605, Work XI; i, 3) a Latin version appeared in the *Hypomnemata Mathematica* (Work XIb). A French translation of the *Wisconstighe Ghedachtenissen* was published by Girard in 1633 in his posthumous edition of the Works of Stevin: *Oeuvres Mathématiques de Simon Stevin* (Work XIII).

Victor Strigel, *Epitome doctrinae de primo motu* (1564).—Heinrich Brucaeus, *De motu primo* (1573—1604). Georg Bachman. *Epitome Doctrinae de primo motu* (1591).—Albert Leoninus, *Theoria motuum coelestium* (1583).—Paolo Donati, *Theoriche overo Speculationi intorno alli Moti Celesti* (1575).—J. A. Magini, *Ephemerides coelestium motuum* (1599—1616).—Jean Bodin, *Universae Naturae Theatrum* (1597).

[7]) Basle, 1566.

[8]) Ernst Zinner, *l.c.*, pp. 275-276.

[9]) *Alae seu Scalae Mathematicae* (1573).

[10]) Leonard Digges, *A Prognosticon everlastinge...* (1576).

[11]) *Cf.* Tychonis Brahei Dani *Opera* VI, p. 217.

[12]) J. B. Benedicti *Diversae Speculationes*, p. 195 and 256 (1585).

[13]) Joh. Kepler, *Mysterium Cosmographicum* (1596).

[14]) W. Gilbert, *De Magnete, Magneticisque Corporibus, et de Magno Magnete Tellure; Physiologia Nova* (1600).

§ 2

In his discussion of the orbits of the heavenly bodies, Stevin follows a special method. He knows that in order to derive these orbits one has to proceed from the observed positions of the sun, moon, and planets in the sky. It would, however, have taken too much time to make such observations with his pupil, Prince Maurice, while this was also beyond the scope of their joint studies. On the other hand, an attempt to derive the orbits from the few observations communicated by Ptolemy and Copernicus would be too difficult. He therefore proceeds from existing printed tables, taking as such the ephemerides computed by Stadius. Since these tables are based upon elements derived from observations, he assumes that they represent the observed motion of the heavenly bodies, and may therefore be used instead of observations for an explanation of the astronomical system. They offer the further advantage that the positions are given for all consecutive days. He calls them *"ervarings dachtafels"* (empirical ephemerides). Though in the Definitions 22 and 23 he distinguishes the empirical ephemerides as determined by instruments, and the computed ephemerides predicted through knowledge of the orbits, he refers to the computed ephemerides as empirical.

Another characteristic of his work is that the theory is presented according to both world-systems. In his First and Second Books the orbits are treated as in the Ptolemaic system, with the earth at rest at the centre; in his Third Book the Copernican theory of the moving earth is introduced. His own opinion on this point is clearly shown by the fact that he calls them the apparent and the true motion (p. 4 of the *Cortbegryp*). The arguments are to be given later on, because the "unknown" motions, to be treated afterwards, show that there is no firm basis as yet for a good theory.

In dealing with the motion of the sun, Stevin first deduces the length of a year by deriving from Stadius' ephemerides the moments of return to the same longitude. From an interval of 52 years between 1554 and 1606 he finds 365d 5h 45m 55s. Finding that this deviates widely from Ptolemy's value (365d 5h 55m 12s, which, however, we know to be too long by 7m), he realizes that the interval of 52 years used was still too short for an exact result. He therefore adopts Ptolemy's value, so that the table of the sun's mean motion, which he adds to his treatment, is identical with Ptolemy's table. Stadius' ephemerides furthermore show that the daily increase of the sun's longitude is smallest in June (57′), greatest in December (1°1′); so the sun moves in an eccentric circle. To find the longitude of its apogee, Stevin determines by trial and error a date in June such that in an equal number of days before and after that day equal arcs of longitude were described. Thus he finds 94°24′, holding for 1554. Another method consists in finding two opposite longitudes of the sun, such that each of the semi-circumferences is covered in the same time; the result was 95°14′. From the data of 1594 in the same way he finds 97°53′, 3°29′ more, from which follows a yearly increase of 5′13″. The interval of 40 years of course is too small to give a reliable result for this increase; hence he compares his longitude of the apogee with Ptolemy's value of 65°30′, 1455 years earlier, from which follows a yearly progress of 1′20″.

This is the method followed throughout for all the moving celestial bodies. In the case of the moon, in Chapter 2, which has been omitted from this abridged edition, a number of different periods have to be derived. First the moon's motion "in her own orbit"; this is how he denotes its motion from apogee through perigee to the next apogee. Stevin perceives that the greatest daily progress (in perigee) is different for full moon and quarters. In order to avoid such irregularities in the derivation of the period, he makes use of three dates of most rapid motion coinciding with full moon (in 1569, 1572, and 1581), and finds a period (the anomalistic month) of (written in sexagesimals) 27;32,56 days, corresponding to 27d 13h 10m 24s. The corresponding daily motion of 13°4′ is in accordance with Ptolemy's value of 13°3′54″56‴. Thereupon he derives the rapid progress in longitude of the lunar apogee; from an interval of 16,393 days (nearly 45 years), in which the apogee made 5 revolutions, a daily increase of 6′35″ is found (Ptolemy gave 6′41″). Adding this motion of the apogee to the first-derived lunar motion relative to the apogee, he gets 13°10′35″ for the daily motion in longitude. From direct comparisons of the longitudes after five intervals of 9 years minus 9 days each, he finds 13°9′ for the moon's daily motion in longitude, which is sufficiently in accordance with the other value.

The mean length of a lunation is derived from two oppositions with an interval of 19 years in which occurred 235 oppositions; the result in days and sexagesimals is 29;32 days (*i.e.* 29d 12h 48m). The corresponding daily progress in elongation (called by Stevin the moon's gain) is found to be 12°11′25″, which is hardly different from Ptolemy's value of 12°11′27″. The same progress, when computed by simply subtracting the sun's daily motion from the moon's, is 12°11′. Finally the return of the latitude is derived from the statement that in 1,089 days the same maximum of latitude returned 40 times at the same longitude; consequently, the daily progress is 13°13′23″, and the daily motion of the node is 3′10″. For all these motions tables are given, "taken from Ptolemy's tables".

The 3rd chapter deals with the motion of Saturn. The retrogradations shown in the ephemerides indicate that Saturn moves on an epicycle, and that the centre of the epicycle describes an eccentric circle (the deferent). In order to eliminate the oscillations due to the epicycle, Stevin only makes use of longitudes in the opposition to the sun, because then the centre of the epicycle is situated behind the planet at the same longitude. To derive the longitude of Saturn's apogee, he makes use of the same method as with the sun; finding a longitude such that the arcs described in the same interval (here about 7 years) before and after this opposition are equal. Thus he finds 268°20′; a special computation is added to make sure that this point of symmetry is the apogee and not the perigee. Tables are then given for the mean motion in longitude of the epicycle's centre, and also of the planet on its epicycle. The latter is found as the difference between the sun's and Saturn's mean motions.

The 4th and the 5th chapter deal with the motions of Jupiter and Mars. The description is said by Stevin to be similar in kind to that for Saturn, and to differ only as to the quantities. Hence we omit them in this edition. The same holds for Venus and Mercury as treated in the 6th and the 7th chapter.

Here, however, some difficulties arise, because there are no oppositions where the epicyclic movement is eliminated. Both coincidences of the planet with the epicycle's centre are conjunctions with the sun, where the planet cannot be observed. What *can* be observed is the greatest elongations on the evening and the morning side. Hence Stevin derives from Stadius' longitudes of the planet and the sun a table of the dates and the values of greatest elongation. Like Ptolemy, he uses elongations from the mean, not from the actual sun. The differences between these values show that the deferent is an eccentric circle. As to the longitudes of apogee and perigee in this circle, he explains that they may be found by looking for a case where at the same longitude the eastern and the western greatest elongations are equal. But he does not pursue this method any further, because he does not have a sufficient number of such data available. Since the construction of such tables, he says, would take more time than it behoves him to spend on it, and the aim is not to derive the planets' orbits with the utmost exactitude, but to understand matters in a general sense, he will use an easier method. From Stadius' tables he derives the conjunctions of the planets with the sun just as if they could have been observed, and he uses them in the same way as Saturn's oppositions. From one case, taken by way of example only, he finds 82° for the longitude of the apogee of the Venus-deferent (which leaves differences partly above 1°). Then he remarks that a more exact determination should have given 76°20′, because this value had been used by Stadius as the basis of his tables. Exactly the same method is followed for Mercury, where 59°51′ for its apogee is taken from Stadius.

After the planets have been discussed, a short chapter deals with the fixed stars. The constancy of their relative distances and alignments since Ptolemy is stated, and the amount of the precession 1°29′ in a century (*i.e.* 53″ a year) is derived from a comparison of the longitudes of Spica, as determined by Ptolemy and as found in Stadius.

The Second Book, entitled *On the finding of the Motions of the Planets by Means of Mathematical Operations*, extends the previous general knowledge by geometrical computations, resulting in numerical values for the eccentricities and dimensions. It presents the method followed by Ptolemy in computing, from three positions of a planet at known moments of observation, the exact place of the earth within the circular orbit of the planet. There is nothing new or peculiar in Stevin's exposition of the method, so that it was not necessary to include it in this edition. Simpler cases are treated by means of plane trigonometry.

The first application deals with the sun; from three longitudes taken from Stadius, Stevin computes an eccentricity of 0.0325 — which he expresses by 325 parts, 10,000 of which are equal to the radius of the solar circle — and a longitude 95°41′ of the apogee. From other such sets of data he finds the values 326 and 318, 95°9′ and 95°14′. He then explains how from these elements the distance of the sun from the earth is computed, as well as the (negative or positive) correction that must be applied to the longitude of the mean sun to get the sun's true longitude. In translations and older astronomical treatises this correction — our modern *aequatio centri* — is called by the Greek term *prosthaphairesis;* Stevin is the only author of the time to render this term by an exact translation into his own language; *voorofachtring,* literally: advance-or-lag.

These results are applied in the derivation of the equation of time. The inequality of the days (intervals between two consecutive meridian passages of the sun) had not been treated by Ptolemy; the Ancients reckoned with the true solar time. In the 16th century, however, roughly regulated clocks had come into use, and these led to the conception of a "mean time", deviating periodically from the true solar time. Stevin says that he has borrowed the treatment of these concepts from Regiomontanus, who said that he took them from the Arabian Geber: "but they seem not to be Arabian findings, but rather remnants from the Age of the Sages" [15]). Besides the eccentricity of the solar circle producing a yearly periodical inequality in the angular velocity of the sun, there is the obliquity of the ecliptic to the equator causing the equatorial progress to be alternately smaller and larger than the ecliptical progress. He finds from a table by Reinhold that the difference between longitude and right ascension reaches a maximum of 2°28′24″ when the sun is about 46° distant from the equinoxes. He adds a geometrical demonstration that this maximum occurs when the sine of the polar distance is equal to the square root of the cosine of the obliquity, *i.e.* when the ecliptical and the "equatorial" longitude (*i.e.* the right ascension) complement one another to 90°. The results of these computations are used to derive the deviation of the natural from the mean days, for the eccentricity on the one hand and for the obliquity on the other.

Next the distance, the diameter, and the parallax of the sun are dealt with, especially with a view to their subsequent use for the eclipses. For the finding of the parallax he assumes two observers, one at a more northern, one at a more southern latitude, "such as now could easily be done through the great Dutch navigations", if they both measure every day the solar altitude. When afterwards they compare their measurements made on the same days at known latitudes under the same meridian, the parallax can be found and the distance deduced. Though not workable at the time, the principle of later determinations of the parallax is clearly indicated. As a fictitious instance he supposes an observed parallax of 2′, and derives the sun's corresponding distance to be 1,147 times and its radius 5 times the semi-diameter of the earth.

The 3rd chapter deals with the moon. In the same way as with the sun, the eccentricity and the longitude of the apogee are computed from three observations (*i.e.* from data of Stadius), taking account of the rapid motion of the apogee. Parallax and distance are also deduced. The latitudes of the moon and the motion of the nodes afford the basis for a computation of the eclipses.

In the 4th chapter Saturn is first dealt with in the same way. With this difference, however, that Stevin does not here derive eccentricity and apogee, as Ptolemy was obliged to do, from three oppositions, but takes the latter from the First Book, the derivation of the eccentricity (0.1170) thus being much simpler. The radius of the epicycle is found to be 1,150 when the radius of the deferent is 10,000. He also gives here the demonstration of Apollonius of the condition for retrogradation of a planet. The other planets are treated shortly, since the demonstrations are the same as for Saturn.

The last chapters of this Book deal with the planets' conjunctions and oppositions, *i.e.* chiefly with the eclipses of sun and moon. Since there is nothing of peculiar character in these chapters, the Second Book has been omitted from this edition.

Exception had to be made for the two "Remarks" at the close of this Book —

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[15]) See Vol. I, p. 7 and 46 and the selection at the end of Vol. III.

which for this reason have been included — because they show Stevin's attitude towards the problems of the heavenly motions. First he remarks that in the last chapters only the conjunctions of sun and moon have been treated, because the motions of the planets are not sufficiently well known for an exact computation of their conjunctions. Then he says that he originally intended to add a sixth chapter on the unknown irregularities detected by Ptolemy and by Copernicus, and a seventh chapter dealing with the motions in latitude. Because of the uncertainty of their explanation he had postponed them till after the Third Book, as being problems that still had to be cleared up. After some time, however, they became so much clearer to him, on the assumption of a moving earth, that he decided to treat them more extensively in a supplement and an appendix to the Third Book.

The Third Book explains the motions of the planets on the assumption of a moving earth. Stevin assumes that this true motion of the celestial bodies had been known in the "Age of the Sages", but that this knowledge was afterwards lost to men, so that Ptolemy did not know of it. Until at last Copernicus had again revealed this system, or a similar one. Stevin's arguments are primarily based on the simplicity and naturalness of the Copernican system: the velocity of the revolutions increases regularly as their size decreases, so that the starry heavens are at rest and the rotation of the small terrestrial globe is the most rapid. The argument is corroborated by the belief that all revolving motions in nature take place in the same direction, from West to East. The prejudice that the heavy earth cannot appear as a luminous star is dispatched by simply assuming that the earth is a heavenly body. Whilst in disproving Ptolemy's fear that buildings will be demolished by the velocity of the motion and the resistance of the air, Copernicus, as a philosophical thinker, stresses the contrast between natural and forcible motions, Stevin, being a practical engineer, refers to everyday experience, such as with a stick in rapidly flowing water.

In the Summary and in the first sections dealing with the general theory of the planets we meet repeatedly with the expression the "heavens" of the planets (the literal translation of Stevin's *hemelen)*. This term implies a structure of the planetary system entirely different from that according to our modern ideas. On the outside we have the highest "heaven", that of the fixed stars, which is the immobile sphere assumed by Copernicus. Arranged inside this are the "heavens" of the planets, which are evidently understood as analogous spheres carrying along in their axial rotation the planets themselves. In the same paragraph the planets are said to move in eccentric circles; this is how they appear in the drawing on page 120, which in the wording of Proposition 1 is called the arrangement of the heavens of the planets. The two expressions used indiscriminately in the subsequent sections are sometimes given side by side, as alternatives, *e.g.* the planets revolving "in the largest circles or heavens . . . ." (page 125).

The belief that the planets are attached to spheres and are carried along in their orbits by a rotation of these spheres was common among Arabian astronomers in the late Middle Ages. In a sense it was opposed to the epicycle-theory. A system of concentric spheres, detached from one another, could only be constructed on the basis of single circular orbits, without epicycles. By removing the epicycles, Copernicus opened the way for this ambiguous concept, and we find it clearly described in his works. What Stevin refers to as the "heaven" of a planet, by Copernicus was called its *orbis* [16]).

The system of the world described in the works of Copernicus thus is not identical with our modern heliocentric theory of planets running their course freely through empty space. Their *orbis* was sphere and circle at the same time. In the first of the seven theses of the *Commentariolus,* in the words *"omnium orbium coelestium sive sphaerarum",* the term is explicitly identified with sphere. The seven theses are followed by an exposition of the order of the orbs: *De ordine orbium,* from the immobile *orbis* of the fixed stars down *via* the planets, from Saturn to Mercury. In his great work *De Revolutionibus* this enumeration is repeated (in Book I, *caput X,* just before the figure): *"ordo sphaerarum sequitur in hunc modum".* In this description he says that between the convex *orbis* of Venus and the concave *orbis* of Mars there is space to take up *"orbem quoque sive sphaeram"* for the earth. In his dedication to Pope Paul III he refers to his work as the Books he wrote *"de revolutionibus sphaerarum mundi".* Accordingly, there can be no doubt that the term *orbis* is regularly used by Copernicus to indicate a sphere. At the same time there are numerous instances where it is used for a circular orbit. In the *Commentariolus,* immediately after the seven theses, he says that the magnitudes of the semi-diameters of the *orbes* will be given in the explanation of "the circles themselves". In the same treatise he speaks of the intersections *circulorum orbis et eclipticae,* called the nodes. In his great work, when describing how some authors added more spheres (up to an eleventh) beyond the outer starry firmament, he says that this number of circles *(quem circulorum numerum)* will be shown to be superfluous. Sphere and Circle therefore are both used as synonymous with *orbis* [17]). There is some vagueness about the substance or the substantiality of these spheres called *orbes.* Frisch on this subject observes [18]): "Copernicus nowhere in his work either explicitly asserts or implicitly denies the reality of the spheres".

There can be no doubt that Stevin's "heaven" of a planet is intended to render in the vernacular what Copernicus denoted by *orbis.* There is, however, a difference: the difference between the theoretical philosopher and the practical engineer. What for Copernicus was an ambiguous geometrical concept, to Stevin is a structure of physical objects and materials. In order to emphasize the spatial character of the heaven he sometimes denotes it by *hemelbol,* "celestial sphere". Being natural objects and substances, they must be acting on one another. The structure is a dynamical system. This opinion is not presented as a systematically worked-out theory, complete with proofs and arguments. With Stevin it is rather a picture spontaneously arisen in the background of his mind, a vague feeling appearing in some arguments. The basic idea, *viz.* that bodies contained in another body are bound to share in the movement of the latter, may appear obvious enough. In the world-system it means that outer spheres by their motion

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[16]) *Orbis* is not identical with the modern concept of orbit; its English equivalent is *orb, e.g.* in the title of O. Mitchell's popular work *The Orbs of Heaven* (London 1853).

[17]) Edward Rosen (in the Introduction to the booklet *Three Copernican Treatises*) summarizes the discussion of all these cases as follows: "When he deals with the planetary theory, he uses *orbis* to mean the great circle in the case of the earth and the deferent in the case of the other planets" (p. 21). "But when he is speaking more generally about the structure of the universe or the principles, *orbis* regularly means sphere" (p. 19).

[18]) *Johannis Kepleri Opera* III, p. 464.

influence the lower spheres contained within them; the primary force thus derives from the highest sphere, the outer firmament.

At first sight the character of these forces exerted by the higher upon the lower spheres looks very peculiar. The heaven of Mars, rotating in two years, would — Stevin says — in the absence of other forces impose the same two years' period of revolution upon the earth's aphelion. He asserts this without providing any argument or proof. No proof *can* be given, because observation contradicts such a rapid motion of the aphelion. Where the motion of a planetary aphelion could be determined, it was less than one degree in several centuries. Evidently, strong other forces must be at work, which prevent any considerable motion of the aphelia. Stevin's statements about these motions consist entirely in theoretical ideas, which are not clearly formulated or systematically developed. This shows how he is struggling with the problem of understanding causes at a time when there were present as yet only the first traces of a causal natural science. His exposition impresses the modern reader as an entirely artificial and fantastic mechanism: a planet like Mars, attached to a sphere carrying it around in two years, at the same time carries the aphelion of the sphere of the earth along with it in the same period of two years, though this sphere itself revolves in one year. He himself sometimes refers to it as a contradiction, saying (p. 133) that Jupiter's heaven performs a revolution in 30 years, which it receives from Saturn, but at the same time in reality rotates in 12 years about an axis of constant direction; the enforced period relates to the aphelion that protrudes outwards, and it is this bulge which in Stevin's theory is drawn along in a rotation in which the sphere itself does not share. When we call this an attempt to understand something of the mechanism of the world, it should be borne in mind that the physical character of these "heavens" plays no part therein and remains indefinite; it is the geometrical forms — in this case the eccentric circles — which determine the motions. Probably Stevin took this idea from his knowledge of the tides, where, in the abstract simplified case of the absence of continents, a wave crest is drawn along by the moon to pass round the earth in the moon's period of 27 days, whilst the waters of the ocean themselves revolve as one body in 23h 56m.

In the first Proposition of the Third Book Stevin effects a significant improvement in the Copernican theory. Copernicus had assigned three motions to the earth. Besides the axial rotation and the orbital revolution there was a third — annual — motion. He was governed by the classical Greek idea of the revolving body being carried around by its fixed connection with the radius vector, so that the direction in space of the earth's inclined axis of rotation would describe a cone. To explain the constant direction of this axis in space, he was obliged to compensate that motion by another conical motion of the axis in the opposite direction, completed in one year.

Stevin is aware that this is an unnecessary complication. He does not believe that two independent motions in nature can compensate one another so exactly. Copernicus was less rigorous in this: he took the two movements to be independent; their small difference explained the precession. Stevin is of the opinion that their combined result should rather be considered as a single primary phenomenon. He looks upon the constant direction of the axis in space as a fundamental property. In this respect he was guided by the researches of William Gilbert on magnetism, published in 1600, shortly before his own work. Just as the magnet in the ship's compass continues to point in the same direction in space, notwithstanding the changing course of the ship, the axis of the earth continues to point in the same direction in space during its annual revolution. Stevin therefore calls this property of the earth *"haer seylsteenighe stilstandt"* (literally: "its loadstony standstill"), which has here been translated by "its magnetic rest". This is not a mere analogy; he quotes Gilbert's opinion that the earth itself is a huge magnet [19]).

Stevin widens the scope of this idea by applying it to the orbits themselves. From his ideas on the action of the "heavens" of the higher planets on the orbits of the lower planets he had deduced that the aphelia would show rapid rotations. In reality, however, they exhibited only minute, scarcely perceptible displacements. He solved this contradiction by imparting a magnetic character to the orbits. The directions of the planetary aphelia may also be said to be subject to a magnetic constancy. This subjection is the force referred to above as keeping the aphelia at rest. Whence does this force proceed? Arguing by comparison with magnets in closed boxes, Stevin derives that the origin of these forces is situated outside the spheres of the planets, in the sphere of the fixed stars. The matter is different for the moon; its apogee has a rapid daily motion of 6′41″ (with a period of nine years), and the origin of the forces is to be sought in the regions of the nearer planets.

It is not only the orbits but the entire spheres constituting the "heavens" of the planets which are subject to a magnetic force. It causes the poles of their axes of rotation (hence also the orbital planes) to keep a constant direction in space. It is the cause that, in spite of the considerable deviation of Mars from the ecliptic, the ecliptic itself (the plane of the earth's motion) keeps its constant position. The constancy of the orbital planes, which the later science of theoretical mechanics styles conservation of moment of momentum, is explained by Stevin as magnetic stability. In his 3rd proposition he speaks of the doubts he had felt with regard to the real cause of the planetary motions; his initial conviction that the motion was transferred from the outer spheres to the inner spheres was disproved by the practical tests. Sometimes he had wondered whether the planets did not run their course freely through empty space "like birds flying around a tower", until finally the principle of magnetic rest suggested itself as the simplest solution of the problem.

Considerable space is devoted by Stevin to the transition from the old to the new system. With regard to the moon the exposition of its course has become more difficult; instead of simply describing a circle about a fixed centre, it now has to revolve about a body which itself revolves in a yearly period. Stevin assumes (in accordance with the Greek epicycle-theory) that in such a case as this the radius vector of the earth to the centre of its circle is the natural zero line for the position of adjacent bodies. Relative to this radius, which changes

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[19]) It is to be noted that the constant direction of the axis in space had already been mentioned by Copernicus and had even been compared to a magnet in his *Commentariolus*. The *Commentariolus* was not printed in the 16th century, but circulated in a few hand-written copies. It is not probable that Stevin had seen one of these; Gilbert's work is quite sufficient as the source of his theory that the earth itself is a magnet.

its direction in space by progressing 360° a year, *i.e.* 59′8″ daily, the small advance of the moon's apogee of 6′41″ daily (360° in 9 years) is a retrogression of 52′27″ daily. In the same way the retrogradation (in the old system) of the nodes of the moon's orbit by 3′11″ daily (360° in 18 years), relative to the revolving terrestrial radius vector, is a retrogression of 1°2′19″. Formally this contradicts Stevin's view that all motions in the universe have the direction from West to East; but no undue weight is to be attached to this, since apogee and node are no bodies, and may depend on exterior forces in some other way. In his short discussion of these points on page 135 Stevin cannot be said to have succeeded in harmonizing them.

The ensuing method of computing the moon's longitude — by first finding the sun's longitude and then subtracting the lag of the apogee — is demonstrated on page 161 to be right. He remarks that the computation according to the old system (*i.e.* using absolute positions in space) is more direct and rapid.

The orbit of the earth in the new system is exactly identical with what used to be the sun's orbit in the old system, with the same period and the same relative positions. A special figure is given on page 148 to show how the two systems result in the same observed longitudes and the same distances for the sun and the earth. The former longitude of the sun's apogee is identical with the new longitude of the earth's perihelion. The new data of the moon's motion are also shown to correspond directly to the old ones. The numerical data formerly derived for the planets and now transferred to the new system are summarized in a list, in which the earth ranks third in the series of the planets.

It is significant evidence of his practical-mindedness as well as of his pupil's that Stevin did not content himself with presenting the general argument that of course all the relative positions and motions of the planets are the same in the two world-systems. They want to see it in the details of each individual case. For this purpose Stevin gives for each of the planets Mars and Venus (as instances of an outer and an inner planet respectively) a drawing in which the connecting lines and the circles for the two systems are combined. The desire to see their relations illustrated in a geometrical drawing, however, was not the only motive. The interchange of the sun and the earth as central bodies presented certain difficulties. Copernicus neither mentioned nor solved these. In his 18th proposition Stevin states that Copernicus evidently supposed the matter to be so clear that no further proof was needed; but that he himself had met with difficulties, which called for a more detailed investigation.

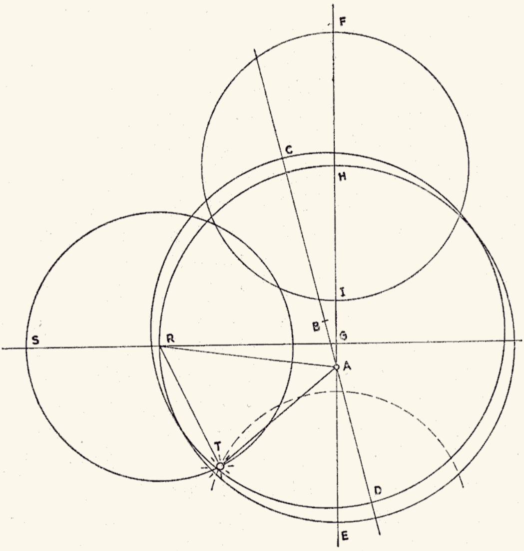
If we suppose that we shall arrive at the new system by simply interchanging in the old system the positions of the sun and the earth and identifying the old deferent with the new planetary orbit, we are mistaken in the same way as when we sometimes say for the outer planets that the motion on the epicycle reflects the sun's motion about the earth. The perfectly uniform course of the planet on its epicycle, however, is not identical with the sun's course (in the other case) about the earth (which is not uniform), but with the sun's uniform motion about the centre of its circle. Hence, the deferent does not correspond to the planet's orbit about the sun, but to its orbit around the centre of the earth's annual circle. When we pass from the old to the new system, the earth at rest is to be replaced by the centre of its orbit, and not by the sun.

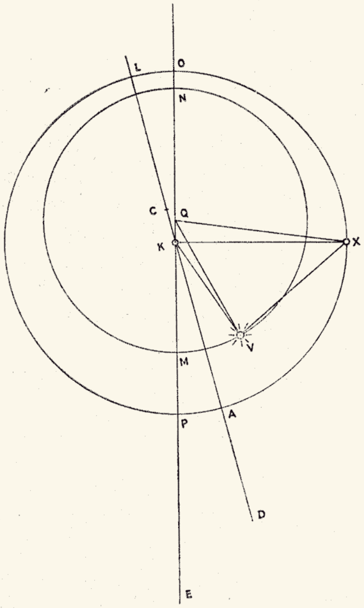
On page 182 Stevin presents the combined drawing for Mars, which is discussed in Proposition 15. Since the reader might easily be thrown into confusion by the tangle of all these distances and circles, he separates the lines and letters belonging to either system into two drawings on page 192. The positions of the sun and the earth are *C* and *A;* in the old system *A* is fixed and the sun moves on a circle through *C* (centre *B*); in the new system *C* is fixed and the earth moves on a circle through *A* (centre *K*). The fundamental law of the epicycle-theory states that the radius vector of the planet on its epicycle is always parallel to the radius vector of the sun on its circle, so that the arc described by the sun is the sum total of the arcs described by the planet on its epicycle and by the epicycle's centre on the deferent. In the old system Mars' deferent is *HRE* (centre *G*), in the new system Mars' orbit is *NVM* (centre *Q*). At conjunction the epicycle's centre was at *H,* Mars at *F;* in the new system Mars was at *N,* the earth at *P.* At a later time the centre had progressed from *H* to *R,* Mars from *F* to *T;* in the new system Mars had passed from *N* to *V,* the earth from *P* to *X* (arc *PX* = arc *HR* + arc *ST*). Owing to the equality of circles and arcs, *RT* and *KX* as well as *AR* and *KV* are seen to be equal and parallel. Then from the equality of triangles *ART* and *KVX, AT* and *XV* are demonstrated to be equal and parallel. This means that Mars is seen from the earth in the same direction and at the same distance according to both systems.

The same demonstration is then given for Venus as an instance of an inner planet. Because the planet's deferent is equal in size to the sun's orbit, with the centres at a small distance, it is more difficult in this case than in that of Mars to disentangle the combined drawing for Venus (on page 196). A separation of the combined drawing into its two components is here all the more necessary; we therefore give these two drawings on page 17.

The positions of the earth and the sun are denoted by *A* and *C;* in the old (geocentric) system, *A* is fixed and the sun moves on a circle (centre *B*) through *C;* in the new (heliocentric) system, *C* is fixed and the earth moves on a circle (centre *K*) through *A.* The eccentricity of the sun's orbit is *AB* = *CK,* that of Venus' orbit is *AG* = *KQ.* For time zero we take an upper conjunction at the apogee; the centre of the epicycle is at *H,* Venus itself at *F;* in the new system Venus is at *N,* the earth at *P.* We have to show that at any other time the lines joining the planet to the observer in the old and the new system are equal and parallel. As to their length, we observe that the orbit of the sun and the deferent of the planet (in the old system) are equal in size to the orbit of the earth (in the new system); the planet's epicycle also is equal to the planet's orbit in the new system. This entails the equality of all semi-diameters drawn from any point of such a circle to its centre; *GR* = *GH* = *BC* = *KP* = *KX,* and *HF* = *RT* = *QN* = *QV.* As to the directions of these lines, we observe that they rotate entirely uniformly. The planet (in the old system) has two motions, one along the epicycle (*e.g.* the arc *ST*) and another with the epicycle along the deferent (*e.g.* the arc *HR*). These two motions combined in the heliocentric system convey the planet by its uniform rotation from *N* to *V,* so that *QV* is equal and parallel to *RT.*

Positions eccentric to these circles (*e.g. A*) do not fall under these headings, so that here an additional computation is needed. For this purpose we compare the acute-angled triangles *AGR* and *QKX. AG* and *QK* (the eccentricity of the planet's orbit) as well as *GR* and *KX* are equal and parallel; so the triangles are equal and similar, and consequently their third sides *AR* and *QX* are equal and parallel. Since the triangles *ART* and *XQV* now have two pairs of sides equal and parallel,





Motion of the planet Venus
according to the geocentric system (above)
and to the heliocentric system (below).

this holds also for the third pair *AT* and *XV*. This means that the planet is seen from *A* (the earth in the old system) and from *X* (the earth in the new system) in the same direction and at the same distance.

For the understanding and the derivation of the motions in longitude Stevin considers the old, untrue system with the earth at rest to be the simplest and most appropriate — probably because it directly represents the observed motion. It is different, however, with the latitudes of the planets. For this reason he omitted the latter from his Second Book, and postponed them until, at the end of the Third Book, he should have treated them according to the true system of the moving earth; for, so he says, in this way we can arrive better at a causal knowledge of the motion in latitude. This reverse way of arguing, *viz.* the derivation of the older imperfect theory from the new, more perfect theory, is an indication of the real underlying character of the problem. Whereas for the three upper planets the epicycle theory was the direct expression of the observed phenomena of the longitudes, this was not the case with the latitudes. Here Ptolemy's theory was an artificial construction; it was complicated because two independent inclinations had to be derived, *viz.* one of the epicycle to the deferent, and one of the deferent to the ecliptic. The new heliocentric system required one angle only, the inclination of the planet's orbit to the ecliptic. Copernicus, assuming that Ptolemy's theory was a good representation of the observed motions in latitude, had to make the inclination variable by assigning an oscillation (between opposition and conjunction) to the orbit. With the epicycle itself, Stevin discarded also its special inclination and stated as the basic structure for the geocentric system: the epicycle in its course along the deferent always has to keep parallel to the plane of the ecliptic.

In dealing first with Saturn, Stevin starts from the values in Ptolemy's tables, which he assumes to represent Ptolemy's observations. In these tables the maximum northern latitude (at a longitude 50° behind the apogee, *i.e.* at longitude 183°) is 3°2′, the maximum southern latitude is 3°5′, the planet in both cases being at the lowest point of its epicycle. Ptolemy had derived 2°26′ for the deferent's inclination, 4°30′ for the epicycle's inclination. From the latitude 3°2′ and the known distances Stevin finds 2°43′ for the inclination of Saturn's orbit to the ecliptic. He shows how easy the computation of Saturn's latitudes is now, since they follow directly from the horizontal distances of Saturn from the earth and the vertical distances of Saturn above or below the ecliptic. Ptolemy's observation that near the nodes Saturn does not show any latitude, which is accidental in his theory, is a necessity in the new theory, because it shows the epicycle at that time to coincide with the ecliptic. In explaining this state of affairs, Stevin cannot refrain from remarking that it forms a strong argument in favour of the moving earth.

The other planets for which the same holds are mentioned in brief statements only, in which their numerical data are given. More space is devoted to Mercury. Stevin begins by expounding Ptolemy's theory of the latitude of Mercury, as an instance of the two inner planets, in the form given by Peurbach and also used by Copernicus. It assumes three oscillations. An oscillation of the deferent about the line of nodes as axis makes its inclination vary between zero and 1°45′ to the South. The epicycle has two oscillations about two perpendicular axes, one axis tangential to the deferent's circumference, the other in the radial direction; when the former is zero, the latter reaches its maxima in opposite senses in apogee and perigee; and conversely the former reaches its maxima in opposite senses at longitudes 90° and 270°, when the latter is zero. Stevin mentions *in margine* their Latin names *deviatio, declinatio, reflexio,* and himself calls them *afweging, afwycking,* and *afkeering;* they have here been translated by "deviation", "declination" and "deflection". Stevin has no use for this complicated theory. The greater simplicity — he says — of the heliocentric system was not realized by Copernicus himself, who slavishly copied Ptolemy's three oscillations, with the numerical values given in the Tables, and incorporated them in his fundamentally different world-system. Stevin draws the former epicycle (representing the planet's orbit) as a small circle at rest about the centre of the larger surrounding circle (the earth's orbit, formerly the deferent) and then has to determine its inclination to the latter. From the "observed" latitudes (in reality, as always in his explanations, taken from Ptolemy's tables) of Mercury at two opposite points at 90° and 270° from the apogee, 1°45′ North and 4°5′ South, he derives (as illustrated in the lower figure on page 248) an inclination 5°32′.

In this short paragraph of Stevin's work the simple construction of the heliocentric system is used as the new basis of computation. Of course it was done in one instance only; much more was not yet possible at the time. His was not the task which Kepler was to accomplish afterwards. He was only interested in explaining the theory of the heavenly motions, not in constructing numerical tables for their computation.

As a Supplement *(Byvough)* Stevin gives what should have been the last chapter of the Second Book, with the treatment of the latitudes in the old geocentric system. He introduces it by presenting this system in a form different from the original one. Originally the two lowest planets, Mercury and Venus, in contrast with the three upper planets have deferents that are completed in a year and thus represent the earth's orbit, whilst the smaller epicycles here represent the planets' orbits. Instead of the size of the circles, Stevin now takes their function in the system to be their specific character. In all cases the circles representing the earth's orbit are to be called epicycles; deferents is to be the name for the planets' circles. Accordingly, for Venus and Mercury the old terms have to be interchanged. He says it in the following way: the circles which are called deferents here are epicycles, and conversely; and he uses these names in the following propositions. Instead of two kinds of planets with different characters, we now have one homogeneous series, with only the size of their orbits regularly decreasing from Saturn to Mercury, the earth finding its place among them. In an illustrative drawing on page 260 Stevin pictures the planetary system in which all the orbits have been provided with terrestrial circles of equal size and all parallel to the ecliptic.

Stevin's task was to show that the phenomena of latitude, too, are the same in the two systems. This was not difficult, since his corrected geocentric system, with the epicycles parallel to the ecliptic, was a formal transformation of the true system. He compares this geocentric system with Ptolemy's and finds that for the upper planets Ptolemy came very near to the truth, since he found the two inclinations (for Saturn 2°26′ and 4°30′) to be nearly equal, the difference being only 2°4′. His criticism (pages 277 and 279) that Ptolemy's tables are not in conformity with his theory is unfounded (*cf.* p. 279, note 3). His statement that for the two lower planets Ptolemy's theory was not successful is true; the basic reason is that the epicycle theory did not fit Mercury.

Stevin knows that his exposition of the motions of the planets is neither exact nor complete. An *Appendix* is therefore added on the "unknown motions" observed by Ptolemy, *i.e.* on motions which formerly were unknown and had not been included in the theory as given in Stevin's three Books. They are Ptolemy's "second inequality" of the moon, and his introduction of a *"punctum aequans"* in the orbits of the planets. Since, in his treatment of the former, Stevin (in his Propositions 2 to 5) renders Ptolemy's discussion rather exactly, it was not necessary to reproduce it here in detail. The other point is dealt with by Stevin in the 6th Proposition of the *Appendix*.

Ptolemy had found that the simple theory of the epicycle's centre uniformly describing an eccentric circle about the earth does not tally with the observed motions of the planets. The motion along the deferent is not uniform, but it seems to be uniform when viewed from the equant (*punctum aequans*) situated at the same distance as the earth from the centre of the deferent, but on the opposite side. This irregularity of the motion along the deferent is the main point in Ptolemy's planetary theory. The radius vector drawn from the equant to the epicycle's centre, though of variable length, rotates perfectly uniformly. Since the planet moves uniformly along the epicycle, the anomaly, reckoned from the apogee of the epicycle as its zero point, also increases uniformly when viewed from the same point. Stevin directs the reader's attention to this point. He first remarks that Ptolemy must have found the simple theory entirely satisfactory as long as he considered only the oppositions to and conjunctions with the sun, *i.e.* the planet in the nearest and the farthest point of the epicycle, where it has the same longitude as the centre of the epicycle. But in the other points it is different. He takes an example where shortly before opposition (anomaly 150°) the planet is observed to have a smaller longitude, hence has progressed farther on its epicycle than had been computed. With the aid of a figure he shows that this can be accounted for if the zero from which the anomaly in the epicycle is reckoned is shifted in a forward direction. Then, in order to see it coincide with the epicycle's centre, Ptolemy had to look not from the centre of the deferent, but from another point, situated nearer to the apogee of the deferent. The precise situation of this point, when computed, would have turned out to be exactly the *punctum aequans*. In this deduction, instead of the most striking property, *viz.* the non-uniform motion of the epicycle along the deferent, the less striking, non-uniform motion of the planet along the epicycle is used to introduce the *punctum aequans*.

This is Stevin's explanation of Ptolemy's theory for the planets Saturn, Jupiter, Mars, and Venus. For the more complicated motion of Mercury he simply reproduces Ptolemy's description of the circles and their motions.

Thereafter Stevin explains how the same "unknown motions", first of the moon (in Proposition 8), then of the planets (in Proposition 9), are dealt with by Copernicus. For the planets Stevin first introduces the "unknown motions" into the geocentric theory. The distance of the earth from the centre of the deferent is diminished by one-fourth of its value; *e.g.* for Saturn, instead of 0.1139, it is taken 0.0854. The one-fourth subtracted (here 0.0285) is taken as the radius of a small circle, which is described by the planet in such a way that in apogee and perigee the effects are subtracted, whereas in a lateral position they are added. Copernicus of course presents this construction on a heliocentric basis; the deferent is now the planet's orbit, with the sun at a distance from the centre of 0.0854 times the orbit's radius; the planet, in addition, describes the small circle (of radius 0.0285) twice in one revolution. Stevin supposes that Copernicus, though he did not say so, had devised the small circle first in the geocentric, and then transferred it to the heliocentric theory. In any case he himself has chosen this method in order to make the matter clearer to his readers.

In the case of Venus the centre of its circle, of radius 0.7194, situated within the circle of the earth with radius 1.0000, does not have a fixed eccentric position, but moves on a small circle with diameter 0.0208 in such a way that its eccentricity varies between once and double this amount. It is described by the centre of Venus' orbit twice a year in a direct sense; the eccentricity is at its minimum when the earth is in the planet's line of apsides, and at its maximum when it is at a distance of 90°. Stevin, as in all these cases, does not make any comparison with observations; his task is solely to expound the theories of Ptolemy and Copernicus; and he simply adds: "by this means, Copernicus says, the longitudes of Venus are always found in the right way".

The still more complicated system of circles for Mercury as devised by Copernicus is expounded correctly by Stevin in his 12th Proposition. The centre of Mercury's orbit describes a small circle with radius 0.0316 every half year in such a way that the eccentricity is greatest when the earth is in Mercury's line of apsides, and smallest when its longitude is 90° different. In addition, the planet moves to and fro linearly along the radius of its orbit; such a linear movement, as Stevin demonstrates in his 11th Proposition, is produced by two circular movements in opposite directions.

The Book closes with an article on the "unknown motion" of the stars, dealing with the precession of the equinoxes. The difference between Ptolemy's value (1° in a century) and the larger values of later authors is thought by Stevin to be perhaps due to wrong equinoxes caused by irregularities and differences of the refraction. He points to the abnormal phenomenon of the sun observed by the Dutch mariners in Novaya Zemlya in 1596-97, which he also ascribes to a refraction, abnormally large in February, caused by the cold nebulous atmosphere. New measurements of stellar altitudes and refractions in the countries of ancient astronomy as well as at its present centres, he says, will be needed to remove these uncertainties. Moreover, for greater precision in measuring the positions of the planets and the stars better instruments are necessary, such as those constructed and used by Tycho Brahe. The present deviations between observations and theory show that our theory is unsatisfactory and must be improved by means of the best observations available.

Little did Stevin suspect that at the very time when he wrote these words, mapping out the programme in a vague and general way, Kepler was already engaged in establishing the true theory of the planetary motions.

DERDE
DEEL DES
WEERELTSCHRIFTS
VANDEN
HEMELLOOP.

CORTBEGRYP
des Hemelloops.

ICK ſal int begin der beſchrijving deſes Hemelloops de ſaeck nemen al ofter gantſch niet af bekent en vvare, en daer na den handel met ſulcken oirden vervolghen, gelijck daerſe haer vermeerdering dadelick me ſchijnt ghenomen te hebben, daer af beſchrijvende drie boucken.

Het eerſte bouck vande vinding der dvvaelderloopen en der vaſte ſterren deur ervarings dachtafels met ſtelling eens vaſten Eertcloots.

Het tvvede vande vinding der dvvaelderloopen deur vviſconſtighe vverckingen met ſtelling eens vaſten Eertcloots, en eerſte oneventheden.

Het derde vande tvveede oneventhedẽ, vvaer in comt Copernicus ſtelling eens roerenden Eertcloots.

SUMMARY

OF THE HEAVENLY MOTIONS

In the beginning of the description of the Heavenly Motions I will assume the matter to be altogether unknown, and then I will proceed with the discussion in the same order in which it seems to have taken its progress in actual fact, describing it in three books.

The first book, of the finding of the motions of the Planets and of the fixed stars by means of empirical ephemerides, on the assumption of a fixed Earth.

The second, of the finding of the motions of the Planets by means of mathematical operations, on the assumption of a fixed Earth, and the first inequalities.

The third, of the second inequalities, in which Copernicus' assumption of a moving Earth is set forth.

EERSTE BOVCK DES HEMELLOOPS VANDE VINDING DER

*.DWAELDERLOOPEN en der vaster sterren deur ervarings dachtafels, met stelling eens vasten Eertcloots. *Motuum Planetarum*

FIRST BOOK

OF THE HEAVENLY MOTIONS

OF THE FINDING OF THE PLANETS' MOTIONS AND THE MOTION OF THE FIXED STARS

by means of Empirical Ephemerides,
on the Assumption of a Fixed Earth

CORTBEGRYP

deſes eerſten Boucx.

DE bepalinghen beſchreven ſijnde ſoo ſal dit eerſte bouck acht onderſcheytſels hebben, vande vinding deur ervarings dachtafels des loops van Son, Maen, Saturnus, Iupiter, Mars, Venus, Mercurius, en der vaſte ſterren, alles met ſtelling eens vaſten Eertcloots als vveerelts middelpunt, vvant hoevvelſe eyghentlick in een rondt Elementa. *draeyt ghelijck d'ander Dvvaelders, nochtans leertmen de * beghinſelen deſer conſt lichtelicker verſtaen deur het ſchijnbaer, dan deur het eyghen, ſoo daer af breeder gheſeyt ſal vvorden in des 3 boucx 7 voorſtel.* Speculationes. *Angaende voorder* ſpieghelinghen vvaer toe de eyghen ſtelling des loopenden Eertcloots bequamer is, daer af ſal ick int boveſchreven derde bouck handelen.*

SUMMARY OF THIS FIRST BOOK

After the definitions have been described, this first book is to comprise eight sections, of the finding by means of empirical ephemerides of the motion of Sun, Moon, Saturn, Jupiter, Mars, Venus, Mercury, and of the fixed stars, all this on the assumption of a fixed Earth as centre of the universe; for though in reality it revolves in a circle, like the other Planets, nevertheless it is easier to understand the elements of this science from the apparent than from the true motion, as will be set forth more in detail in the 7th proposition of the 3rd book. As to further theories, for which the true assumption of the moving Earth is better suited, I will deal with those in the third book referred to above.

BEPALINGHEN.

ANghesien in des driehouckhandels 4 bouck, beschreven sijn de bepalinghen der Hemelsche ronden boghen en punten die wy daer behoufden, soo houden wy de selve hier voor bekent: Sulcx dat nu ghestelt sullen worden de resterende bepalinghen int volghende noodich.

1 BEPALING.

Den tijt van dat de Sonnens middelpunt uyt het middachront gaet, tot dattet vveder daer in comt, is een *natuerlicken dach. En die natuerlicke dagen vvorden ooc oneven ghesey t. En sulcke tijt int ghemeen *oneven tijt.

Dies naturalis. *Dies inæquales.* *Tempus inæquale.*

Dese tijt eens natuerlicken dachs is van een keer des *evenaers, met noch sulcken boochsken des selfden, alsser deurlijdt te wijle de Son met haer eygen loop daerentusschen te rugh ghegaen is. Nu by aldien al die boochskens evegroot waren, soo souden de natuerlicke daghen al evelanck sijn, t'welck niet en ghebeurt. De reden van dier boochskens onevenheyt is tweederhande: Ten eersten deur dien de Sonwechboochskens op dien tijt vande Son beschreven niet evegroot en sijn, om de *uytmiddelpuntícheyts wille. Ten anderen al warense evegroot, nochtans soo en gaense met gheen even boochskens des evenaers deur *t'middachront, om de scheef heyt of afwijcking des *Duysteraers. En hoe wel dit verschil op een dach ongevoelelick is, nochtans op veel daghen t'samen cant merckelick sijn.

Aequatoris. *Excentricitatem.* *Meridianū.* *Zodiaci.*

2 BEPALING.

*Natuerlick jaer is dẽ tijt der Sonnens omloop, vviens begin en einde een ghenomen punt is dat altijts evevvijt vande lentsne blijft, gheduerende 365 daghen 5 uyren, met noch een onseker ghedeelte.

Annus naturalis.

Ptolemeus heeft dat onseker ghedeelte boven de 5 uyren, ghestelt op 55 ① 12 ②: *Albategni* op 46 ① 24 ②. Ander hebben ander uytcomst bevonden.

3 BEPALING.

Egips jaer is dat 365 daghen begrijpt.

Dese Egipsche jaren verleken by de natuerlicke verloopen alle vier jaren bycans een dach.

DEFINITIONS

Since in the 4th book of the treatise on trigonometry [1]) the definitions of the heavenly circles, arcs, and points which we there needed have been described, we here assume them to be known, so that the remaining definitions needed hereafter will now be given.

1st DEFINITION.

The time from the moment when the centre of the Sun leaves the meridian up to the moment when it returns thereto is a natural day. And these natural days are also said to be unequal. And such time in general: unequal time.

This time of a natural day is of one revolution of the equator, together with an arc thereof such as it passes through while the Sun with its proper motion has meanwhile gone back. If all these arcs were equal, all the natural days would have the same length, which does not happen. The reason of the inequality of these arcs is twofold. Firstly, that the arcs of the Sun's orbit described in that time by the Sun are not equal, because of eccentricity. Secondly, even if they were equal, yet they would not pass with equal arcs of the equator through the meridian, because of the obliquity or deviation of the Ecliptic. And although this difference is insensible in one day, yet it may be perceptible in a large number of days taken together.

2nd DEFINITION.

A natural year is the time of the Sun's revolution, whose beginning and end is an adopted point which always remains at the same distance from the vernal equinox, this being 365 days and 5 hours, with an uncertain fraction in addition.

Ptolemy [2]) has put this uncertain fraction over and above the 5 hours at 55m 12s; *Albategni* [3]) at 46m 24s. Others have found other results.

3rd DEFINITION.

An Egyptian year is one that comprises 365 days.

These Egyptian years, when compared with the natural years, shift nearly one day in every four years.

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[1]) Work XI; i, 14. See Vol. II B, p. 755.

[2]) *Syntaxis* III, 1 (Translation of Manitius, I, p. 146).

[3]) Abu-'Abdallâh Muḥammad ibn Jâbir ibn Sinân al-Battâni, al-Ḥarrâni, al-Ṣâbi (born near Harran in Mesopotamia, 850?—Damascus 928?). His principal work, *al-Zîdj*, translated into Latin in the early part of the 12th century, and into Spanish somewhat later, was printed at Marburg (*De Scientia Stellarum*, 1536) and at Nuremberg (*De Motu Stellarum*, 1537).

4 BEPALING.

Iuliaensche jaren sijn vvelcker drie 365 dagen vervanghen, het vierde 366, hebbende dan de maent Februarius 29 daghen, daerse anders maer 28 en heeft.

Om t'verloop der Egipsche jaren weerom te innen, en t'natuerlick jaer naerder te commen, soo heeft *Iulius Cesar* den voorschreven dach alle vier jaren in Februario veroirdent, welck gedaente van jaren Iuliaensche genaemt werden.

5 BEPALING.

Soo men neemt het natuerlick jaer te hebben soo veel even dagen en even uyren met haer gedeelte, alser oneven dagen en oneven uyren met haer gedeelte in sijn: Sy vvorden * evedaghen oock middeldagen ghenoemt. En sulcken tijt int ghemeen * eventijt ende middeltijt.

Dies æquales & dies medij. Tempus æquale Tempus medium.

Doirsaeck des naems middeldaghen, is van weghen datse middelmatich sijn tusschen de langher en corter natuerlicke daghen der 1 bepaling.

6 BEPALING.

Planetæ.

* Dvvaelders sijn seven vveereltlichten, die int uyterlick ansien schijnen sonder regel te loopen al offse dvvaelden, met naem Saturnus, Iupiter, Mars, Son, Venus, Mercurius, Maen.

Hoewel den Eertcloot eyghentlick me een Dwaelder is, nochtans anghesien wy om de redenen vooren int Cortbegrijp verhaelt, die nemen vast te staen, soo en wortse hier onder de Dwaelders niet ghetelt.

7 BEPALING.

Excentricus circulus. Concentricus.

* Uytmiddelpuntichront noemtmen diens middelpunt buyten den Eertcloot staet. Maer * middelpuntichront diens middelpunt oock des Eertcloots middelpunt is.

Laet

4th DEFINITION.

Julian years are such that three of them contain 365 days, the fourth 366, the month of February then having 29 days, whereas otherwise it has only 28.

In order to recover the shifting of the Egyptian years and come nearer to the natural year, *Julius Caesar* ordained the above-mentioned day to be added every four years in February, which kind of years were called Julian years.

5th DEFINITION.

If the natural year is taken to have as many equal days and equal hours with their fraction as there are unequal days and unequal hours with their fraction in it, they are called equal days and also mean days. And such time in general: equal time and mean time.

The origin of the name mean days is that they are the mean between the longer and shorter natural days of the 1st definition.

6th DEFINITION.

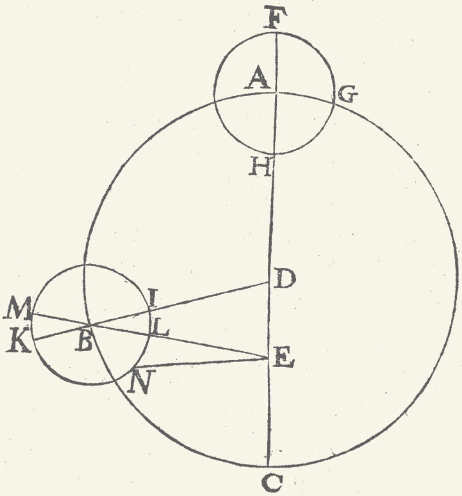
Planets are seven luminaries which in outward appearance seem to move without rule, as if they were wandering; their names are Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon.

Although the Earth is in reality also a Planet, yet because for the reasons related hereinbefore in the Summary we assume it to be fixed, it is not reckoned here among the Planets.

7th DEFINITION.

Eccentric circle we call a circle whose centre is outside the Earth. But concentric circle, a circle whose centre is also the centre of the Earth.

Laet A B C een rondt ſijn diens middelpunt D, en E den Eertcloot: Dit ſoo weſende, A B C heet uytmiddelpuntichront. Maer ſoo men naem het punt D den Eertcloot te ſijn, t'ſoude dan middelpuntich heeten.



8 BEPALING.

De lini van des uytmiddelpuntich ronts middelpunt totten Eertcloots middelpunt heet * uytmiddelpunticheytlijn. *Linea excentricitatis.*

Als inde form der 7 bepaling de lini D E.

9 BEPALING.

Uytmiddelpuntichronts * verſtepunt noemtmen dat verſt vanden Eertcloot is: En * naeſtepunt dat den Eertcloot naeſt ſtaet. *Apogeum.* *Perigeum.*

Laet inde form der 7 bepaling de uytmiddelpunticheyt D E op beyden ſijden voortghetrocken worden tot inden omtreck an A, en C, Twelck ſoo ſijnde, want het punt A, inden omtreck A B C alderverſt vanden Eertcloot E is, en C aldernaeſt, ſoo wert A des uytmiddelpuntichronts A B C verſtepunt genoemt, en C het naeſtepunt.

10 BEPALING.

Dvvaeldervvech is een uytmiddelpuntichront daer in een Dvvaelder loopt. *Deferens vel Via Planetæ.*

11 BEPALING.

Weſende op een punt als middelpunt, inden omtreck

Let *ABC* be a circle, whose centre shall be *D,* and *E* the Earth. This being so, *ABC* is called eccentric circle. But if the point *D* were taken to be the Earth, it would be called concentric.

8th DEFINITION.

The line from the centre of the eccentric circle to the centre of the Earth is called line of eccentricity.

Thus, in the figure of the 7th definition the line *DE.*

9th DEFINITION

Apogee of the eccentric circle we call the point farthest from the Earth, and perigee the point nearest to the Earth.

In the figure of the 7th definition let the line of eccentricity *DE* be produced on either side to the circumference, at *A* and *C.* This being so, because the point *A* on the circumference *ABC* is farthest from the Earth *E,* and *C* nearest to it, *A* was called the apogee of the eccentric circle *ABC,* and *C* the perigee.

10th DEFINITION.

Planet's orbit is an eccentric circle in which a Planet moves.

eens uytmiddelpuntichronts, beſchreven een cleen ront, daer in men neemt een Dvvaelder te loopen: Dat cleender heet * inront, en t'grooter * inrontvvech.

Epiryclus. Deferens epiryclum.

Laet inde form der 7 bepaling op eenich punt als A des uytmiddelpuntichronts A B C beſchreven worden het cleender rondt F G H daer in men neemt een Dwaelder te loopen, t'ſelve cleender heet inront, en t'grooter A B C inrontwech.

12 BEPALING.

Inronts verſtepunt noemtmen dat verſt vanden Eertcloot is: En * naeſtepunt dat die naeſt is: Maer * inronts middelverſtepunt, dat verſt van ſijn vvechs middelpunt is: En * middelnaeſte punt dat die naeſt is.

Apogeum epirycli vel aux vera. Perigeum vel oppoſitũ augis veræ, Aux epirycli media. Oppoſitum augis mediæ.

Laet inde form der 7 bepaling de middellijn C A voortghetrocken worden tot F inden omtreck des inronts: Tſelve punt F boven alle punten des inronts ten verſten vanden Eertcloot E ſijnde, men noemet inronts verſtepunt, en H naeſt weſende heet naeſtepunt. Maer om nu te verclarẽ de gedaente vant middelverſtepunt, en middelnaeſte punt, ſoo laet het inront ghecommen ſijn tot een ander plaets ſijns wechs, ick neem met ſijn middelpunt an B, daer na ſy getrocken de rechte D B K, ſniende den omtreck ten naeſten an I, en ten verſten an K: Sghelijcx de rechte E L B M, ſniende den omtreck ten naeſten an L, en ten verſten an M: Dit ſoo weſende, M is om de voorgaende redenen verſte punt, en L naeſtepunt: Maer K verſt van D is middelverſtepunt, en I naeſt D middelnaeſtepunt: Sulcx dat deſe twee verſtepunten M, K, het inront daer weſende, verſcheyden plaetſen hebben: Maer als het inronts middelpunt an des wechs verſtepunt A, of naeſtepunt C is, dan ſijn verſtepunt en middelverſte punt een ſelve, alſoo oock ſijn naeſtepunt en middelnaeſtepunt. D'oirſaeck des naems van middelverſtepunt en middelnaeſtepunt is deſe: Want K een verſtepunt is, t'welck uyt D gheſien altijt eenvaerdelick everas voortgaet, maer M alſnu raſſcher dan tragher, tuſſchen welck de loop van K als middelmatich ſijnde, wort middelverſtepunt gheheeten.

13 BEPALING.

Des inronts boochſken tuſſchen de tvvee verſtepunten noem ick * verſtepuntensbooch: T'ander tuſſchen de tvvee naeſte punten, naeſtepuntensbooch.

Aequatio centri in epiryclo.

Als t'boochſken K M inde form der 7 bepaling tuſſchen t'verſtepunt K, en het middelverſtepunt M heet verſtepuntensbooch, maer I L naeſtepuntensbooch.

11th DEFINITION.

When about a point as centre, on the circumference of an eccentric circle, there be described a small circle, on which a Planet is taken to move, the smaller circle is called epicycle and the larger, deferent.

In the figure of the 7th definition, let there be described about any point of the eccentric circle *ABC*, such as *A*, the smaller circle *FGH*, on which a Planet is assumed to move; this smaller circle is called epicycle, and the larger *ABC*, deferent.

12th DEFINITION.

The epicycle's apogee we call the point which is farthest from the Earth, and perigee that which is nearest to it. But the epicycle's mean apogee: the point which is farthest from the centre of its orbit, and the mean perigee: that which is nearest to it.

In the figure of the 7th definition let the diameter *CA* be produced to *F* on the circumference of the epicycle. This point *F* being farthest of all points of the epicycle from the Earth *E*, it is called the epicycle's apogee, and *H*, being nearest, is called perigee. But in order to explain the nature of the mean apogee and the mean perigee, let the epicycle have arrived at another point of its orbit, I assume with its centre in *B*. Thereafter let there be drawn the straight line *DBK*, intersecting the circumference at the nearest point in *I* and at the farthest in *K*; likewise the straight line *ELBM*, intersecting the circumference at the nearest point in *L* and at the farthest in *M*. This being so, for the above reasons *M* is the apogee and *L* the perigee. But *K*, being farthest from *D*, is the mean apogee, and *I*, being nearest to *D*, is the mean perigee, so that these two apogees *M*, *K*, when the epicycle is in this position, are in different places. But when the epicycle's centre is in the orbit's apogee *A* or perigee *C*, the apogee and the mean apogee are one and the same point, and so are the perigee and the mean perigee. The origin of the name mean apogee and mean perigee is as follows: because *K* is an apogee which, viewed from *D*, always moves at uniform velocity, but *M* moves now faster, now slower, between which the motion of *K*, as being a medium value, is called the mean apogee.

13th DEFINITION.

The epicycle's arc between the two apogees I call arc of the apogees, the other between the two perigees, arc of the perigees.

Thus, the arc *KM* in the figure of the 7th definition, between the apogee *M* [1]) and the mean apogee *K* [1]), is called arc of the apogees, but *IL*, arc of the perigees.

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[1]) In the Dutch text, for *K* read *M* and for *M* read *K*.

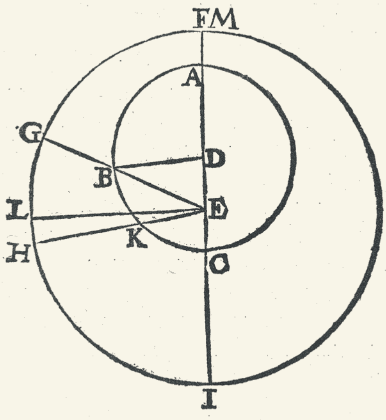
14 BEPALING.

Wechs eerſte halfront noem ick de booch vant verſtepunt tottet naeſtepunt, na'tvervolgh der trappen, dats van t'begin totten 180, d'ander booch tvveede halfront.

15 BEPALING.

Schijnbaer Dvvaelder is een punt daer hy ſchijnt te vveſen, doch eygentlick niet en is: En Schijnbaerloop een booch die hy ſchijnt te loopen, doch eyghentlick niet en loopt.

Laet A B C den Dwaelderwech beteyckenẽ, diens middelpunt D, en E den Eertcloot, op welcke als middelpunt beſchrevẽ ſy den Duyſteraer F G H. Voort ſy ghetrocken deur E D des duyſteraers middellini F I, ſniende den Dwaelderwech in A als verſtepunt, en in C als naeſtepunt: En den Dwaelder ſy inde wech ant punt B, deur t'welck uyt E getrocken is de rechte lini E B G. Dit ſoo weſende, den Dwaelder eyghentlick an B ſijnde, ſal uyt den Eertcloot E geſien, inden duyſteraer ſchijnen te weſen tuſſchen de vaſte ſterren an G, en daerom heetet ſelve punt G ſchijnbaer Dwaelder. Maer ſo hy van B voortloopt, ick neem tot K, en deur K ghetrocken de rechte lini E K H, den Dwaelder die eyghentlick gheloopen heeft den booch B K, ſal uyt den Eertcloot E gheſien ſchijnen gheloopen te hebben den booch G H, die metten eyghen loop B K onghelijck is: En daerom heet de ſelve booch G H (daermen anders oock wel voor neemt den houck B E K of G E H) des Dwaelders ſchijnbaerloop.



VERVOLGH.

Deur dit voorbeelt van ſchijnbaer Dwaelder en ſijn ſchijnbaer loop met ſtelling eens uytmiddelpuntighe wechs, is oock dergheliјcke openbaer met ſtelling eens inronts.

Voort, deur t'ghene hier gheſeyt is van ſchijnbaer Dwaelder en ſchijnbaer loop, is kennelick genouch watmen verſtaen ſal met ſchijnbaer booch, ſchijnbaer verſtepunt als F, ſchijnbaer naeſtepunt als I, en dierghelijcke, welcke hier na alſt de ſake vereyſcht ghenoemt ſullen worden.

16 BEPA-

14th DEFINITION.

The first semi-circle of the orbit I call the arc from the apogee to the perigee, in the order of the degrees, *i.e.* from the beginning to 180 degrees; the other arc, second semi-circle.

15th DEFINITION.

Apparent Planet is a point where it appears to be, but is not in reality, and Apparent Motion is an arc through which it appears to move, but does not move in reality.

Let *ABC* denote the Planet's orbit, whose centre shall be *D,* and *E* the Earth, about which as centre let there be described the Ecliptic *FGH.* Further let there be drawn through *ED* the ecliptic's diameter *FI,* intersecting the Planet's orbit in *A* as apogee and in *C* as perigee. And let the Planet be in the orbit at the point *B,* through which is drawn from *E* the straight line *EBG.* This being so, the Planet, which is in reality at *B,* will, viewed from the Earth *E,* appear to be in the ecliptic between the fixed stars at *G,* and on this account this point *G* is called apparent Planet. But if it moves on from *B,* I assume to *K,* and through *K* there be drawn the straight line *EKH,* the Planet, which has in reality moved through the arc *BK,* will, viewed from the Earth *E,* appear to have moved through the arc *GH,* which is different from the proper motion *BK.* And on this account this arc *GH* (for which is sometimes also taken the angle *BEK* or *GEH)* is called the Planet's apparent motion.

SEQUEL.

From this example of an apparent Planet and its apparent motion on the assumption of an eccentric orbit the same is also evident on the assumption of an epicycle.

Furthermore, from what has been said here about the apparent Planet and apparent motion it is sufficiently obvious what is to be understood by apparent arc, apparent apogee, *e.g. F,* apparent perigee, *e.g. I,* and the like, which will be mentioned hereinafter, when required.

16 BEPALING.

Middeldvvaelder is een verdocht punt inden duysteraer eenvaerdelick voortgaende, en altijt ant schijnbaer verstepunt vvesende, als den vvaren Dvvaelder gheen inront hebbende an sijn vvechs verstepunt is: Maer een inront hebbende, als des selfden inronts middelpunt ant verstepunt is: En sijn loop heet middelloop.

Als by voorbeelt wanneer den Dwaelder inde form der 15 bepaling is ant verstepunt A, soo isser een verdocht punt ant schijnbaer verstepunt F altijt eenvaerdelick voortgaende, sulcx dat wanneer den waren Dwaelder weerom ghecommen is ant verstepunt A, ghedaen hebbende een volcommen keer, soo sal dat punt commen sijn an t'schijnbaer verstepunt, oock ghedaen hebbende een volcommen keer, met soo veel meer als daerentusschen het schijnbaer verstepunt geloopẽ heeft: Sulck punt heet Middeldwaelder, en sijn loop middelloop: De selve hier verclaert sijnde op den tijt eens volcommen keers des waren Dwaelders, wy sullen om alles noch breeder uyt te legghen, voorder verclaring doen op deel eens keers, tot welcken einde ick aldus segh: Den waren Dwaelder ghecommen sijnde ick neem van A tot B, soo sal den middeldwaelder volghende t'ghestelde daerentusschen gheloopen hebben een booch ghelijck met A B, welcke sy F L (te weten L soo ghestelt, dat E L evewijdeghe sy met D B) en noch soo veel meer als daerentusschen den loop des schijnbaer verstepunts bedraecht, t'welck sy van M tot F: Inder voughen dat M L is den middelloop overcommende in tijt met des waren Dwaelders wechloop A B, en heet middelloop, uyt oirsaeck dat soodanighe even loopen middelmatich sijn tusschen de schijnbaer rasscher en tragher.

Deur dit voorbeelt van Middeldwaelder en middelloop met stelling eens dwaelderwechs, is oock derghelijcke openbaer met stelling eens inronts, want nemende des inronts middelpunt als voor Dwaelder, soo sal al t'ghene vooren ghaseyt is daer op overcommen.

17 BEPALING.

Additio. Voordering is t'ghene den vvaren Dvvaelder in den duysteraer schijnbaerlick voorder is dan den Middeldvvaelder: En * achtering t'gene hy schijnbaerlick meer achtervvaert is. En deser tvvee ghemeene naem vvort * voorofachteringh gheseyt.

Diminutio.

Prosthaphaeresis.

Als by voorbeelt den waren Dwaelder inde form der 15 bepaling geloopen hebbende van A tot B, en schijnbaerlick sijnde an G, soo sal daerentusschen den Middeldvvaelder geloopen hebbẽde booch F L gelijck met A B: Twelck soo wesende, dẽ Dwaelder is in dẽ duysteraer an G schijnbaerlick meer achterwaert dan den Middeldwaelder L, soo veel als de booch G L, en daerom heet de selve booch G L achtering, welcke achtering overal int eerste halfront des

wechs

16th DEFINITION.

Mean Planet is an imagined point which moves uniformly in the ecliptic and is always at the apparent apogee when the true Planet, if it has no epicycle, is at its orbit's apogee; but if it has an epicycle: when the centre of this epicycle is at the apogee; and its motion is called mean motion.

Thus, when in the figure of the 15th definition the Planet is at the apogee *A*, there is an imagined point in the apparent apogee *F* which always moves uniformly, so that when the true Planet has returned to the apogee *A*, and has performed a complete revolution, this point will have arrived at the apparent apogee, having also performed a complete revolution, with the addition of the distance meanwhile passed through by the apparent apogee. This point is called Mean Planet, and its motion mean motion. This having been expounded here with regard to the time of a complete revolution of the true Planet, in order to set forth everything even more fully we shall give a further explanation with regard to part of a revolution, to which end I say as follows. When the true Planet has come, I assume, from *A* to *B*, according to the supposition the Mean Planet will meanwhile have passed through an arc similar to *AB*, which shall be *FL* (to wit, *L* being so placed that *EL* shall be parallel to *DB*), with the addition of the distance meanwhile passed through by the apparent apogee, which shall be from *M* to *F*. In such a manner that *ML* is the mean motion corresponding in time to the distance *AB* passed through by the true Planet, and it is called mean motion because such uniform motions are the mean between the apparently faster and slower ones.

From this example of the Mean Planet and mean motion on the assumption of a Planet's orbit the same is also evident on the assumption of an epicycle, for when we take the epicycle's centre for the Planet, all that has been said above will apply thereto.

17th DEFINITION.

"Voordering" (advance) is the amount by which the true Planet is apparently in advance of the Mean Planet in the ecliptic, and "achtering" (lag) is the amount by which it apparently lags behind. And the common name of these two is "voorofachtering" (advance-or-lag [1]).

Thus, when in the figure of the 15th definition the true Planet has moved from *A* to *B* and is apparently at *G*, meanwhile the Mean Planet will have passed through the arc *FL*, similar to *AB*. This being so, the Planet in the ecliptic at *G* apparently lags behind the Mean Planet *L*, as much as the arc *GL*, and for this reason this arc *GL* is called lag, which lagging takes place all through

[1]) This expression, being the English equivalent of Stevin's Dutch word, will be used here instead of Ptolemy's Greek word *prosthaphairesis*.

wechs ghebeurt. Maer als sulcx met voordering valt, t'welck overal int tweede halfrondt ghefchiet, men noemet voordering, en soodanighe twee int ghemeen voorofachtering.

Merckt noch datmen soo wel den houck D B E, als de booch G L, achtering des Dwaelders noemt, om dat de ghetalen haerder tr. even sijn, uyt oirsaeck dat G L booch is des houcx G E L, even sijnde met D B E.

Tot hier toe is voorbeelt beschreven van een Dwaelder in een wech loopende, maer bovē dien een inrondt hebbende, hy heeft tweederley voorofachtering, d'eene van wegen sijn inrondts middelpunt, t'welck een voorofachtering ontfangt van ghedaente ghelijck de bovefchreven des Dwaelders in sijn wech, d'ander van wegen sijn loop int inrondt, en de voorofachtering veroirsaeckt deur dese twee voorofachteringen t'samen, wort int gemeen des Dwaelders voorofachtering ghefeyt.

Laet tot breeder verclaring inde form der 7 bepaling den Dwaelder int inrondt sijn an N, alwaer hy benevens de achtering die hy van weghen sijn inrondts middelpunt ontfangt, voordering crijcht des houcx B E N, welck by d'ander D B E vervought (te weten vergaert alsse beyde eennamich sijn maer van malcander ghetrocken verscheennamich wesende als hier) datter uyt comt is des Dwaelders voorofachtering

Merckt noch t'ghetal der trappen des verstepuntensbooch als hier M K, of des naestepuntensbooch als I L, altijt t'overcommen met het getal der voorofachtering vant middelpunt des inrondts als D B E, om dat I L beschreven is opt middelpunt B.

18 BEPALING.

Eerste voorofachtering is die des middelpunts vant inrondt. Tvveede voorofachtering die des Dvvaelders int inrondt. Gheeffende voorofachtering die veroirsaeckt vvort deur t'vermenghen van d'eerste en tvveede.

Den Dwaelder sonder inrondt maer een voorofachtering hebbende, te weten die hy door de middeluytpunticheyt sijns wechs crijcht, en behouft gheen onderscheyden namen van eerste tweede noch gheeffende voorofachtering deser bepaling, welcke alleen de Dwaelders toecommen in inronden loopende, wiens eerste en tweede beyde voordering sijnde, haer somme is gheeffende voordering, ende beyde achtering wesende, haer somme is gheeffende achtering, maer d'een voordering d'ander achtering sijnde, soo is haer verschil mette naem van t'grootste ghetal der twee de gheeffende voorofachtering.

19 BEPALING.

Tghene d'een Dvvaelder op een ghestelde tijt voorder loopt als d'ander, of daer op vvint, vvort * Dvvaelder-vvinst gheheeten. *Motus & stantia.*

Desen tijt wort dickwils verstaen te beghinnen op de saming tweer Dwaelders, sulcx dat mette schijnbaerbooch na t'vervolgh der trappen vandē traech-

sten

the first semi-circle of the orbit. But if this is positive, which happens all through the second semi-circle, it is called advance, and these two together, advance-or-lag.

Note also that both the angle *DBE* and the arc *GL* are called lag of the Planet, because the numbers of their degrees are equal, owing to *GL* being the arc of the angle *GEL,* which is equal to *DBE.*

Up to this point the example has been described of a Planet moving in an orbit; but if it also has an epicycle, it has two kinds of advance-or-lag, one owing to its epicycle's centre, which receives an advance-or-lag of a similar nature to that described above with regard to the Planet in its orbit, the other owing to its motion on the epicycle, and the advance-or-lag caused by these two together is generally called the Planet's advance-or-lag.

With a view to a fuller explanation, in the figure of the 7th definition let the Planet be on the epicycle at *N,* where, in addition to the lag it receives owing to its epicycle's centre, it receives the advance of the angle *BEN;* when the latter is combined with the other, *DBE* (to wit, added when they have the same sign, but one subtracted from the other when they have the opposite sign, as here), the result is the Planet's advance-or-lag.

Note also that the number of degrees of the arc of the apogees, such as here *MK,* or of the arc of the perigees, such as *IL,* always corresponds to the amount of the advance-or-lag of the centre of the epicycle, such as *DBE,* because *IL* has been described about the centre *B.*

18th DEFINITION.

The first advance-or-lag is that of the centre of the epicycle. The second advance-or-lag is that of the Planet on the epicycle. The total advance-or-lag is that which is caused by combining the first and the second.

The Planet without an epicycle, having only one advance-or-lag, to wit, that which it receives owing to the eccentricity of its orbit, does not need the different names of first, second or total advance-or-lag of this definition, which are only to be assigned to the Planets which move on epicycles; when its first and second are both advances, their sum is the total advance, and when both are lags, their sum is the total lag; but when one is an advance and the other a lag, their difference, with the sign of the larger of the two, is the total advance-or-lag.

19th DEFINITION.

That by which one Planet moves in advance of another in a given time, or gains thereon, is called the Planet's gain.

This time is frequently understood to begin at the conjunction of two Planets, ſten Dwaelder totten ſnelſten, int ghemeen Dwaelderwinſt beteyckent wort, welcke men ſoo wel berekent met middeldwaelders als ware.

20 BEPALING.

Conjunctione. Oppoſitione.

Tvvee vveereltlichten een ſelve ſchijnbaer duyſteraerlangde hebbende vvorden in ★ ſaming gheſeyt. Maer een ſchijnbaer half rondt verſchillende, in * tegheſtant.

21 BEPALING.

Conjunctiones & oppoſitiones mediæ.

Der Middeldvvaelders ſaminghen en tegheſtanden vvorden haer * middelſaminghen ende middeltegeſtanden gheſeyt.

22 BEPALING.

Inſtrumenta machematica.

Ervarings dachtafels der Dvvaelders ſijn vvaer in oirdentlick van dach tot dach beſchreven ſtaen de plaetſen der Dvvaelders ſoomenſe deur * vviſconſtuygen daer toe bequaem dadelick bevonden heeſt, met tijt haerder ſaminghen, ſoo mette vaſte ſterren als onder malcander, voort haer anclevende als duyſteringhen, grootheyt des verduyſtert deels, over vvelcke ſijde verduyſtert, vvanneer beginnende, vvanneer eindende, en dierghelijcke.

23 BEPALING.

Ephemerides.

Berekende dachtafels der dvvaelders, ſijn die deur kennis des Hemelloops berekent vvordẽ, vvaer in oirdentlick van dach tot dach beſchreven ſtaen, der Dvvaelders plaetſen, ſoo men meynt datſe in toecommende tijden ſijn ſullen: Voort duyſteringhen van Son en Maen, met haer anclevende.

Sulcke berekende dachtafels gaen nu veel in druck uyt, als van *Ioannes Stofflerus, Eraſmus Rheinoldus, Leovitius, Stadius, Maginus, Martinus Everarti*, en dierghelijcke.

24 BEPALING.

Tempus periodicum.

Keertijt eens Dvvaelders noemtmen, in vvelcke deur een groote menichte der gagheſlaghen keeren van ſijn ronso that by the apparent arc, in the order of the degrees, from the slowest Planet to the fastest the Planet's gain is generally denoted, which is calculated with mean as well as true Planets.

20th DEFINITION.

When two luminaries have the same apparent ecliptical longitude, they are said to be in conjunction. But when they differ by an apparent semi-circle: in opposition.

21st DEFINITION.

The Mean Planets' conjunctions and oppositions are called the Planets' mean conjunctions and mean oppositions.

22nd DEFINITION.

Empirical ephemerides of the Planets are tables in which are described in regular order, from day to day, the positions of the Planets as they have been found in practice by means of suitable mathematical instruments, with the times of their conjunctions, both with the fixed stars and among themselves, further their related phenomena, such as eclipses, size of the eclipsed part, on which side they are eclipsed, when the eclipse begins, when it ends, and the like.

23rd DEFINITION.

Calculated ephemerides of the Planets are tables which are calculated by means of knowledge of the Heavenly Motions, in which are described in regular order, from day to day, the positions of the Planets as they are expected to be in the future; further eclipses of Sun and Moon, with their related phenomena.

Such calculated ephemerides are now frequently printed, *e.g.* those of *Johannes Stofflerus, Erasmus Rheinoldus, Leovitius, Stadius, Maginus, Martinus Everarti,* and the like [1]).

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[1]) Johannes Stöffler (Blaubeuren 1452—*ibid.* 1531), professor of mathematics at Tübingen. Author of the *Tabulae Astronomicae* (Tubingen 1500 and 1514). Jointly with Pflaum he published ephemerides of the planets for 1499—1531: *Almanach Nova* (Ulm, 1499). Later he published alone the sequel: *Ephemeridum opus . . . a capite anni 1532 in alios 20 proxime subsequentes. . . elaboratum* (Tübingen, 1531 and subsequent editions in 1533 and 1548).

Erasmus Reinhold the Elder (Saalfeld 1511—*ibid.* 1553), professor of mathematics at Wittenberg. Shortly after his ephemerides for 1550—1551 (Tübingen, 1550) appeared his celebrated *Prutenicae Tabulae coelestium motuum.* (Tübingen 1551 and several subsequent editions up to 1585), dedicated to the Duke Albrecht von Preussen. They were based on the work of Copernicus and remained the best until the publication of the Rudolphine tables; they formed the basis for the Gregorian reform of the calendar.

Cyprian Leowitz (Leowitz in Bohemia 1524—Lauingen 1574), mathematician to Count Otto Heinrich, author of *Ephemeridum novum atque insigne opus ab A. 1556—1606 accuratissime supputatum* (Augsburg 1557).

Johannes Stadius (Leonhout near Antwerp 1527—Paris 1579), professor of mathematics at Louvain and Paris, author of *Ephemerides novae et exactae ab A. 1554 ad A. 1570* (Cologne 1556; later editions up to 1591 contain an extension up to the year 1606). They were based on the *Prutenicae Tabulae.*

Giovanni Antonio Magini (Padua 1555—Bologna 1617), professor of mathematics, physics and astronomy at Bologna, author of *Ephemerides coelestium motuum ab A. 1581—*

ronden, ghevonden vvort de rasheyt van yder rondts loop op bekende tijt.

MERCKT.

Den duysteraer sal hier ghelijck den evenaer en meer ander ronden, ghedeelt worden in 360 trappen, sonder twaelf teyckens te ghebruycken elck van 30 tr. of namen van dien na d'oude ghewoonte, waer af ick de reden inden Anhang deses Hemelloops verclaren sal.

24th DEFINITION.

The period of revolution of a Planet we call the time in which from a great multitude of observed revolutions is found the velocity of the motion in each circle in a known time.

NOTE.

The ecliptic, like the equator and other circles, will here be divided into 360 degrees, without using the twelve signs, each of 30 degrees, or their names as used of old, the reason of which I will set forth in the *Appendix* to these Heavenly Motions [1]).

1620 (Venice, 1582), later extended to 1608—1630 (Frankfurt 1608 and 1610); a supplement appeared in 1614 (Venice) and 1615 (Frankfurt).

Martin Everart or Everaerts (born at Bruges), mathematician and surgeon. *Ephemerides meteorologicae anni 1583* (Antwerp 1582); *Ephemerides novae et exactae* (Leiden 1597), relating to the years 1590—1610. In new editions, they were extended up to 1615 (Heidelberg, 1600 and 1602).

[1]) This explanation is not found in the *Appendix*.

EERSTE
ONDERSCHEYT DES EERSTEN BOVCX, VANDE VINding des Sonloops deur ervarings dachtafels.

Eer ick comme totte Sonloop int besonder, sal beschrijven t'volghende eerste Voorstel vande vinding der Dvvaelderloopen haer en d'ander int ghemeen angaende.

FIRST CHAPTER

OF THE FIRST BOOK

Of the Finding of the Sun's Motion by Means of Empirical Ephemerides

Before I come to the Sun's motion in particular, I will describe the following first Proposition of the finding of the Planets' motions, relating to her and the others in common.

1 VOORSTEL.

Te verclaren hoet schijnt dat de Menschen eerst begosten tot kennis vande loop der * Dvvaelders te gheraken, of daer toe souden meugen beginnen te commen, sooder gantsch gheen af en vvaer.

* Planetarum.

WAnt het kennelick is datmen om een const wel en grondelick te verstaen, behoort an te vanghen met haer uyterste beginselen: Soo sal ick mijn ghevoelen segghen hoet schijnt dat sy deden die eerst begosten de ghedaenten des Dwaelderloops te leeren, of hoemen soude meughen doen datter gantschelick niet af beschreven en ware. Om dan van dese Hemelsche stof eerst deur aertsche by voorbeelt te spreken, ick segh dat ghelijck ymant die in Caert wil brenghen een Lantschap dat noyt caertsche wijse gheteyckent en was, of daer hem gheen teyckening noch onderrichting af ter handt ghecommen en is, soude moeten het Landtschap of self dadelick besien, of seker onderricht hebben vande gene diet dadelick gesien hadden: Alsoo eenen die de manier des loops der Dwaelders wil verstaen en beschrijven, moet eerst haer loop of self ghesien hebben, of daer af sekerlick onderricht sijn van hemlien diet deur dadelicke ervaring weten: En sulcx hebben de voorganghers ghedaen, welcke alsoose eertijts ernstelick gasloughen de plaetsen der Hemelsche lichten, ende siende tusschen de groote menichte der vaste sterren, beneven Son en Maen noch seker vijf beweeghlicke, diens loop int uyterlick ansien seer ongheregelt scheen, nu ras, dan slap, somwijlen stil staen, en ettelicke mael te rugh keeren, sy hebben hun begeven tottet ondersoucken der oirsaken dier ongeregeltheyt, beginnende met daghelicx seer nau ga te slaen, en op te teyckenen haer schijnbaer plaetsen tusschen de vaste sterren, oock Maenduysteringhen, en Sonduysteringhen, met haer omstandighen, als tijt van haer begin tottet eynde, hoe groot t'verduysterde deel was, en op welcke sijde verduystert, onder wat duysteraerlangde en breede t'middel der duystering gheschiede, en tot wat plaets des Eertcloots sy dat gageslaghen hadden. Het streckte oock tot noch meerder sekerheyt des handels, op te teyckenen den tijt met ander omstandighen vande duysteringhen der vaste sterren, te weten als sy die vande Dwaelders bedeckt saghen, en oock de duysteringhen van d'ander Dwaelders onder malcander. uyt de boveschreven schijnbaer plaetsen der Dwaelders, merckten sy haer daghelicksche veranderingen in langde en breede, welcke sy dachtafelsche wijse opteyckenden, daer by noch voughende de boveschreven anclevinghen van der Dwaelders duysteringen: En creghen alsoo de nacommelinghen metter tijt, benevens haer eygen gageslagen dachtafels, oock die haerder voorgangers van seer veel jaren: Inde selve hadden sy bequame middel om de ghedaente des eyghen loops der Dwaelders t'ondersoucken, ghelijck oock souden ghehadt hebben *Hypparchus*, *Ptolemeus*, en hun nacommers, by aldiense t'haerder handt ghecommen waren.

Maer verloren blijvende, en sedert gheen ander soo ghemaeckt wesende als de saeck vereyscht, ick sal om desen handel te verclaren, in die plaets nemen eenighe berekende nu in druck uytgaende, als die van *Stadius*, want hoewelse op effen tijt berekent sijn: daermen de ervarings dachtafels op oneven of natuerlicke maeckt, en datse daer benevens niet genouch mette saeck en overcommen,

1st PROPOSITION.

To set forth how man first seems to have begun to acquire knowledge of the motions of the Planets, or might begin to acquire it, if there were none at all.

Because it is obvious that, in order to understand a science well and thoroughly, one should start with its first principles, I will give my opinion how it seems that those proceeded who first began to learn the nature of the planetary motions, or how one might set about it if there were no description of it at all. In order first to speak of these heavenly matters by means of terrestrial examples, I say that just as a man who wishes to map out a region of which no map has ever been drawn or of which no drawing or report has come into his hands, would either have to inspect the region actually himself or to have reliable information from those who had actually inspected it: thus a man who wants to understand and describe the manner of the motion of the Planets must either first have seen their motion himself or must have been reliably instructed on it by those who know it from practical experience. And this is what our predecessors did, who — when formerly they seriously observed the positions of the heavenly luminaries and saw, among the great multitude of the fixed stars, besides the Sun and the Moon at least five more moving stars, whose motion to all outward appearance seemed very irregular, now fast, now slow, sometimes stopping, and several times returning — started to examine the causes of this irregularity, beginning by observing every day very closely and noting their apparent positions among the fixed stars, also Lunar eclipses and Solar eclipses, with their circumstances, such as the time from their beginning to their end, how large was the eclipsed part and on which side it was eclipsed, at what ecliptical longitude and latitude the middle of the eclipse took place, and in what place on the Earth they had observed this. It also tended to greater certainty in the treatment to note the time and other circumstances of the eclipses of the fixed stars, to wit, when they saw them covered by the Planets, and also the eclipses of the other Planets among themselves. From the apparent positions of the Planets described above they noted their daily changes in longitude and latitude, which they recorded in the manner of ephemerides, adding thereto the circumstances described above, of the Planets' eclipses. And thus, in due time, the successors obtained, besides the ephemerides observed by themselves, also those of their predecessors of a great many years. In these they had suitable means for examining the nature of the true motion of the Planets, as *Hipparchus, Ptolemy,* and their successors would also have had, if these had come into their hands.

But because these remained lost and no others have since been made such as the matter requires, in order to set forth this subject I will take instead some calculated tables, now printed, namely those of *Stadius* [1]); for though they are calculated for exact time, whereas empirical ephemerides are made for unequal or natural time, and though moreover they do not agree sufficiently with the facts,

[1]) See note p. 45.

eensdeels deur misrekeninghen dieder vallen, ten anderen om dat den Hemelloop nu niet ghenouch bekent en is, doch salt voorbeeltsche wijse meughen bestaen, om daer me mijn voornemen te verclaren: Te weten hoemen voor t'eerste vermoedt de Dwaelders te loopen in uytmiddelpuntige ronden, en eenighe boven dien noch in inronden. Ten anderen hoemen daer deur vindt der Dwaelders middelloopen: Voort de schijnbaer duysteraerlangden der verstepunten of naestepunten van weghen en inronden.

Den Dwaelderloop dus verre deur platte verstanelicke redenen uyt den rouwen bekent sijnde, soo sal ick daer na int tweede bouck en d'ander volghende voorder commen tottet ondersoucken deser stof deur wisconstighen handel, na d'oirden int Cortbegrijp des Hemelloops verclaert: Want datmen de boveschreven dinghen ten eersten begint te soucken na de wijse van *Hypparchus*, en *Ptolemeus* ter handt ghecommen, en deur hemlien ons achtergelaten (daer wy hun danck af schuldich sijn) te weten deur seker wisconstighe wercking, ghegront op een Dwaelders drie gagheslaghen schijnbaer duysteraerlangden, de natuerlicke reden schijnt te willen dat d'eerste ondersouckers daer me niet en begosten, maer dat veel andere dinghen voor moesten gaen, deur welcke men leerde dattet gaslaen van sulcx tot soo seker gront vande kennis des Hemelloops strecken soude, voort dat die wisconstighe werckinghen daer na tot meerder overvloet en sekerheyt by d'ander vervought wierden. En alsoo sal ick die oock beschrijven, na dat de saeck eerst deur wercking met ervarings dachtafels verstaen sal sijn: Merckt noch dat nadien den Eertcloot int uyterlick ansien schijnt stil te staen, en dat den anvang der leering des Hemelloops op sulcke stelling begost heeft, oock also bequamelicker en verstaenlicker begint, soo sullen wy t'selve hier na volghen, beschrijvende op sulcken gront het eerste en tweede bouck, maer int derde den Hemelloop mette natuerlicke stelling eens roerenden Eertcloots.

Voort is te weten dat de beschrijving des Hemelloops *Ptolemeus* ter handt gecommen, na sijn eygen seggen seer eenvoudich was, namelick de Maenloop, gelijck vande Son in een uytmiddelpuntichront, en d'ander Dwaelders alleenelick in uytmiddelpuntige inronden: Maer *Ptolemeus* achtende dat sulcx niet genouch met sijn dadelicke ervaringē overeen en quam, heeft de spiegeling deur hem daer toe verdocht, ghemengt mette voorschreven eenvoudige stelling, en daer af een werck gemaeckt: Sgelijcx heeft oock *Copernicus* op de stelling eens roerendē Eertcloots gedaen: Doch ick en sal hier die vermengde wijse niet volgen, maer den Hemelloop met d'eerste eenvoudige stelling alleē beschrijvē, op dat wy die voor t'eerste alsoo hebbē, gelijckse *Ptolemeus* ter handt quam, en van die naverdochte oneventhedē wesende onnatuerlick duyster en gemist, sal ick in een Anhang besonderlick handelen, op dat voor den leerlinck te claerlicker blijcke watter inde verbetering des Hemelloops gesocht wort, om alsoo op een vaster voet na bequamer * spieghelinghen te meughen trachten, want dit de bequaemste wech is die my nu te voren comt, om op den cortsten tijt mette meeste claerheyt, sijn VORSTELICKE GHENADE te doen verstaen t'ghene my vandē loop der Dwaelders bekent is, oock mede om opentlick te doē blijcken, hoe noodich ervarings dachtafels en * Gaslagers sijn, om tot sulcken kennis deser const te gheraken, alsser inden Wijsentijt af gheweest heeft.

Theorias.

Observatores.

MERCKT.

Eer ick voorder comme sal noch dit segghen: Te weten dat mijn voornemen niet en is met groote sekerheyt te bewijsen der Dwaelders toecommende ware on the one hand owing to miscalculations, on the other hand because the Heavenly Motions are not sufficiently known, yet by way of example it will be permissible to explain therewith my intention: to wit, how firstly the Planets are supposed to move on eccentric circles, and some moreover on epicycles. Secondly, how thus the Planets' mean motions are found. Further the apparent ecliptical longitudes of the apogees or perigees of the orbits and epicycles.

The Planets' motions thus far being roughly known by means of simple intelligible reasons, I will thereafter, in the second book and the others, proceed to examine this matter by mathematical means, in the order set forth in the Summary of the Heavenly Motions. For though the above things are now at once examined after the manner of *Hipparchus*, and as they came into *Ptolemy's* hands and were left to us by them (for which we owe them thanks), to wit by certain mathematical operations, based on three observed apparent ecliptical longitudes of a Planet, it is natural to suppose that the first investigators did not begin in this way, but that many other things had to precede, from which people learned that the observation of this would tend to produce a sure basis of knowledge of the Heavenly Motions, further that those mathematical operations were thereupon added to the others for greater completeness and certainty. And thus I will also describe them, after the matter has first been understood by means of empirical ephemerides. Note also that since to outward appearance the Earth seems to stand still and the beginning of the knowledge of the Heavenly Motions started from this assumption, and also begins thus more suitably and understandably, we shall imitate this here, describing on this basis the first and the second book, but in the third the Heavenly Motions on the natural assumption of a moving Earth.

Further it is to be noted that the description of the Heavenly Motions, as it came into *Ptolemy's* hands, according to his own reports was very simple, namely, the motion of the Moon, like that of the Sun, in an eccentric circle and the other Planets only in eccentric epicycles. But *Ptolemy*, being of opinion that this was not sufficiently in agreement with his actual experience, combined the theory devised by him for this purpose with the above-mentioned simple assumption and wrote a treatise about it. The same was also done by *Copernicus* on the assumption of a moving Earth. But I will not here follow this mixed method, but will describe the Heavenly Motions only on the first simple assumption, so that we may first have it as it came into *Ptolemy's* hands; and with those inequalities devised afterwards, which are unnaturally obscure, and are erroneous, I will deal in particular in an Appendix, in order that it may appear all the more clearly to the pupil what is sought for in the improvement of the Heavenly Motions, thus to be able to strive on a firmer basis after more suitable theories, because this appears to me the most suitable method for making his PRINCELY GRACE understand in the shortest possible time and with the greatest clearness what is known to me about the motions of the Planets, and also for showing manifestly how needful are empirical ephemerides and Observers for acquiring such knowledge of this science as existed in the Age of the Sages.

NOTE.

Before I go on, I will also say this: to wit, that my intention is not to prove with great certainty the Planets' future true positions, but only to set forth the de ware plaetſen, maer alleen te verclaren de manier des loops, nemende voorbeelden die beſt te pas commen, ghewis of onghewis, uyt oirſaeck dat den handel int gheheel na mijn gevoelen om der onbekende tweede oneventheden wille, alſnu gheen ghenouchſaem vaſte gront en heeft, en een nieuwe ghewiſſer vereyſcht, die ſoo haeſt niet gheleyt en ſal connen worden, eenſdeels om datter geen geſlacht van volck en is die in haer aengeboren tael hun heel ernſtelick daer in t'ſamen oeffenen, en vervolghens niet ſoo veel Gaſlaghers en connen ghevonden worden als de ſaeck vereyſcht, ghelijck daer af breeder gheſeyt is onder de 6 bepaling des eerſten boucx vant Eertclootſchrift. Ten anderen dat bovendien ſulcke verbetering oock tijt vereyſcht.

T B E S L V Y T. Wy hebben dan verclaert hoet ſchijnt dat de menſchen eerſt begoſten tot kennis vande loop der Dwaelders te gheraken, of daer toe ſouden meughen beghinnen te commen, ſooder gantſch gheen af en waer, na den eyſch.

2 VOORSTEL.

Deur ervarings dachtafels de lanckheyt des natuerlick jaers te vinden.

Men beghint billichlick mettet onderſoucken der lanckheyt des natuerlick jaers, omdatmen een ſeker bepaelde tijt behouft, waer in alder Dwaelders en Hemelen loopen berekent worden. Op dat wy dan totte ſaeck commen, t'is noodich datmen om t'volghende lichtelick te verſtaen, hebbe de voorſchreven dachtafels van *Stadius*, of immers ander in haer plaets, welcke wy nemen al oft ervarings dachtafels waren, want ſonder die ſoude alles duyſterder vallen. De ſelve dan by der handt weſende, ick ſouck in eenich jaer, ick neem het eerſte, weſende t'jaer 1554 op welcke middach de Son de Lentſne ten naeſten was, en bevinde opten 11 Maerte, want doen waſſe onder den 359 tr. 59 ①, t'welck alleenlick 1 ① vande Lentſne is. Ick ſouck daer na opt volghende jaer 1555, wanneer de Son weerom was inden voorſchreven 359 tr. 59 ①, en bevinde den 11 Maerte na middach te 5 uyren 36 ①, want op den middach waſſe na tuytwijſen des dachtafels inden 359 tr 45 ①, ſulcx datter noch gebreect den tijt des loops van 14 ①: Om welcke te vinden ick ſie inde dachtafel dat de Son doen op een dach liep 1 tr. dats 60 ①, daerom ſegh ick, 60 ① geven 24 uyren, wat de boveſchreven 14 ①? comt alſvooren 5 uyr 36 ①. Maer vanden 11 Maert 1554, totten 11 Maert 1555, ſijn 365 daghen, daerom t'jaer ſoude na die rekening dueren 365 daghen 5 uyr 36 ①.

Dit is aldus eerſt voorbeeltſche wijſe metten loop van een Son keer berekent, op dat alles claerder en grondelicker verſtaen worde. Maer angheſien men op veel keeren of jarẽ, meer ſekerheyt heeft dan op een of weynich (want op duyſent jaren een uyre ghemiſt, en maeckt op een jaer maer $\frac{1}{1000}$ uyrs, daer anders op een jaer een uyr ghemiſt, voor t'ſelve een yder jaer een heele uyr bedraecht) ſoo ſullen wy nu daer toe ſoo veel jaren nemen alſſer inde dachtafels ſijn.

Ick ſouck dan opt laetſte jaer, weſende het 1606, wanneer de Son weerom was inden 359 tr. 59 ①, en bevinde den 10 Maerte na middach 11 uyr 47 ① 48 ②, want op den middach waſſe inden 359 tr. 30 ①, ſulcx datter noch ghebrack manner of their motion, taking the examples that are most appropriate, either certain or uncertain, because in my opinion the treatment as a whole does not have a sufficiently firm basis, on account of the unknown second inequalities, and requires a new and more certain foundation, which it will not be possible to lay so soon, on the one hand because there is no nation whose members together practise this science very earnestly in their native language, and further because there cannot be found as many observers as are required for the matter, as has been stated more fully in the 6th definition of the first book of Geography. Secondly, because such an improvement also takes time.

CONCLUSION. We have thus set forth how men first seem to have begun to acquire knowledge of the motion of the Planets, or might begin to acquire it, if there were none at all; as required.

2nd PROPOSITION.

To find the length of the natural year by means of empirical ephemerides.

The work is suitably started by investigating the length of the natural year, because a certain well-defined time is needed in which the motions of all the Planets and the Heavenly Bodies are calculated. To come to the matter, it is necessary, in order to understand the sequel easily, to have the above-mentioned ephemerides of *Stadius,* or others instead for that matter, which we use as if they were empirical ephemerides, for without them everything would be more obscure. These tables therefore being to hand, I seek in some one year — I take the first, which is the year 1554 — at what noon the Sun was nearest to the Vernal Equinox, and find on 11th March, for then it was at 359°59′, which is only 1′ from the Vernal Equinox. I then seek in the following year 1555 when the Sun was again at the above 359°59′, and find 11th March, after noon at 5h 36 m, for at noon according to the ephemeris it was at 359°45′, so that the time of the motion of 14′ is still wanting. In order to find this, I see in the ephemeris that the Sun then moved in one day 1°, that is 60′; therefore I say: 60′ give 24 hours, what do the above 14′ give? This gives, as above, 5h 36m. But from 11th March 1554 to 11th March 1555 there are 365 days; therefore, according to this calculation the length of the year would be 365d 5h 36m.

This has thus first been calculated by way of example for the motion of one revolution of the Sun in order that everything may be understood more clearly and thoroughly. But since the certainty is greater for many revolutions or years than for one or few (for one hour's error on a thousand years only makes 1/1000 hour on a year, whereas one hour's error on one year amounts to a whole hour for every year), we shall now take for this as many years as there are in the ephemerides.

I then seek in the last year, which is 1606, when the Sun was again at 359°59′, and find 10th March, after noon, at 11h 47m 48s, for at noon it was at 359°30′, so that the time of the motion of 29′ was still wanting. In order to find this, I see in the ephemeris that the Sun then moved in one day 59′; therefore I say: 59′ give 24 hours, what do the above 29′ give? This gives, as above, 11h 47m 48s. Now from 11th March of the year 1554 to 10th March at 11h 47m 48s of the year 1606, 52 revolutions have taken place, which the brack den tijt des loops van 29 ① : Om welcke te vinden, ick ſie inde dachtafel dat de Son doen op een dach liep 59 ①, daerom ſegh ick, 59 ① gheven 24 uyren, wat de boveſchreven 29 ① ? comt alſvooren 11 uyren 47 ① 48 ②. Nu vanden 11 maerte int jaer 1554, totten 10 Maerte 11 uyr 47 ① 48 ② int jaer 1606, ſijn gheſchiet 52 keeren, waer over de Son gheloopen heeft 18992 daghen 11 uyren 47 ① 48 ② (te weten 52 mael 365 min 1 (ick ſegh min 1 om dattet is vanden 11 Maerte totten 10 Maerte) dats 18979, met noch 13 dagen der 13 ſchrickeljaren, die in Februarius vervought worden) Dit ſoo ſijnde, ick ſegh, 52 keeren duyren 18992 dagen 11 uyren 47 ① 48 ②, hoe langh ſal 1 keer duyren? Comt voor de begheerde lanckheyt des jaers na deſe rekening, 365 daghen 5 uyren 45 ① 55 ②.

TBESLVYT. Wy hebben dan deur ervarings dachtafels de lanckheyt des natuerlick jaers ghevonden, na den eyſch.

3 VOORSTEL.

De Sonnens middelloop op een ghegeven tijt te vinden, en daer af een tafel te beſchrijven.

1 MERCK.

Int 2 Voorſtel is gheſeyt het jaer bevonden te weſen van 365 daghen 5 uyren 45 ① 55 ②, waer op men als gront ſoude moghen voortvaren, int maken van nieuwe tafels des middelloops der Son: Doch want my t'ſelve moeylick ſoude vallen, dat oock daerbenevens dit beſluyt vande lanckheyt des jaers (ghelijck oock alle ander na den wijſentijt) weynich ſekerheyts heeft, en dat alles maer voorbeeltſche wijſe en gheſchiet, om de redenen van dies breeder verclaert int 1 voorſtel, ſoo ſal ick om ſulcke moeyte te ſchuwen, nemen de lanckheyt des jaers by *Ptolemeus* beſchreven, en de tafelen by hem daer op berekent. Deſe lanckheyt des jaers heeft hy na de wijſe alſvooren bevonden van 365 daghen 5 uyren 55 ① 12 ②, die in ander verdeeling ſonder uyren te noemen, doen 365 daghen 14 ① 48 ②, ofte anders 365 $\frac{37}{150}$ daghen.

En ſulcx als hier in dit merck gheſeyt is vande Sonloop, dergheli jcke ſal int volghende oock alſoo ghedaen worden met *Ptolemeus* tafels der middelloopen van d'ander Dwaelders, die ick nemen ſal in plaets van nieuwe te maken.

TGHEGEVEN. Het is den tijt van een dach. TBEGHEERDE. Men wil daer op de Sonnens middelloop gevonden hebben. TWERCK. Ick ſegh, op 365 $\frac{37}{150}$ daghen, loopt de Son 360 tr. deur het 1 merck deſes voorſtels, wat op 1 dach? Comt voor t'begheerde 59. ① 8. 17. 13. 12. 31.

2 MERCK.

Wy hebben hier een voorbeelt gheſtelt, int welcke de Sonnens eyghenloop eens dachs ghevonden wert door een reghel van drien, waer me kennelick is hoe men deur dergheli jcke wercking, de Sonloop ſoude vinden van alle ghegheven tijt, maer want die wercking moeyelick valt, ende dat boven dien ons ſulcx in deſen handel dickwils te vooren comt, ſoo wordender tafels ghemaeckt, om van alle ontmoetende tijt den loop met lichticheyt te vinden, welck maeckſel ick beſchrijven ſal als volght.

MAECK-

Sun performed in 18,992d 11h 47m 48s (to wit: 52 × 365 minus 1) (I say minus 1, because it is from 11th March to 10th March), that is 18,979 plus 13 days of the 13 leap years, which are added in February). This being so, I say: 52 revolutions take 18,992d 11h 47m 48s, how long will 1 revolution take? According to this calculation the required length of the year is 365d 5h 45m 55s.

CONCLUSION. We have thus found the length of the natural year by means of empirical ephemerides; as required.

3rd PROPOSITION.

To find the Sun's mean motion in a given time, and to describe an ephemeris thereof.

1st NOTE.

In the 2nd Proposition it has been said that the year has been found to be 365d 5h 45m 55s, on which basis we might proceed to make new ephemerides of the Sun's mean motion. But because this would be difficult for me, while moreover there is little certainty in this conclusion as to the length of the year (just as in all other things subsequent to the Age of the Sages), and everything is only done by way of example, for the reasons set forth more fully in the 1st Proposition, I will, in order to eschew this trouble, take the length of the year described by *Ptolemy,* and the tables calculated by him thereon. This length of the year was found by him in the above manner to be 365d 5h 55m 12s, which by another division, without mentioning hours, makes 365;14,48,[1]) or otherwise $365\frac{37}{150}$ days.

And the same as has been said in this note for the Sun's motion will also be done hereafter for *Ptolemy's* tables of the mean motions of the other Planets, which I will take instead of making new ones.

SUPPOSITION. The time is one day. WHAT IS REQUIRED. The Sun's mean motion is required to be found. PROCEDURE. I say: in $365\frac{37}{150}$ days the Sun moves 360°, by the first note of this proposition; what does it move in one day? The required value is 0°;59,8,17,13,12,31 [2]).

2nd NOTE.

We have here given an example in which the Sun's proper motion of one day was found by the rule of three, from which it is evident how by a similar operation the Sun's motion might be found in any given time, but because this operation is difficult, while moreover we shall often meet with it in this work, tables are made for easily finding the motion for any time that may occur, the construction of which tables I will describe as follows.

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[1]) In sexagesimals of a day, hence meaning $365 + \frac{14}{60} + \frac{48}{3600}$ days.

[2]) In angular (or analogous) quantities written sexagesimally, the successive sexagesimal subdivisions (which Stevin indicated by writing the digits 1, 2, . . . in small circles) in accordance with modern practice are separated by commas. The whole numbers are followed by a semi-colon. If there are only few subdivisions, the ordinary notation in degrees, minutes, and seconds is used.

MAECKSEL DES TAFELS VANDE MIDDELLOOP DER SON.

VANDE boveſchreven loop eens dachs, dats van 24 uyren doende
59 ① 8. 17. 13. 12. 31.
Ghenomen het $\frac{1}{24}$, comt voor 1 uyr 2 ① 27. 50. 43. 3. 1.
Diens dobbel voor 2 uyr 4 ① 55. 41. 26. 6. 2.
Ende ſoo voortgaende men crijcht den loop van al d'ander uyren tot 23 toe: Daer na tot den loop van een dach, vergaert den loop van noch een dach, men heeftſe van twee: Ende ſoo vervolghende men crijchtſe van meer dagen, oock van Egipſche jaren tot 8 10, ghelijckſe inde nabeſchreven tafel ſtaen.

CONSTRUCTION OF THE TABLE OF THE SUN'S MEAN MOTION.

If of the above-mentioned motion of one day, that is 24 hours,
being 0°;59, 8,17,13,12,31
there be taken $\frac{1}{24}$, we get, for 1 hour 0°; 2,27,50,43, 3, 1
The double of this is, for 2 hours 0°; 4,55,41,26, 6, 2
And, proceeding in this way, we get the motion of all the other hours, up to 23. If thereupon to the motion of one day there be added the motion of another day, we have the motion of two days. And, proceeding in this way, we get it of more days, also of Egyptian years up to 810, as they appear in the following table.

[*The following table (pp. 20-21) has not been reproduced here*]

GHEBRVYCK DES TAFELS.

Laet begheert sijn de Sonnens eyghen loop op 879 Egipsche jaren 66 dagen 2 uyren. T'WERCK. Soo inde tafel de 879 Egipsche jaren 66. dag. 2 uyr. metten middelloop van dien al in een reghel ghevonden wierden, wy souden ten eersten den begheerden middelloop hebben, sonder eenighe vergaring van ghedeelten te behouven: Maer dat niet wesende, wy moeten verscheyden sticken by malcander voughen die t'samen dat heel maken: Tot desen eynde neem ick ten eersten uyt de tafel de jaren die de begheerde 879 naest sijn, als 810, die stellende met haer middelloop alleenelick tot ② toe (als totte saeck ghenouch sijnde, om dat de rest dient tottet maecksel der tafel, niet tottet ghebruyck van dien, soo int laetste deel der nabeschrevẽ waerschouwing opt maecksel der tafel, breeder verclaert sal worden) En de ghestalt der wercking sal dusdanich sijn:

810. jaren. 163 tr. 4. 12.

Nu ghebreken my noch 69 jaren, daer toe neem icker (hoe wel men die noch andersins soude meughen nemen) 45 en 15: Daer na 60 dag. en 6 dag. ende ten laetsten de twee uyr: De selve altemael oirdentlick vervought onder de boveschreven 810 jaer, ende alles vergaert na t'behooren, soo sal de ghestalt der wercking sijn als hier onder.

810 jaer.	163 tr.	4.	12.
54 jaer.	346 tr.	52.	17.
15 jaer.	356 tr.	21.	11.
60 dag.	59 tr.	8.	17.
6 dag.	5 tr.	54.	50.
2 uyr.	0 tr.	4.	56.
	931 tr.	25.	43.

uyt dese 931 tr. ghetrocken al de heele ronden dieder in sijn elck van 360 tr. comt twee ronden, die ick verlaet, ende rest 211 tr. welcke mette rest doen 211 tr. 25 ① 43 ②, doch verlatende de 43 ②, comt ten naesten voor den begheerden loop op den ghegheven tijt 211 tr. 26 ①.

T'BESLVYT. Wy hebben dan de Sonnens middelloop op een ghegeven tijt ghevonden, en daer af een tafel beschreven, na den eysch.

WAERSCHOVWING OPT MAECKSEL DER TAFEL.

Hier valt te bedencken dat de tafel van een dach vermeerderende, vereyscht ghemaeckt te worden altijt deur menichvulding of vergaring, ende niet deur deeling of aftrecking: Als by voorbeelt, my bekent geworden sijnde den loop van 18 jaren, doende 168 tr. 49. 52. 9. 9. 45. 0. soo ick daer me wil vinden den loop van 4 mael 18, dats van 72 jaren, ick menichvuldighe den voorschreven loop met 4, eñ comt (heele ronden verlaten) 315 tr. 9. 28. 36. 39. 0. 0. T'welck alsoo deur menichvulding wel gaet, ghelijck boven gheseyt is: Maer soo ons deur ander wech eerst bekent hadde geweest de selven loop van 72 jaren, ende datmen deur verkeerde wech der voorgaende, dats deelende dien loop 315 tr. 9. 28. 36. 39. 0. 0. deur 4, soude meenen te vinden den loop van 18 jaren, ten soude niet volgen, als blijckt, want sulck vierendeel den loop van 18 jaren niet uyt en brengt, ende dat om bekende oirsaken, te weten datmen al menichvuldighen-

USE OF THE TABLE.

Let it be required to find the Sun's proper motion in 879 Egyptian years 66 days and 2 hours. PROCEDURE. If in the table the 879 Egyptian years 66 days and 2 hours, with the mean motion thereof, were all found in one line, we should at once have the required mean motion, without having to add together any parts. But this not being so, we have to add different parts which together form the whole. To this end I first take from the table the years which are nearest to the required 879, namely 810, recording them with their mean motion only to seconds (this being sufficient for the matter, because the rest serves for the construction of the table, not for its use, as will be set forth more fully in the last part of the caution to be given hereafter about the construction of the table). And the form of the procedure will be as follows:

810 years 163° 4′12″

Now there are still 69 years short; for this I take (though one might also take them differently) 54 [1]) and 15. Thereafter 60 days and 6 days, and finally the two hours. When these are all placed in the right order below the above-mentioned 810 years and everything is properly added together, the form of the operation will be as shown below.

810 years	163° 4′ 12″
54 years	346° 52′ 17″ [2])
15 years	356° 21′ 11″
60 days	59° 8′ 17″
6 days	5° 54′ 50″
2 hours	0° 4′ 56″
	931° 25′ 43″

When from these 931° I take all the complete circles which are contained therein, each of 360°, I get two circles, which I discard, and the remainder, 211°, which together with the rest make 211°25′43″; but when I discard the 43″, I get approximately 211°26′ for the required motion in the given time.

CONCLUSION. We have thus found the Sun's mean motion in a given time, and described a table thereof; as required.

CAUTION ABOUT THE CONSTRUCTION OF THE TABLE.

It should be borne in mind that the table, progressing by days, requires to be made always by multiplication or addition, and not by division or subtraction. For example, when I have found the motion of 18 years, making 168;49,52,9,9,45,0 [3]), when I wish to find by means of this the motion of 4 times 18, that is of 72 years, I multiply the above-mentioned motion by 4, and this makes (whole circles being discarded) 315;19,28,36,39,0,0. Which is thus done correctly by multiplication, as has been said above. But if by other means this motion of 72 years had first been known to us, and if by the converse method to the above, *i.e.* dividing that motion of 315;19,28,36,39,0,0 by 4, we thought we might find the motion of 18 years, this would not follow, as is

[1]) For 45 in the Dutch text read 54.
[2]) For 146 in the Dutch text read 346.
[3]) This example has been taken from the tables of the moon (Book I, second chapter, p. 47 of the Dutch text).

dighende heele ronden verlaet, diemen al deelende daer by soude moeten doen, maer de selve onbekent sijnde en can niet bequamelick te wegheghebrocht worden. Tis oock openbaer waerom dattet werck deur de boveschreven deeling gheen hindernis en crijcht deur deeling des dach in uyren, ghelijckt oock en soude in de ghedeelten des tijts van 365 daghen, namentlick om dat daer in gheen heel ront en can wesen.

VOORT, anghesien wy van cleender ghedeelten dan ① als ② ③ &c. in dadelicke ervaringhen weynich of gheen sekerheyt en hebben, als t'sijnder plaets verclaert is, so mocht ymant dencken, waerom dese Sonloopen inde tafel cleender dan met ① beschreven sijn tot ⑥ toe, te weten waerom datmen niet en seyde den loop eens dachs te wesen alleenelick van 59 ①, achterlatende de rest: Ofte waerom datmen even tot ⑥ comt, ende niet tot noch cleender gedeeltē: De reden daer af can deur t'voorgaende openbaer sijn: Want na dien der ouden voornemen was, te beschrijven een tafel van 810 jaren, begrijpende 295650 dagen, ende dat de Sonloop der selve gevonden wort deur versaming van soo veel dagelicsche Sonloopen (want van elcke tijt des tafels haer loop deur reghel van drien te vinden, als eens gedaen is int vinden des loops van 1 dach, men soude alsoo wel een tafel meughen maken alleen tot ② toe, maer sulck maecksel soude veel te moeyelick vallen, als boven gheseyt is) daer uyt soude volghen datter inde Sonloop vande voorschreven 810 jaren, gemist soude sijn over de 295650 mael 8② (dieder boven de 59 ① sijn, bedragende over de 657 tr. Daerom de Sonloop der tafels in ① te einden en waer uyt oirsaeck van sulcx niet behoorlick. Ende om de selve reden salmen oock verstaen, dattet niet behoorlick en waer die te einden in ②, ③, of ④, want in ④ eindende, ende eenighe ⑤ daer an ghebrekende, ofte overschietende, het soude inde groote tijden eenige ② feyls connen geven, want 30 ⑤ genomen 295650 mael, maken over de 40②, welcke deur vergaring van verscheyden gedeelten, feyl van eenighe ① soude connen veroirsaken. Inder voughen dat in sulck ansien, de Sonloopen des tafels tot ⑤ souden moeten commen, om int gebruyck van dien sekerheyt van ① te hebben: Doch tot noch meerder gewisheyt, ende om Sonloopen te meughen berekenen op grooter tijden dan 810 jaren, so hebben de ouden, ghelijckt schijnt, tot ⑥ ghecommen, ende met goede reden tot gheen cleender ghedeelten, als onnoodich sijnde.

MERCKT.

Ghelijck hier een tafel gemaeckt is mette gevonden eygenloop der Son eens dachs; Alsoo sullen int volgende tafels gemaeckt worden mette gevonden loop eens dachs van d'ander dwaelders, ende haer Hemelen: Doch wy en sullen aldaer noch manier des maecksels, noch des ghebruycx van dien, beschrijven, als gelijck sijnde ande voorgaende. Men sal oock verstaen de boveschreven waerschouwing over de ghelijcke volghende tafelen ghemeen te wesen.

4 VOORSTEL.

Deur ervarings dachtafels te vinden de schijnbaer * duysteraerlangde van des Sonvvechs verstepunt en naestepunt *Longitudinē Zodiaci apogei & perigaei.*

Alsmen

apparent, because such fourth part does not give the motion of 18 years, such from well-known causes, to wit that, in multiplying, whole circles are discarded, which, in dividing, would have to be added thereto; but since they are unknown, this cannot be brought about adequately. It is also evident why the procedure of the above-mentioned division is not impaired by the division of the day into hours, just as it would not be in parts of the time of 365 days, namely because there cannot be a whole circle therein.

Further, as we have little or no certainty in practical experience of parts smaller than 1′, such as higher-order sexagesimals, as has been set forth in its proper place, one might think why these Sun's motions have been described in the table in parts smaller than 1′, down to sixth-order sexagesimals, to wit, why it was not said that the motion of one day was only 59′, discarding the rest. Or why we go precisely up to sixth-order units, and not to even smaller parts. The reason for this may be evident from what precedes. For as it was the intention of the Ancients to describe a table of 810 years, containing 295,650 days, and the Sun's motion therein is found by addition of as many daily Sun's motions (for by finding for each time of the table its motion by the rule of three, as has once been done in finding the motion of one day, one could thus make a table only up to 1″, but the construction of this would be much too difficult, as has been said above), it would follow therefrom that in the Sun's motion in the above-mentioned 810 years there would be an error of more than 295,650 times 8″ (which there are in addition to the 59′), which is more than 657°. On this account it would not be right to end the Sun's motion in the tables at 1′. And for the same reason it is also to be understood that it would not be right to end them at the second, third or fourth order of fractions, for if we stopped at the fourth and there should be some of the 5th order short or in excess, this might produce an error of some seconds in long periods, for if we take 295,650 times 30 fifth-order units, this makes more than 40″, which through the addition of different parts might cause an error of several times 1′. In such a way that in this respect the Sun's motions in the table would have to go up to the 5th order if we are to have a certainty of 1′ in using it. But for the sake of even greater certainty, and in order that they might be able to calculate Sun's motions in periods greater than 810 years, the Ancients apparently went up to the 6th order, and with good reason not to smaller parts, this being unnecessary.

NOTE.

Just as here a table has been made by means of one day's proper motion of the Sun as found, thus tables will be made hereafter by means of the motion of one day, as found for the other planets and their Heavens [1]). But we shall there describe neither the manner of making the table nor its use, since they are identical with the preceding case. It is also to be understood that the above-mentioned caution applies to the similar following tables as well.

4th PROPOSITION.

To find, by means of empirical ephemerides, the apparent ecliptical longitude of the apogee and perigee of the Sun's orbit.

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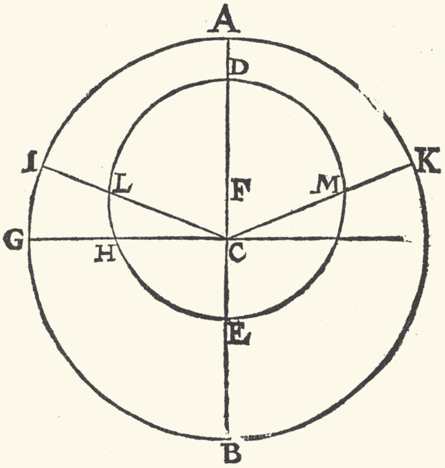
[1]) *Cf.* Third Book, Chapter 1; also *Introduction*, p. 11.

Alſmen de dachtafels van *Stadius* (die wy om de redenen des 1 voorſtels nemen al offe uyt ervaringhen beſchreven waren) wel deurſiet, men bevint onevenheyt inde ſchijnbaer Sonloop, te wetē op d'een tijt wel 3 of 4 ① ſdaechs meer als op d'ander, en daerbenevens dattet een jaer voor t'ander na, den loop ontrent het middel van Iunius altijt ten ſlapſten is, daghelicx van 57 ①. Als by voorbeelt op den 15 Iunius int jaer 1555, bevinde ick de Son onder den 92 tr. 39 ①, en op den 16 daer na onder dē 93 tr. 36 ①: Sulcx datſe op dien dach vanden 15 Iunius totten 16, geloopen heeft 57 ①: en ſo veel looptſe oock van yder 15 Iunius totten 16 op elck der volgende ende voorgaende jaren. Maer ontrent half December iſſe alle jare ten ſnelſten van ontrent 1 tr. 1 ①. Nu dit aldus jaerlicx ghebeurende, het gheeft den eerſten onderſouckers der oirſaken deſer dinghen billichlick vermoen, dat de Son in een rondt draeyt, diens middelpunt des Eertcloots middelpunt niet en is, maer daer buyten. Om t'welck deur een form breeder te verclaren, laet A B den duyſteraer ſijn, diens middelpunt dats dē Eertcloot C, en D E de Sonwech, diens middelpunt F, t'verſtepunt D, t'naeſtepunt E: Laet daer na van C totten duyſteraer an G, getrockē worden C G rechthouckich op A B, en ſniende de Sonwech in H. Dit ſoo weſende, men ſiet d'oirſaeck waerom de Son in haer wech loopende van D tot H meer dan eē vierendeel ronts, nochtans inden duyſteraer ſchijnbaerlick van A tot G alleenelick een vierendeelronts: Sghelijcx waerom ſy in haer wech eyghentlick loopende van H tot E min dan een vierendeelronts, nochtans inden duyſteraer ſchijnbaerlick van G tot B een volcommen vierendeelronts. Men ſiet oock de oirſaeck waerom de Son over even ſchijnbaer boghen als A G en G B oneven tijden loopt, te weten langer van D tot H, dats de ſchijnbaerloop A G, dan van H tot E, dats de ſchijnbaerloop G B. Maer in des ſchijnbaerloops twee half ronden A G B, B A, looptſe even ſoo lang als in des eyghen loops twee half ronden D H F, E D. Hier uyt is openbaer dat de Son ant verſtepunt D weſende, int middel van haer ſchijnbaer traechſte loop moet ſijn, en an t'naeſtepunt E int middel van haer ſchijnbaer ſnelſte. Maer om dat middel, t'welck t'begeerde deſes voorſtels is, op een ſeker voet te ſoucken, ick ſalder eerſt deſe verclaring af doen. Laet inden duyſteraer gheteyckent worden de twee punten I en K, alſoo dat de booch A I even ſy an A K: Daer na ſy ghetrocken van C tot I inden duyſteraer de lini C I, ſniende de Sonwech in L, ſghelijcx C K ſniende de Sonwech in M. Dit ſoo ſijnde, tis openbaer dat ghelijck de ſchijnbaer booch A I over d'een ſijde, even is mette ſchijnbaer booch A K over d'ander, alſoo is de Sonwechbooch D L over d'een ſijde, oock even mette Sonwech booch D M over d'ander: En vervolghens alle twee ſchijnbaerboghen die over beyde ſijden van



If we look through the ephemerides of *Stadius* (which for the reasons mentioned in the 1st proposition we use as if they had been described from experience) attentively, we find inequality in the Sun's apparent motion, to wit at one time as much as 3′ or 4′ a day more than at another, and moreover that year in and year out its motion is always slowest about the middle of June, being 57′ daily. Thus, for example, on 15th June of the year 1555 I find the Sun at 92°39′, and on the 16th after that at 93°36′, so that in that day, from 15th June to 16th, it moved 57′; and the same is also its motion from every 15th June to the 16th in each of the following and preceding years. But about the middle of December its motion is fastest every year, namely about 1°1′. Since this happens every year, it gives the first investigators of the causes of these things reasonable conjecture that the Sun revolves in a circle the centre of which is not the centre of the Earth, but outside it. In order to explain this more fully by means of a figure, let *AB* be the ecliptic whose centre is the Earth *C*, and *DE* the Sun's orbit, whose centre is *F*, its apogee *D*, its perigee *E*. Thereafter let there be drawn from *C* to the ecliptic, at *G*, *CG* at right angles to *AB* and intersecting the Sun's orbit in *H*. This being so, we see the cause why the Sun in its orbit, while moving from *D* to *H* more than a quarter circle, yet in the ecliptic appears to move from *A* to *G* only a quarter circle. Likewise why, moving in reality in its orbit from *H* to *E* less than a quarter circle, yet in the ecliptic it appears to move from *G* to *B* a complete quarter circle. We also see the cause why the Sun passes through equal apparent arcs such as *AG* and *GB* in unequal times, to wit, longer from *D* to *H*, *i.e.* the apparent motion *AG*, than from *H* to *E*, *i.e.* the apparent motion *GB*. But through the two semi-circles *AGB*, *BA* of the apparent motion it moves just as long as through the two semi-circles *DHE*, *ED* of the proper motion. From this it is evident that the Sun, when it is at the apogee *D*, must be in the middle of its apparently slowest motion, and at the perigee *E* in the middle of its apparently fastest motion. But in order to seek this middle — which is the thing required in this proposition — on a sure basis, I will first give the following explanation of it. Let there be marked in the ecliptic the two points *I* and *K*, in such a way that the arc *AI* shall be equal to *AK*. Thereafter there shall be drawn from *C* to *I* in the ecliptic the line *CI*, intersecting the Sun's orbit in *L*; likewise *CK*, intersecting the Sun's orbit in *M*. This being so, it is evident that just as the apparent arc *AI* on the one side is equal to the apparent arc *AK* on the other side, the arc of the Sun's orbit *DL* on the one side is also equal to the arc of the Sun's orbit *DM* on the other side. And consequently the two apparent arcs which are equally large den van A evegroot sijn, gheven oock twee Sonwechbogen over beyde sijden vā D evegroot. Hier uyt volght dat alsmen inde ervarings dachtafels een schijnbaer plaets sulcx vint, dat haer twee loopen op even tijden over beyde sijden evegroot sijn, de selve plaets de Sonnens schijnbaer verstepunt moet wesen.

Het soucken van dien gaet aldus toe: Ick verkies eenighe maent van Iunius (daer in men den schijnbaerloop over al ten traechsten vindt) als neem ick Iunius int jaer 1554, ondersouck daer me het voornemen op eenighen dach die by raming ontrent het middel des traechsten loops der Son is, ick neem op dē 10, alwaer de Son bevonden wort onder den 88 tr. 2 ①: Maer binnē drie maenden daer te vooren (welcke drie maenden, of tijt waer me de Son ontrent een vierendeel haers wechs loopt, ick liever neem als ander, om de redenen die daer af int volgende Merck verclaert sullen worden) te weten den 10 Maerte, wasse onder den 359 tr. doende die booch 89 tr. 2 ①: En binnen drie maenden na den 10 Iunius, dats totten 10 September, alwaer de Son is onder den 176 tr. 39 ①, wort sulcken booch bevonden van 88 tr. 37 ①, welcke niet even sijnde mette 89 tr. 2 ①, soo en was onder den 88 tr. 2 ① het verstepunt niet.

Daerom dit alsoo ondersocht op een ander dach, ick neem op den 15 Iunius, alwaer de Son bevonden wort onder den 92 tr. 49 ①, ick vinde de booch op drie maenden daer te vooren van 88 tr. 53 ①, en op drie maenden daer na van 88 tr. 46 ①, welcke twee noch niet even sijnde, en de laetste booch cleender dan d'eerste, ghelijck inde voorgaende ondersoucking, tis teycken datmen noch voorder in Iunius moet commen: Latet ondersocht worden op den 19 dach, alwaer ick de Son vinde onder dē 96 tr. 39 ①, en den booch op drie maenden daer te voor van 88 tr. 46 ①, maer op drie maendē daer na van 88 tr. 51 ①, welcke twee noch niet even sijnde, en de laetste nu grooter dan d'eerste, tis teycken dat ick te verre in Iunius gecommen ben, en deysen moet: Daerom ondersouck ick dergelijcke op dē 16, alwaer ick de Son bevinde onder dē 93 tr. 46 ①, en de booch op d'eerste drie maenden van 88 tr. 51 ①, maer op de laetste drie van 88 tr. 48 ①: Twelck noch niet effen commende, ick versouck dergelijcke op dē 17 van Iunius, en bevinde dan de Son indē 94 tr. 43 ①, en d'eerste booch op drie maenden van 88 tr. 48 ①, de laetste van 88 tr. 50 ①.

Sulcx dat ick tot hier toe bevonden heb t'verstepunt te moeten wesen tusschen den 93 tr. 46 ①, en den 94 tr. 43 ①. Maer om nu noch naerder te commen, ick sie dat de laetste booch op den 16 van Iunius te cleen bevondē wiert, en op den 17 te groot, t'welck teycken is, t'begheerde tusschen beyden te moeten schuylen, te weten op den 16 Iunius met eenighe uyren: Daerom dit ondersouckende eerst neem ick op den 16 Iunius met noch 12 uyren, daer na met meer, ick vinde mette 16 uyren de Son te wesen onder den 94 tr. 24 ①, en d'eerste booch op drie maenden van 88 tr. 49 ①, maer de laetste van 88 tr. 49 ① 20 ②, t'welck weynich verschilt, doch diet noch nauwer begeerde, mocht met gedeelten van uyren wercken, Dit soo sijnde, ick houde den boveschreven 94 tr. 24 ① voor de begheerde schijnbaer duysteraerlangde des verstepunts: Waer toe vergaert 180 tr. comt voor begheerde schijnbaer duysteraerlangde des naestepunts 274 tr. 24 ①. Merckt dat *Copernicus* dit verste punt t'sijnder tijt stelde onder den 96 tr. 40 ①: Doch dit comt aldus na t'inhoudt deser dachtafels op die manier ghewrocht.

Maer beneffens de selve, soo isser noch een ander die tot proef van d'eerste verstrecken can, welcke ick oock verclaren sal als volght:

Hier vooren is geseyt de Son schijnbaerlick even soo lang te loopen int een halfront A G B, als int ander B A, waer uyt volght, dat alsmen inde dachtafels on either side of *A* also give two arcs of the Sun's orbit which are equally large on either side of *D.* From this it follows that if we find in the empirical ephemerides an apparent point such that the motions on both sides of it in equal times are equally large, this point must be the Sun's apparent apogee.

The seeking of this takes place as follows. I choose some month of June (where the apparent motion is always found to be slowest), for example June of the year 1554, investigate therewith the object in view on some day which is estimated to be near the middle of the slowest motion of the Sun, for example on the 10th, when the Sun is found at 88°2′. But three months previously (which three months, or time in which the Sun moves about one-fourth of its orbit, I take rather than any other, for the reasons that will be set forth about it in the succeeding Note), to wit 10th March, it was at 359°, that arc being 89°2′. And three months after 10th June, *i.e.* on 10th September, when the Sun is at 176°39′, this arc is found to be 88°37′; and this not being equal to 89°2′, the apogee was not at 88°2′.

Therefore, when this is examined on another day, for example, 15th June, when the Sun is found to be at 92°49′, I find the arc three months previously to be 88°53′ and three months afterwards 88°46′, and these two not yet being equal and the last arc smaller than the first, as in the preceding investigation, this is a sign that we have to be further still in June. Let it be examined on the 19th day, where I find the Sun at 96°39′, and the arc three months previously 88°46′, but three months afterwards, 88°51′; and these two still not being equal, and the last now larger than the first, this is a sign that I have come too far in June and have to go back. I therefore examine the same thing on the 16th, where I find the Sun at 93°46′, and the arc three months previously, 88°51′, but three months afterwards, 88°48′, and this still not being equal, I examine the same thing on 17th June, and then find the Sun at 94°43′, and the first arc three months previously 88°48′, and the last 88°50′.

Thus I have hitherto found that the apogee must be between 93°46′ and 94°43′. But in order to come nearer still, I see that the last arc on 16th June was found to be too small and on the 17th too large, which is a sign that the required point must be between the two, to wit on 16th June with a few hours added. Therefore, investigating this, I first take 16th June with 12 hours in addition, thereafter with more; I find that with 16 hours the Sun is at 94°24′, and the first arc three months previously 88°49′, but the last 88°49′20″, which is a slight difference, but who should wish it even closer might operate with parts of hours. This being so, I take the above-mentioned 94°24′ to be the required apparent ecliptical longitude of the apogee. When to this is added 180°, we get for the required apparent ecliptical longitude of the perigee 274°24′ (Note that *Copernicus* in his day put this apogee at 96°40′). But this results from the data of these ephemerides, made in the said manner.

But in addition to this there is still another manner that may serve as proof of it; which I will also set forth as follows:

It has been stated above that the Sun apparently moves as long in the one semi-circle *AGB* as in the other *BA,* from which it follows that when we have found in the ephemerides two apparent places of the Sun which are 180° apart, twee ſchijnbaer Sonplaetſen ghevonden heeft 180 tr. van malcander, ſulcx dat de Son over elck halfront eveveel tijts geloopen heeft, d'een dier twee plaetſen t'ſchijnbaer verſtepunt d'ander t'ſchijnbaer naeſtepunt te moeten weſen. Om deur ſulcke wech t'verſtepunt te vinden, ick vergaer 180 tr. totten voorſchreven 94 tr. 24 ① des ſchijnbaer verſtepunts, comt gelijck bovẽ geſeyt is 274 tr. 24 ① voor ſchijnbaer naeſtepunt: Ick ſouck daer na inde dachtafels wanneer tot die plaets de ſchijnbaer Son was, en bevinde int ſelve jaer 1554 in December dẽ 16 dach 8 uyr 16 ①: By aldiẽ nu den tijt des Sonloops dier 180 tr. te weten vandẽ 16 Iunius 16 uyr, tottẽ 16 December 8 uyr 16 ①, even waer ande Sonloop des ander halfronts: Of anders geſeyt dat den ſelven loop duerde den helft des jaers doende 182 daghen 14 uyren 58 ①, ſoo ſoude d'eerſtghevonden plaets des verſtepunts de begheerde weſen: Maer vanden boveſchreven 16 Iunius 16 uyr totten 16 December 8 uyr 16 ①, ſijn 182 daghen 16 uyr 16 ①, t'welck alleenelick 1 uyr 18 ① te veel ſijnde, ſoo en iſt niet verre van daer. Doch om t'begeerde naerder te commen, ick verſouck dergheliјcke tot een ander plaets, nemende t'verſtepunt te ſijn daer de Son ſchijnbaerlick was op den 18 Iunius, te weten inden 95 tr. 41 ①, en volghende daer me de boveſchreven manier van wercking, vinde over den loop des halfronts o uyr 25 ① te luttel, ſulcx dat na deſe wijſe t'verſtepunt moet weſen tuſſchen den 94 tr. 24 ①, en den 95 tr. 41 ①: Maer om de ſaeck noch naerder te commen, ick verſouck dergheliјcke tot een plaets tuſſchen de twee voorſchreven, ick neem op den 17 dach 13 uyren van Iunius, op welcken tijt ick inde dachtafels merck de Son geweeſt te hebben onder den 95 tr. 14 ①, en volghende daer me de boveſchreven manier van werking, vinde den loop des halfronts van o uyr 2 ① te cleen, t'welck als voor even meughende genomen worden, ſoo ſoude na deſe wijſe t'verſtepunt ſijn onder den 95 tr. 14 ①, dat nu d'ander bevonden wiert onder den 94 tr. 24, ① die alleenelick verſchillen 50 ①, weſende in deſen gevalle van cleenderacht, en connende ſpruyten uyt der dachtafels onvolcommentheyt.

VERVOLGH.

Ghelijck hier de twee boghen elck van drie maenden loops evegroot vallen, alſoo moeten om de voorgaende redenen (de dachtafels wel ſijnde) alle ander ſoodanighe twee boghen op even tijden gheloopen oock evegroot ſijn, als by voorbeelt op 1 maent doende 31 dagen voor en na den 17 Iunius 1554, was de Son bycans eveverre vant verſtepunt, te weten over d'een ſijde 29 tr. 37 ①, over d'ander 29 tr. 39 ①. Ende op twee maenden van 61 daghen waſſe over d'een ſijde 58 tr. 31 ①, over d'ander 58 tr. 33 ①. Angaende dit verſchilken van 2 ①, dat ſpruyt openbaerlick uyt onvolcommenheyt der tafels. Maer tot ander plaetſen can ſulck verſchil groot ghevonden worden: Als by voorbeelt op den 14 September 1554, was de Son onder den 180 tr. 36 ①, en op 3 maenden daer te vooren, hadſe 3 tr. 5 ① ſchijnbaerlick min geloopen dan op 3 maenden daer na, want op den 14 Iunius, te weten daer te vooren 3 maenden doende 92 daghen, waſſe onder den 91 tr. 51 ①: Maer 92 daghen na den 14 September, te weten op den 15 December waſſe onder den 273 tr. 2 ①: Deerſte ſchijnbaerloop doende 89 tr. 21 ①, is als vooren 3 tr. 5 ① cleender dan de tweede, doende 92 tr. 26 ①.

MERCKT.

Hier boven is gheſeyt, dat ick verclaren ſoude de reden waerom het nemen des tijts waer me de Son een vierendeelronts van t'verſtepunt comt, ſekerder

beſluyt

in such a way that the Sun has passed through each semi-circle in the same time, one of these two places must be the apparent apogee, the other the apparent perigee. In order to find the apogee in this way, I add 180° to the above-mentioned 94°24′ of the apparent apogee, which makes, as said above, 274°24′ for the apparent perigee. I then look up in the ephemerides when the apparent Sun was in that place and find this to be in the said year 1554 in December, the 16th day, at 8h 16m. If the time of the Sun's motion through those 180°, to wit from 16th June at 16h to 16th December at 8h 16m, were equal to the Sun's motion of the other semi-circle; or in other words: if the said motion took half a year, making 182d 14h 58m, the place of the apogee first found would be the one required. But from the above-mentioned 16th June at 16h to 16th December at 8h 16m there are 182d 16h 16m, which, being only 1h 18m too much, is not far amiss. But in order to come nearer to the required value, I examine the same thing in another place, taking the apogee to be where the Sun was apparently on 18th June, to wit at 95°41′, and carrying out therewith the above method of operation, I find the interval over the semi-circle to be 0h 25 m short, so that the apogee must be between 94°24′ and 95°41′. But in order to come nearer still to the matter, I examine the same thing in a place between the two mentioned above, for example, on 17th June at 13h, at which time I find in the ephemerides that the Sun was at 95°14′, and carrying out therewith the above method of operation, I find the interval over the semi-circle to be 0h 2m short, and since this can be considered equal, in this way the apogee would be at 95°14′, so that, since the other was found to be at 94°24′, they differ only by 50′, which in this case is insignificant and may be due to the imperfection of the ephemerides.

SEQUEL.

Just as here the two arcs, each of the motion of three months, are equal, for the above reasons (the ephemerides being correct) all other two such arcs passed through in equal times must also be equal; for example, one month — making 31 days — before and after 17th June 1554 the Sun was almost the same distance from the apogee, to wit, on the one side 29°37′, on the other side 29°39′. And at two months — 61 days — (before and after the said date) it was on the one side 58°31′, on the other 58°33′. As to this small difference of 2′, this is evidently due to the imperfection of the ephemerides. But in other places this difference may be found to be large. For example, on 14th September 1554 the Sun was at 180°36′, and 3 months before it had moved apparently 3°5′ less than 3 months after, for on 14th June, to wit, 3 months — making 92 days — before, it was at 91°51′ [1]). But 92 days after 14th September, to wit, on 15th December, it was at 273°2′. The first apparent motion, being 89°21′, as before is 3°5′ less than the second, being 92°26′.

NOTE.

Above it has been said that I would set forth the reason why a more certain conclusion is gained by taking the time in which the Sun has reached a point

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[1]) Instead of 91°51′, a value of 91°15′ has been used in the subsequent computations.

besluyt gheeft als ander tijt waer me sy naerder of verder vant verstepunt is: Om nu daer toe te commen ick segh by voorbeelt aldus: Op den 12 Maerte 1555, was de Son onder den 0 tr. 45 ①, en drie maenden daer te vooren, te weten 90 dagen (tis wel waer dat d'alder meeste sekerheyt het recht vierendeel jaers soude sijn, doch t'voorneme̅ sal hier me genouch conne̅ verclaert worde̅) hadse 3 tr. 7 ① schijnbaerlick meer gheloopen, dan op 3 maenden daer na. Dit groot verschil ons meer versekerende dan cleen dat den 180 tr. 36 ① wijt vant schijnbaer verstepunt is, wort bemerct deur de boveschreven neming des tijts waer me de Son een vierendeelronts loopt: Want soo wy, by voorbeelt geseyt, maer ghenomen en hadden 2 daghen voor en na den 14 September, wy souden den loop sulcker twee boghen evegroot bevinden, elck van 1 tr. 58 ①, sonder kennis of wy ant verstepunt waren of niet. Maer dattet nemen van twee even tijden waer me de Son verder dan een vierendeelronts vant verstepunt comt, ooc achterlick sijn, wort daer deur verstaen, datmen daer me comt tot twee cleene boochskens niet wijt van t'naestepunt, waer af de reden de selve is als vant verstepunt.

Noch is te weten, dat al hebben wy hier boven gheseyt van t'nemen des tijts waer me de Son een vierendeelronts vant verstepunt is, daer by verstaetmen sulcx niet alleenelick te wesen het vierendeel jaers als boven ghenomen wiert, maer alle tijden die de Son daer brengen, als een of meer heele jaren met noch een vierendeel daer toe: Oock drie vierendeelen jaers alleen, of met een of ettelicke heele jare̅ daer toe, op alle welcke de Son een vierendeelronts van t'verstepunt comt, uytghenomen soo veel als t'verstepunt daerentusschen mocht verloopen sijn, t'welck op weynich jaren van gheender acht en is.

Noch staet te ghedencken dat t'ghene hier in dit merck geseyt is de Sonloop angaende, dergelijcke oock plaets te houden met d'ander Dwaelders daer int volghende afghehandelt sal worden, wantmen daer int soucken van haer verstepunten, om de boveschreven redenen oock de meeste sekerheyt heeft mettet nemen des tijts waer me den Dwaelder een vierendeelronts vant verstepunt comt.

TBESLVYT. Wy hebben dan deur ervarings dachtafels ghevonden de schijnbaer duysteraerlangde vande Sonwechs verstepunt en naestepunt, na den eysch

5 VOORSTEL.

Deur ervarings dachtafels den loop vande Sonvvechs ⋆ verstepunt te vinden.

Apogæum.

Eerst gesocht hebbende gelijck int 4 voorstel de schijnbaer duysteraerlangde van des Sonwechs verstepunt, en die int jaer 1554 bevonden inden 94 tr. 24 ①, ick souck op de selve wijse waer die op eenich volghende jaer, ick neem 1594 gheweest heeft, en bevindese inden 97 tr. 53 ①, waer onder de Son bevonden wiert op den 20 Iunius, want drie maenden daer te vooren wasse onder den 9 tr. en drie maenden daer na onder den 186 tr. 46 ①, wiens twee boghen even sijn doende elck 88 tr. 53 ①. Sulcx dat vant jaer 1554, tot 1594, makende 40 jaren, so is t'verstepunt op dien tijt verloope̅ volgende dese dachtafels en voor t'begeerde 3 tr. 29 ①, want so verre ist vanden 94 tr. 24 ① totten 97 tr. 53 ①. En hier me wort bekent des verstepunts loop op alle gheghevẽ tijt: Als a quarter circle away from the apogee than by taking another time in which it is nearer to or further away from the apogee. To arrive at this explanation, I say, for example, as follows: On 12th March 1555 the Sun was at 0°45′, and in three months before this, to wit, 90 days (it is true indeed that the greatest certainty would be the exact quarter of a year, but the intention can be sufficiently set forth in this way), it had apparently moved 3°7′ more than in three months after. That this large difference gives us greater certainty than the small one does, about this longitude of 180°36′ being far away from the apparent apogee, is perceived from the above-mentioned ascertaining of the time in which the Sun moves through a quarter circle. For if, for example, we had taken only 2 days before and after 14th September, we should find the motion of two such arcs to be equal, each 1°58′, without knowing whether we were at the apogee or not. But that the taking of two equal times in which the Sun reaches two points further than a quarter circle away from the apogee would also be less accurate, is understood from the fact that thus we get two small arcs not far away from the perigee, the cause of which is the same as with the apogee.

It is also to be noted that, though we have spoken above of the taking of the time in which the Sun has reached a point a quarter circle away from the apogee, it should be understood that this is not only the quarter of a year as was taken above, but all those times which bring the Sun there, *e.g.* one or more whole years with a quarter added. Also three quarters of a year alone, or with one or several whole years added, at all of which times the Sun reaches a point a quarter circle away from the apogee, except for the amount the apogee should meanwhile have moved, which is of no significance in a few years.

It is also to be borne in mind that what has been said in this note with regard to the Sun's motion also applies to the other Planets which will be dealt with hereafter, for there, in seeking their apogees, the greatest certainty is for the above-mentioned reasons also reached by taking the time in which the Planet has reached a point a quarter circle away from the apogee.

CONCLUSION. We have thus found, by means of empirical ephemerides, the apparent ecliptical longitude of the apogee and perigee of the Sun's orbit; as required.

5th PROPOSITION.

To find, by means of empirical ephemerides, the motion of the apogee of the Sun's orbit.

First having sought, as in the 4th proposition, the apparent ecliptical longitude of the apogee of the Sun's orbit, and having found it in the year 1554 at 94°24′, I look up in the same way where it was in some later year — for example, 1594 — and find it at 97°53′, at which the Sun was found to be on 20th June, for three months before it was at 9° and three months after at 186°46′, which two arcs are equal, since they are each 88°53′. Thus, according to these ephemerides, from the year 1554 to 1594, making 40 years, the apogee in that time (as required) has moved 3°29′, for that is the distance from 94°24′ to 97°53′. And in this way the motion of the apogee in any given time becomes known. For example, in by voorbeelt om die te hebben op een jaer,ick fegh 40 jaren gheven 3 tr.29 ①, wat een jaer ? comt 5 ① 13 ②.

Merckt dat int berekenen defer dachtafels heeft moeten ghemift fijn, doch het voorbeelt can dienen om te betoonen de wijfe hoemen met beter ervaringhen doen fal,maer om de faeck naerder te commen, men foude d'eerfte ervaring meughen nemen over veel langher tijt gefchiet,als die *Ptolemeus* befchrijft int 4 hooftftuck fijns 3 boucx 463 jaren na t'overlijden vanden grootẽ *Alexander*,alwaer hy feght des Sonwechs verftepunt ghevonden te hebben onder des duyfteraers 65 tr. 30 ① (en hoe wel ick volghende fijn gheftelde vinde 65 tr. 30 ①,doch latet voorbeeltfche wijfe fijn foo hy feght) Voor d'ander ervaring ghenomen den bovefchreven 97 tr. 53 ① ghebeurt op den 20 Iunius 1594, t'welck was 1918 jaren na *Alexander*, foo heeft het verftepunt van d'een tijt tot d'ander bedraghende ontrent 1455 jaren,gheloopen 32 tr. 23 ①, want foo verre ift vanden 65 tr. 30 ① totten 97 tr. 53 ①: En alfmen hier me wil vinden den loop eens ghegheven tijts,ick neem eens jaers, men feght 1455 jaren gheven 32 tr. 23 ①, wat 1 jaer ? comt 1 ① 20 ②: En dit laetfte getal (in plaets vant eerfte 5 ① 13 ②) fal ick ghebruycken in voorbeelden daer van defen loop gehandelt wort.

TBESLVYT. Wy hebben dan deur ervarings dachtafels den loop vande Sonwechs verftepunt ghevonden, na den eyfch.

6 VOORSTEL.

Deur ervarings dachtafels de Sonnens loop in haer vvech te vinden.

Dẽ loop vande Sonwechs verftepunt wort op een jaer bevonden deur het 5 voorftel van	1 ① 20.②.
Die ghetrocken vande middelloop der Son oock op een jaer, te weten een Egips doende deur het 3 voorftel	359 tr.45 ①.
Blijft voor begheerde loop der Son in haer wech op een Egips jaer	359 tr.43 ① 40.②.

Ende is openbaer dat alfoo ghevonden fal worden den loop van alle voorgeftelden tijt.

TBEWYS. Angefien de Son op een jaer inden duyfteraer fchijnbaerlick een keer doet, en dat daerentuffchen haer wech felf 1 ① 20 ② geloopen heeft, foo moetfe in haer wech die 1 ① 20 ② min gheloopen hebben.

TBESLVYT. Wy hebben dan deur ervarings dachtafels de Sonnens loop in haer wech ghevonden, na den eyfch.

7 VOORSTEL.

Deur ervarings dachtafels de Sonvvechs afvvijcking vanden * evenaer te vinden.

Aequatore.

De Sonnens dagelickfche fchijnbaer duyfteraerbreeden, en worden in defe dachtafels nevens haer fchijnbaer duyfteraerlangden niet befchreven, maer in fommige dachtafels deur gemeene reghel met een tafel gevonden: Doch by aldienfer ftonden, ghelijckfe in ware ervarings dachtafels fijn (t'welckmen leerings halven fich mach inbeelden foo te wefen) men foude die grootfte breeden order to have that in one year, I say: 40 years give 3°29′; what does one year give? It gives 5′13″.

Note that in the calculation of these ephemerides errors must have been made, but yet the example may serve to show the way in which to proceed with better experiences. But in order to get nearer to the matter, we might take the first experience a much longer time ago, such as *Ptolemy* describes in the 4th chapter of his 3rd book, 463 years after the death of the great *Alexander,* where he says he has found the apogee of the Sun's orbit at 65°30′ of the ecliptic (and though, following his supposition, I find 65°30′, let it be as he says, by way of example[1]). When for the other experience the above-mentioned 97°53′ is taken, which happens on 20th June 1594, which was 1918 years after *Alexander,* the apogee has moved 32°23′ from one time to the other — amounting to about 1455 years — for this is the distance from 65°30′ to 97°53′. And if from this we wish to find the motion in a given time — for example, in one year — we say: 1455 years give 32°23′; what does 1 year give? It gives 1′20″. And it is this latter value (instead of the first 5′13″) which I shall use in examples in which this motion is dealt with.

CONCLUSION. We have thus found, by means of empirical ephemerides, the motion of the apogee of the Sun's orbit; as required.

6th PROPOSITION.

To find, by means of empirical ephemerides, the Sun's motion in its orbit.

The motion of the apogee of the Sun's orbit is found to be in one year, by the 5th proposition,	1′20″
This being subtracted from the mean motion of the Sun, also in one year, to wit an Egyptian year, making by the 3rd proposition	359°45′
There remains, for the required motion of the Sun in its orbit in an Egyptian year	359°43′40“

And it is evident that thus the motion in any suggested time will be found.

PROOF. Since in one year the Sun apparently performs one revolution in the ecliptic, while its orbit itself has meanwhile moved 1′20″, it must have moved those 1′20″ less in its orbit.

CONCLUSION. We have thus found, by means of empirical ephemerides, the Sun's motion in its orbit; as required.

7th PROPOSITION.

To find, by means of empirical ephemerides, the deviation of the Sun's orbit from the equator.

The Sun's daily apparent equatorial latitudes[2] are not described in these ephemerides in addition to its apparent ecliptical longitudes, but in some ephemerides are found by a common rule with a table. But if they were there, as they are in true empirical ephemerides (which we may imagine to be the case for the sake of instruction), the greatest latitudes would be found twice a year,

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[1]) The sense of the passage in parentheses is not clear; there must be a clerical error in it. The quotation refers to *Syntaxis* III, 4 (Manitius, I, p. 167).

[2]) Stevin's *duysteraerbreeden* (ecliptical latitudes) is an error for *evenaerbreeden.*

den jaerlicx tweemael vinden, d'eene ontrent den 12 van Iunius na t'Noorden, d'ander ontrent den 12 van December na t'Zuyden, en dat in dese dachtafels volghende de *Copernicus* stelling van 23 tr. 28 ①. En datmen hier op noch vorder lette, men soude mercken sulcx altijts te gebeuren wesende de Son schijnbaerlick 90 tr. vanden Lentine, waer uyt men besluyt de Sonwechs afwijcking te wesen vande selve 23 tr. 28 ①, om dat sulcke breetheyts booch ghetrocken op de lanckheyts booch doende een vierendeelronts, voor grootheyt haers tegenoverhoucx verstreckt, als blijckt inden handel der clootsche drichoucken.

TBESLVYT. Wy hebben dan deur ervarings dachtafels de Sonwechs afwijcking vanden evenaer ghevonden, na den eysch.

8 VOORSTEL.

Deur ervarings dachtafels te makē berekende * dachtafels des Sonloops van toecommende tijden.

* *Ephemerides.*

Alsmen in *Stadius* dachtafels (die wy om de boveschreven redenē hier houden al offe deur ervaringhen ghevonden waren) siet na t'ghelijck vervolgh der schijnbaer Sonplaetsen van d'een tijt by d'ander verlekē, om deur de Sonplaetsen des voorleden tijts, te oordeelen vande Sonplaetsen des toecommenden, men merckt datmen overal van vier tot vier jaren, de Son op ghelijcke daghen bycans onder een selve duysteraerlangde vint. Als by voorbeelt int jaer 1554 den 1 van Ianuarius was de Son onder den 290 tr. 36 ①. En vier jaer daer na, te weten den 1 Ianuarius 1558 onder den 290 tr. 37 ①. Voort inde jaren 1562, 1566, 1570, 1574, 1578, telcken op den 1 Ianuarius wiertse bevonden onder dē 290 tr. 37 ①, 290 tr. 36 ①, 290 tr. 37 ①, 290 tr. 37 ①, 290 tr. 38 ①, en alsoo met ander dierghelijcke.

Hier uyt machmen besluyten, dat by aldien de dachtafels niet voorder gageslagen waren dan neem ick tottē 1 van Ianuarius 1578, en datmē na dien voorleden tijt wilde maken dachtafels van toecommenden tijt, mē soude de 4 volghende jaren meughen maken met sulcken voortganck, als de vier voorgaende, seggende de Son op den 2 van Ianuarius int Iaer 1574 te sullen sijn, tot sulcken schijnbaer plaets alsse was den 2 Ianuarius int jaer 1570, te weten onder den 291 tr. 38 ①: Maer op dē 3 Ianuarius onder den 292 tr. 40 ①, en so voort, welcke dachtafelsche wijse opgeteyckent, men crijcht berekende dachtafels des Sonloops van toecommende tijden, na t'begheerde.

1 MERCK.

By aldien de lancheyt des natuerlic jaers effen waer vā 365 dagē 6 uyren, gelijck het Iuliaens jaer inhout, daer en soude gantschelick geen verandering vallen van 4 tot 4 jaren: Maer t'wiert in dese dachtafels deur het 2 voorstel bevonden alleenelick van 365 dagen 5 uyren 45 ① 55 ②, daerom gebreecter alle jare 14 ① 5 ② van een uyr, op welcke de Sonloop bedraecht nagenouch 35 ②, die doen te vier jaren 2 ① 20 ②, en so veel soudemen na die rekening alle vier jarē tot yder dach der voorgaende moeten toedoen. Maer de langde des jaers genomē na *Ptolemeus* rekening op 365 dagen 14 ① 48 ②, waer an totte 356 $\frac{1}{4}$ dagē alleenelick gebreken 12 ② eens daechs, op welcke de Sonloop bedraecht nagenouch oock 12 ②, die doen te vier jaren 48 ②, en so veel soudmē na *Ptolemeus* rekeninge des jaers, alle vier jaren tot yder dach der voorgaende moetē toedoē.

one about 12th June towards the North, the other about 12th December towards the South, and such in these ephemerides according to *Copernicus'* assumption of 23°28′. And if we observed this further, we should perceive that this always happens when the Sun is apparently 90° from the Vernal Equinox, from which we conclude that the deviation of the Sun's orbit is the said 23°28′, because the arc of the circle of latitude at 90° longitude represents the magnitude of its opposite angle, as appears from the work on spherical triangles.

CONCLUSION. We have thus found, by means of empirical ephemerides, the deviation of the Sun's orbit from the equator; as required.

8th PROPOSITION.

To make, by means of empirical ephemerides, calculated ephemerides of the Sun's motion in future times.

If in *Stadius'* ephemerides (which for the above-mentioned reasons we here use as if they had been found by experience) we look at the similar sequence of the apparent positions of the Sun of one time as compared with another, in order to judge from the positions of the Sun in the past what will be the positions of the Sun in the future, we perceive that every four years the Sun is found on similar days almost at the same ecliptical longitude. Thus, for example, in the year 1554, the 1st of January, the Sun was at 290°36′. And four years later, to wit, on 1st January 1558, it was at 290°37′. Further in the years 1562, 1566, 1570, 1574, 1578, each time on 1st January, it was found at 290°37′, 290°36′, 290°37′, 290°37′, 290°38′, and the same in other similar years.

From this it may be concluded that if the ephemerides did not contain observations beyond, for example, 1st January 1578, and if from this past time it was desired to make ephemerides of the future, the 4 following years could be made by the same procedure as the four preceding years, saying that the Sun would be on 2nd January of the year 1574 in the same apparent place as it was on 2nd January of the year 1570, to wit, at 291°38′. But on 3rd January at 292°40′, and so on; and when these values are recorded in the manner of ephemerides, we get calculated ephemerides of the Sun's motion in future times; as required.

1st NOTE.

If the length of the natural year were exactly 365d 6h, as the Julian year comprises, there would be no change at all every four years. But in these ephemerides it was found by the 2nd proposition to be only 365d 5h 45m 55s; therefore every year the hour is short of 14m 5s, in which the Sun's motion amounts to almost 35″, which makes in four years 2′20″, and this amount according to this calculation we should have to add every four years on every day to the preceding value (of the Sun's longitude). But if the length of the year were taken according to *Ptolemy's* calculation to be 365;14,48 [1]), where in 365 ¼ days a day is short of only $12 \times \frac{1}{3600}$ of a day, in which the Sun's motion also amounts to almost 12, in four years this makes 48, and this amount would according to *Ptolemy's* calculation of the year have to be added every four years to the longitudes of the preceding years.

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[1]) Ptolemy here uses sexagesimals of a day: $0;0,12 = \frac{12}{3600} = 4\,\text{m}\,48\text{s}$.

2· MERCK.

Merckt wijder datmen hier siet de oirsaeck van t'verloop des tijts, te weten waerom den 1 Maerte die nu int voorjaer comt, met lancheyt van tijt inde somer soude vallen, daer na inden Herbst, en so voorts, ten waer dat somwijlen voorcommen wierde met afcorting van daghen ghelijck int jaer 1582 met 10 daghen ghedaen is, want volgende de rekening der lanckheyt des jaers van *Ptolemeus*, soo beloopet op de 300 jaren een dach, t'welck na de rekening des jaers van anderen meer bedraecht.

3 MERCK.

Soo yemant de dachtafels van *Stadius*, voorder ondersocht op de voet als boven, hy soude faute bevinden: Laet by voorbeelt hier ghenomen worden al de Sonplaetsen op den 1 Iauuarius van 4 tot 4 jaren, als hier na volght:

1554.	290	tr.	36 ①
1558.	290	tr.	37 ①
1562.	290	tr.	37 ①
1566.	290	tr.	36 ①
1570.	290	tr.	37 ①
1574.	290	tr.	37 ①
1578.	290	tr.	38 ①
1582.	290	tr.	39 ①
1586.	290	tr.	39 ①
1590.	290	tr.	40 ①
1594.	290	tr.	40 ①
1598.	291	tr.	31 ①
1602.	291	tr.	32 ①
1606.	291	tr.	7 ①

Alwaermen een tamelick vervolgh siet tot opt jaer 1594 (doch niet ghenouch vermeerderende) maer van daer voort opt jaer 1598, is by de 51 ① onbehoorlick verschil, en by de 25 ① vant jaer 1602 tot 1606. Doch ten is gheen feyl inde boveschreven ghemeenheyt der reghel, maer openbaerlick deur misrekening of misdrucking.

TBESLVYT. Wy hebben dan deur ervarings dachtafels ghemaeckt berekende dachtafels des Sonloops van toecommende tijden, na den eysch.

Tot hier toe is beschreven mijn voorghenomen anvang vande kennis des Sonloops diemen deur ervarings dachtafels crijcht, op welcke men als geleyde gront nu voorder met wisconstighe stof soude meughen handelen, maer ick sal eerst verclaren dergelijcken anvang deur ervarings dathtafels van d'ander Dwaelders, en daer na het 2 bouck beschrijven vande wisconstighen handel der Dwaelders int ghemeen, volghende t'voornemen verhaelt int Cortbegrijp des Hemelloops.

TWEE-

2nd NOTE.

Note further that we here see the cause of the shifting of time, to wit, why the 1st of March, which now comes in spring, would in the course of time fall in summer, thereafter in autumn, and so on, unless this were prevented sometimes by an omission of days, as was done in the year 1582 by 10 days, for according to the calculation of the length of the year by *Ptolemy* this runs into one day in 300 years, which, however, is more according to the calculation of the year by others.

3rd NOTE.

If anyone were to examine the ephemerides of *Stadius* further on the same basis as above, he would find errors. Let there, for example, be taken here all the Sun's positions on the 1st of January every four years, as follows below:

1554	290°36′	1582	290°39′
1558	290°37′	1586	290°39′
1562	290°37′	1590	290°40′
1566	290°36′	1594	290°40′
1570	290°37′	1598	291°31′
1574	290°37′	1602	291°32′
1578	290°38′	1606	291° 7′

In the above we see a fair sequence up to the year 1594 (but insufficiently increasing), but thence to the year 1598 there is an inadmissible difference of 51′, and of 25′ from the year 1602 to 1606. But this is not an error in the above-mentioned general rule, but is evidently due to miscalculation or misprints.

CONCLUSION. We have thus made, by means of empirical ephemerides, calculated ephemerides of the Sun's motion in future times; as required.

Up to this point have been described my intended beginnings of the knowledge of the Sun's motion which is acquired by means of empirical ephemerides, upon which as foundations we might now proceed further with mathematical material. But I will first set forth similar beginnings by means of empirical ephemerides of the other Planets, and thereafter describe the 2nd book, of the mathematical treatment of the Planets in general, according to the intention related in the Summary of the Heavenly Motions.

[*The second chapter of the First Book, on the finding of the Moon's motion by means of empirical ephemerides, has not been reproduced*]

DERDE ONDERSCHEYT DES EERSTEN BOUCX, VANDE VINding van Sarurnusloop deur ervarings dachtafels.

15 VOORSTEL.

Te verclaren hoemen deur ervarings dachtafels uyt den rouvven merckt den tijt van Saturnus omloop: Mette ghedaente van sijn deysing en stilstant: Oock dat hy in een inront drayt.

Tvoornemen sijnde t'ondersoucken de ghedaente van Saturnus loop deur ervarings dachtafels (in wiens plaets wy de berekende van *Stadius* gebruycken, om de redenē verclaert int 1 voorstel) ick let op de derde pilaer hem angaende. Ghenomen dan dat my ten eersten voorvalt den 1 Ianuarius vant jaer 1554, op welcken dach ick hem vinde onder den 342 tr. 27 ①: Op een jaer daer na, te weten den 1 Ianuarius 1555, vinde hem gheweest te hebben onder den 353 tr. 53 ①, t'welck op dat jaer 11 tr. 26 ① ghevoordert is. En sghelijcx vinde ick hem den 1 Ianuarius opt volghende jaer 1556 ghevoordert te sijn noch 11 tr. 51 ①. En soo voortgaende tot opt jaer 1584, bevinde hem dan op den 1 Ianuarius weerom gecomē te wesen wat over de plaets daer hy int jaer 1554 begost, te weten onder den 347 tr. 54 ①, hebbende over dien keer ghedaen by de dertich jaren. En sgelijcx ondersouckende tot ander plaetsen, bevinde hem overal ontrent de 30 jaren een keer te doen.

Daer na voorder acht nemende op de ghedaente sijns loops in yder jaer, ick bevinde hem d'eenmael te verrasschē, d'andermael te verslappen, ja somwijlen stil te staen, en dat noch meer is ettelickemael te deysen. En op de saeck nauwer lettende, men bevint het middel dier deysingen altijt te gebeuren wesende Saturnus ontrent teghestant der Son, en hoe hy de Son naerder comt, hoe hy met voortganck meer versnelt. Om hier af by voorbeelt te spreken, int jaer 1569 int begin van Iunius, siet men hem daghelicx 1 ① loopē, daer na dagelicx 2 ①, daer na noch meer, tottet begin vā September, loopende daer dagelicx 8 ①, en daer ontrent ten snelstē gheweest hebbende, begint int laetste desselven maēts weerom te vertragen dagelicx meer en meer, ja sulcx dat hy eintlick vanden 18 tottē 24 Ianuarius 1570 stille staet onder den 202 tr. 20 ①. En daer na begint hy te deysen, t'welc eygentlick anvangende op dē 21 Ianuarius, duert totten 11 Iunius, vervanghende 141 dagen, diens helft wesende 70 dagen, soo valt het middel op

THIRD CHAPTER

OF THE FIRST BOOK

of the Finding of Saturn's Motion by Means of Empirical Ephemerides

15th PROPOSITION.

To set forth how the time of Saturn's motion is roughly found by means of empirical ephemerides; with the nature of its retrogradation and standstill; also that it moves on an epicycle.

The intention being to investigate the nature of Saturn's motion by means of empirical ephemerides (instead of which we use the calculated ephemerides of *Stadius,* for the reasons set forth in the 1st proposition), I note the third column relating thereto. Let us therefore assume that I chance first upon 1st January of the year 1554, on which day I find it at 342°27′. One year later, to wit on 1st January 1555, I find it to have been at 353°53′, which is an advance of 11°26′ in that year. And in the same way I find it on 1st January of the following year 1556 to have advanced by 11°51′ more. And continuing like this up to the year 1584, I find that on 1st January it has come back to a little beyond the place where it started in the year 1554, to wit at 347°54′, having performed this revolution in about thirty years. And when I investigate this similarly in other places, I find that it always performs one revolution in about thirty years.

When thereafter I further attend to the nature of its motion in every year, I find it to move faster at one time, slower at another, nay sometimes to stop and, what is more, to retrograde several times. And when we observe the matter more closely, we find that the middle of these retrogradations always occurs when Saturn is nearly in opposition to the Sun, and the nearer it comes to the Sun, the faster becomes its motion. To give an example thereof: in the year 1569, at the beginning of June, we see it move 1′ daily, thereafter 2′ daily, thereafter even more, up to the beginning of September, when it moves 8′ daily, and having moved fastest at about that time, it begins to move slower again in the latter part of this month, more and more with every day, even in such a way that at last from 18th to 24th January 1570 it stands still at 202°20′. And thereafter it begins to retrograde, which, starting in reality on 21st January, continues to 11th June, which makes 141 days, one half of which is 70 days; thus the middle falls on 1st April, and not long from that date, to wit, on 30th March, only 2 days before, Saturn and the Sun were in opposition, so that, del op den 1 April, en niet seer verre van daer, te wetē den 30 Maerte, alleenelick 2 dagen daer te voorē, wast tegestant van Saturnus en de Son. Sulcx dat gelijck gheseyt is, het middel der deysing gebeurt altijt wesende Saturnus ontrent teghestant der Son. Maer hoe hy de Son naerder comt, hoe hy meer versnelt, inder voughen dat de snelste loop altijt ghebeurt ontrent saming, als inde boveschreven snelste loop van September, daer was hy in saming (genomen datmen hem deur ervaring hadde connen sien) op den 25 der selver maent. Men siet oock dat van yder teghestant tot teghestant, van saming tot saming, van begin der deysing tot begin der deysing, is over al een jaer met ontrent noch een halve maent: Als vande teghestant op den 17 Maerte 1569, totten eerstvolgenden teghestant ghebeurende op den 30 Maerte 1570, is een jaer met ontrent een halve maent, op welcken tijt overal een deysing gheschiet, geduerende als vooren ontrent de 140 daghen, te weten twee of drie daghen meer of min: Wiens ghedaente wy deur de dachtafels verclaren wilden.

Dit boveschreven gheeft vermoeden Saturnus in gheen weeh te loopen als de Son, waer uyt soodanighe deysing niet volghen en can, maer in een inront: Sulcx dat hy ontrent teghestant der Son altijt is an des selven inronts naestepunt, alwaer hy om sijn loop int inront meer achterwaert gaende, dan hem den loop vant middelpunt des inronts inden inrontwech voorwaert brengt, soo wort daer uyt de deysing veroirsaeckt. Maer ontrent saming mette Son ant verstepunt wesende, sal daer soo veel rasscher moeten loopen dan des inronts middelpunt, als sijn voorwaert looping int inront veroirsaeckt.

Saturnus dan om sulcke oirsaken d'eenmael voorwaert d'ander achterwaert loopende, daer volght uyt datter by soodanige verandering een tijt van stilstant moet wesen: En weerom verkeert gheseyt, nadienmen dese voortlooping, stilstant, en deysing siet, soo merckt men d'oirsaeck waerom datter een inront geseyt wort. TBESLVYT. Wy hebben dan verclaert hoemen deur ervarings dachtafels uyt den rouwen merckt den tijt van Saturnus omloop, mette ghedaente van sijn deysing en stilstant: Oock dat hy in een inront draeyt, na den eysch.

16 VOORSTEL.

Te verclaren hoemen deur ervarings dachtafels merckt Saturnus inronts vvech uytmiddelpuntich te vvesen.

By aldien Saturnus inronts wech middelpuntich waer, daer uyt soude volghen dat hy op alle twee even tijden, d'een voor ontrent sijn tegestant der Son, d'ander daer na, evegroote schijnbaer bogen moest loopen, dat teghen d'ervaring strijt. Om t'welck deur een form te verclaren, laet het ront A B C Saturnus inronts wech beteyckenē waert meugelick middelpuntich sijnde, te weten soo dat sijn middelpunt D des Eertcloots middelpunt is, daer na sy op A als middelpunt beschreven het inront E F, diens verstepunt E, en naestepunt F, voort op D als middelpunt den duysteraer G H, en G sijn schijnbaer verstepunt van E, Daer na sijn gheteyckent de twee punten I, K, inden inrontwech evewijt van A, en de twee punten van L, M, int inront evewijt van F, voort sy van D deur L tot inden duysteraer ghetrocken de lini D L N, sghelijcx van D deur M tot inden duysteraer de lini D M O. Dit soo wesende, ick sal hier me t'voornemen verclaren.

as has been said, the middle of the retrogradation always occurs when Saturn is nearly in opposition to the Sun. But the nearer it comes to the Sun, the faster it moves, in such a way that the fastest motion always occurs nearly at conjunction; thus, in the above-mentioned fastest motion of September, it was in conjunction (assuming that we could have seen it by observation) on the 25th of this month. We also see that from opposition to opposition, from conjunction to conjunction, from the beginning of one retrogradation to the beginning of the next retrogradation, it is always one year with about half a month in addition. Thus from the opposition on 17th March 1569 to the next opposition, occurring on 30th March 1570, it is one year and about half a month, in which time a retrogradation always occurs, which takes, as before, about 140 days, to wit, two or three days more or less; the nature of which we wished to set forth by means of the ephemerides.

The above makes us suspect that Saturn does not move in an orbit like that of the Sun, from which such retrogradation cannot follow, but on an epicycle, in such a way that when it is nearly in opposition to the Sun, it is always at the perigee of this epicycle; and because its motion on the epicycle is more backwards than the motion of the centre of the epicycle on the deferent takes it forward, the retrogradation is caused by this. But when it is at the apogee, nearly when in conjunction with the Sun, it will have to move so much faster there than the epicycle's centre as is caused by its progress on the epicycle.

Since from these causes Saturn thus moves now forwards, now backwards, it follows therefrom that with such change there must be a time of standstill. And, *vice versa,* because we see this forward motion, standstill, and retrogradation, we perceive the reason why it is said that there is an epicycle. CONCLUSION. We have thus set forth how the time of Saturn's motion is roughly found by means of empirical ephemerides, with the nature of its retrogradation and standstill; also that it moves on an epicycle; as required.

16th PROPOSITION.

To set forth how it is found, by means of empirical ephemerides, that Saturn's deferent is eccentric.

If Saturn's deferent were concentric, it would follow therefrom that in any two equal times, one before it is nearly in opposition to the Sun and the other after this, it must move through equal apparent arcs, which is contrary to experience. In order to set this forth by means of a figure, let the circle *ABC* designate Saturn's deferent, being — if possible — concentric, to wit, so that its centre *D* is the centre of the Earth. Thereafter let there be described about *A* as centre the epicycle *EF,* whose apogee is *E* and perigee *F;* further about *D* as centre the ecliptic *GH,* and *G* the [1]) apparent apogee of *E.* Thereafter let there be marked the two points *I, K,* on the deferent at equal distances from *A,* and the two points *L, M,* on the epicycle at equal distances from *F.* Further let there be drawn from *D* through *L,* up to the ecliptic, the line *DLN;* likewise from *D* through *M,* up to the ecliptic, the line *DMO.* This being so, I shall herewith set forth my intention.

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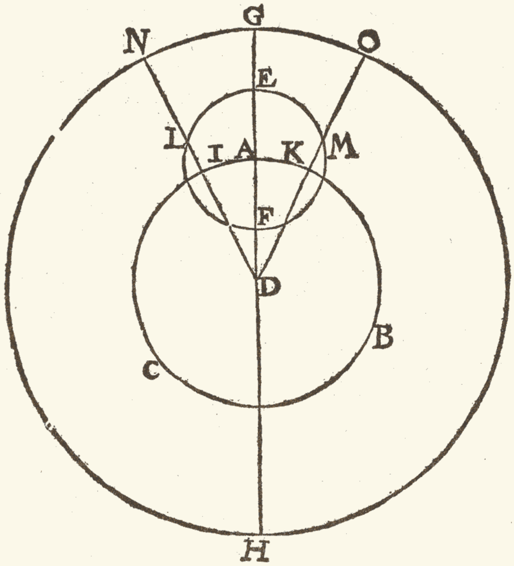
[1]) For *syn* in the Dutch text read *het.*



Het inronts middelpunt eerst geweest hebbende an I, en alsdan Saturnus an M, en schijnbaerlick an O, soo sy het inronts middelpunt daer na gecommen van I tot A, en op den selven tijt Saturnus van M tot F ant naestepunt, alwaer hy dan om de voorgaende redenen sijn sal in teghestant der Son: Daer na sy gheleden een ander tijt even mette voorgaende, en sal daeren tusschen het inronts middelpunt moeten gecommen sijn van A tot K, een booch even an A I, en Saturnus van F tot L, een booch even an F M, en N sal dan Saturnus schijnbaer placts wesen, soo verre van G, als O van G, om de evenheyt der houcken G D O, G D N: Sulcx dat ghelijck wy verclaren wilden, by aldien Saturnus inronts wech middelpuntich waer, als dese, daer uyt soude volgen dat hy op alle twee even tijden, d'een voor sijn teghestant der Son, d'ander daer na, evegroote schijnbaer boghen moest loopen, Maer dat strijt teghen d'ervaring, daerom den inrontwech is voor uytmiddelpuntich te houden: Teghen d'ervaringh te strijden wort aldus betoont: Laet ons tot voorbeelt nemen eenighe teghestant als die gebeurt is int jaer 1569 den 17 Maerte, wesende Saturnus onder den 186 tr. 29 ①: Maer op eenighen tijt daer te vooren, ick neem 7 jaren, te weten den 17 Maerte int jaer 1562, was hy onder den 88 tr. 14 ①, tusschẽ welcke een booch is van 98 tr. 15 ①: Maer 7 jaren daer na, te weten op dẽ 17 Maerte 1576, was hy onder den 271 tr. 44 ①, waer op den booch (te weten vanden 186 tr. 29 ① af) doet 85 tr. 15 ①, die 13 tr. verschilt vande eerste booch 98 tr. 15 ①. TBESLVYT. Wy hebben dan verclaert hoemẽ deur ervarings dachtafels merckt Saturnus inrontswech uytmiddelpuntich te wesen, na den eysch.

17 VOORSTEL.

Deur ervarings dachtafels te vinden de schijnbaer * duysteraerlangde van Saturnus inrontvvechs verstepunt, en naestepunt.

* *Longitudinem zodiaci.*

Deur het 16 voorstel bemerckt sijnde dat den inrontwech middelpuntich is, daer reste doen voorder te soucken sijn verstepunts schijnbaer duysteraerlangde: Tot desen einde heeftmen verdocht, dat gebeurende een tegestant van Saturnus en de Son, als des inronts middelpunt is in sijn wechs verstepunt of naeste-

The epicycle's centre first having been in *I,* and then Saturn in *M,* and apparently in *O,* let the epicycle's centre thereafter have come from *I* to *A,* and in the same time Saturn from *M* to *F* at the perigee, where it will then for the above reasons be in opposition to the Sun. Thereafter let another time equal to the preceding have elapsed, then the epicycle's centre must meanwhile have come from *A* to *K,* an arc equal to *AI,* and Saturn from *F* to *L,* an arc equal to *FM,* and *N* must then be Saturn's apparent position, as far from *G* as *O* from *G,* because of the equality of the angles *GDO, GDN;* in such a way that, as we wished to set forth, if Saturn's deferent were concentric, as this one, it would follow therefrom that in any two equal times, one before its opposition to the Sun and the other thereafter, it must move through equal apparent arcs. But this is contrary to experience, therefore the deferent is to be assumed to be eccentric. That it is contrary to experience is proved as follows: Let us take as example an opposition such as took place in the year 1569 on 17th March, when Saturn was at 186°29′. But some time before, for example 7 years, to wit, on 17th March in the year 1562, it was at 88°14′, between which there is an arc of 98°15′. But 7 years thereafter, to wit, on 17th March 1576, it was at 271°44′, so that the arc (to wit: from 186°29′) makes 85°15′, which differs 13° from the first arc of 98°15′.

CONCLUSION. We have thus set forth how it is found, by means of empirical ephemerides, that Saturn's deferent is eccentric; as required.

17th PROPOSITION.

To find, by means of empirical ephemerides, the apparent ecliptical longitude of the apogee and perigee of Saturn's deferent.

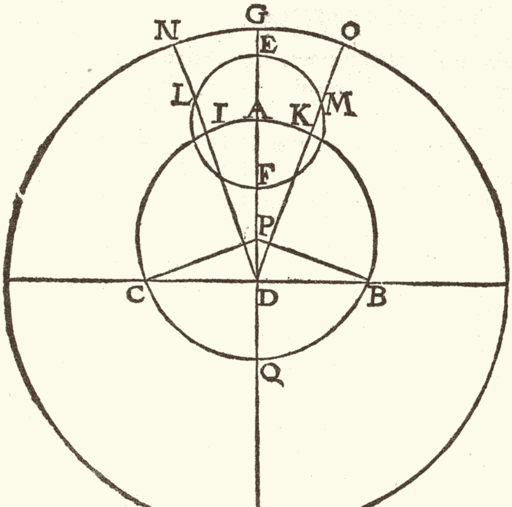
It having been found from the 16th proposition that the deferent is eccentric [1]), it remained to find further the apparent ecliptical longitude of its apogee. To this end it was imagined that, Saturn and the Sun being in opposition, when the

[1]) For *middelpuntich* in the Dutch text read *uytmiddelpuntich.*

naestepunt, dat hy dan op alle twee even tijden, d'eene voor sijn tegheſtant, d'ander daer na, evegroote ſchijnbaer boghen moet loopen: En weerom verkeert, alsmen inde ervarings dachtafels een tegheſtant ſulcx vint, dat hy op alle twee even tijden, d'eene voor die tegheſtant, d'ander daer na, evegroote bogen loopt, dat des inronts middelpunt in die tegeſtant an ſijn wechs verſtepunt of naeſtepunt moet weſen, waer me de ſchijnbaer duyſteraerlangde des ſelvē verſtepunts of naeſtepunts bekent is.

Maer om van t'gene tot hier toegeſeyt is breeder verclaring te doen, ſoo laet de volghende form ghelijck ſijn met die des 15 voorſtels, en de letters vande ſelve beteyckening, uytghenomen dat deſe wech A B C uytmiddelpuntich is, te weten dat des Eertcloots middelpunt D, nu niet en ſy des inrontwechs middelpunt, maer P: En des inronts middelpunt A is an ſijn wechs verſtepunt, weſende Saturnus an des inronts naeſtepunt F in tegheſtant der Son. Dit ſoo ſijnde, ick ſie de twee ſchijnbaer loopen G N, G O, op even tijden noch evegroot te moeten weſen, om de evenheyt der twee houckē G D N, G D O, veroirſaeckt deur de evenheyt van F L met F M. Oock mede dat derghelijcke ghebeuren moet, weſende des inronts middelpunt met ſulcke gedaente an ſijn wechs naeſtepunt gheteyckent met Q.

Merckt noch dat hier vooren geſeyt is, Saturnus altijt ant inronts naeſtepunt te weſen ontrent ſijn tegeſtant der Son: Maer om hier nu eygentlicker te ſpreken, ſoo is te wetē dat hy an ſijn inrondts naeſtepunt niet heel volcommelick en is dan in tegheſtant weſende des inrontsmiddelpunt niet alleenelic an ſijn wechs naeſtepunt, maer boven dien noch de Son in haer wechs verſtepunt of naeſtepunt, want nadienſe inden duyſteraer ſchijnbaerlick oneventlic loopt, en Saturnus in ſijn inront eenvaerdich, ſoo en can hy op alle tegeſtant der Son niet ant inronts naeſtepunt weſen, maer wel in alle tegheſtant der middelſon, die deur de bepaling eenparich loopt: En daerom ſullen wy hier na om eyghentlicker te ſpreken, hem ſegghen altijt ant naeſtepunt of verſtepunt te weſen, in ſijn tegeſtant of ſaming der middelſon.



Dit ſoo ſijnde, en om nu te commen tottet ſoucken der ſchijnbaer duyſteraerlangde van Saturnus inrontwechs verſtepunt of naeſtepunt, t'is kennelick deur de voorgaende redenen, dat ick inde ervarings dachtafels een ſijnder tegeſtanden der middelſon ſoodanich moet vinden, dat hy op alle twee even tijden, d'een epicycle's centre is at the apogee or perigee of its deferent, it must move through equal apparent arcs in any two equal times, the one before its opposition, the other thereafter. And conversely: if we find in the empirical ephemerides an opposition such that in any two equal times, the one before that opposition, the other thereafter, it moves through equal arcs, the epicycle's centre at that opposition must be at the apogee or perigee of its deferent, with which the apparent ecliptical longitude of this apogee or perigee is known.

But to set forth more fully what has been said up to this point, let the following figure be similar to that of the 16th [1]) proposition, the letters having the same meaning, except that here the deferent *ABC* is eccentric, to wit, that the centre of the Earth *D* shall not now be the deferent's centre, but *P*. And the epicycle's centre *A* is at the apogee of its deferent whilst Saturn is at the epicycle's perigee *F* in opposition to the Sun. This being so, I see that the two apparent motions *GN, GO* must still be equal in equal times, because of the equality of the two angles *GDN, GDO,* caused by the equality of *FL* and *FM*. Also that it must happen similarly when the epicycle's centre with similar figures is situated at the perigee of its deferent denoted by *Q*.

Note also that it has been said above that Saturn is always at the epicycle's perigee when it is nearly in opposition to the Sun. But to speak more truly, it is to be known that it is not perfectly at its epicycle's perigee except when not only the epicycle's centre is in opposition to the perigee of its deferent, but moreover the Sun is at the apogee or perigee of its orbit; for since in the ecliptic it moves apparently non-uniformly, and Saturn on its epicycle uniformly, it cannot at every opposition to the Sun be at the epicycle's perigee, but it can at every opposition to the Mean Sun, which by the definition moves uniformly. And therefore, to speak more truly, we shall hereinafter say that it is always at the perigee or apogee when it is in opposition to or conjunction with the Mean Sun.

This being so, and to come now to the finding of the apparent ecliptical longitude of the apogee or perigee of Saturn's deferent, it is evident for the above reasons that I must find in the empirical ephemerides one of its oppositions to the mean sun such that in two equal times, the one before and the other

[1]) For 15th in the original read 16th.

d'een daer voor d'ander daer na, evegroote schijnbaer boghen loopt. Om die te soucken, ick slae het bouck open, en valt my ten eerstē voor, neem ick, het jaer 1569, waer in ick sijn tegestant der Son bevinde geschiet te sijn op den 17 Maerte, wesende Saturnus onder den 186 tr. 29 Ⓘ (t'is waer datmen in plaets der ware Son soude behooren de middelson te nemen, volghende de voorgaende redenen, maer t'is tijts genouch daer me te wercken int laetste, alsmen siet datmen deur neming der ware Son na ghenouch by t'begheerde comt) Hier me versouck ick hoe de boveschrevē twee bogen op even tijden (tot welcke tijden ick om de redenen vant merck des 4 voorstels, verkies 7 jaer, wesende byna het vierendeel eens keers duerende ontrent 30 jaren deur het 14 voorstel) d'een voor, d'ander na teghestant, met malcander overcommen, en bevinde hem op 7 jaren daer te vooren (dats vanden 17 Maerte 1562 onder den 88 tr. 14 Ⓘ, totten 17 Maerte 1569 onder den 186 tr. 29 Ⓘ) gheloopen te hebben 98 tr. 15 Ⓘ: Maer op 7 jaren daer na (dats vanden 17 Maerte 1569 onder den 186 tr. 29 Ⓘ totten 17 Maerte 1576 onder den 271 tr. 44 Ⓘ) vinde ick hem geloopen te hebbē alleenelick 85 tr. 15 Ⓘ, t'welck veel verschillende vande eerste booch 98 tr. 15 Ⓘ, te weten 13 tr. soo is t'schijnbaer verste of naestepunt wijt vande boveschreven 186 tr. 29 Ⓘ, oock van sijn teghepunt den 6 tr. 29 Ⓘ: Doch een van beyden en can ten hoochsten maer 90 tr. van daer sijn, daerom salmen derghelijcke tusschen sulcke bekende palen meughen ondersoucken op de teghestanden vande volghende of voorgaende jaren, voor of na het jaer 1569: En gecommen sijnde, neem ick, opt jaer 1572, sie aldaer teghestant te gheschien op den 23 April, wesende Saturnus onder den 222 tr. 51 Ⓘ, alwaer de rekening gemaeckt als boven op 7 jaer voor en na, vinde eintlick noch verschil der twee boghen van 9 tr. 42 Ⓘ: Daerom ondersouck ick der ghelijcke tot een ander plaets, te weten meer voorwaert, om dat d'eerste booch te groot was, als by de tegestant dieder gebeurde int jaer 1576 op dē 10 Iunius onder dē 268 tr. 20 Ⓘ, alwaer de rekening gemaeckt als boven, op 7 jaer voor en na, vinde eintlick genouchsaem evenheyt, te wetē alleenelic verschil vā 4 Ⓘ, hier van cleēder acht.

Dit aldus soo na commende mette teghestant der eyghen Son, ick souck nu waer op dien tijt de middelson was, om op haer teghestant nauwer rekening te maken: Neem tot dien einde de Sonwechs verstepunt na luyt des 4 voorstels te wesen onder den 94 tr. 24 Ⓘ, alwaer de schijnbaer Son en de middelson deur de 16 bepaling t'samen commende, en dat op den 16 Iunius, sy crijgen soo veel verschil als in 6 daghen loops vanden 10 totten 16 Iunius veroirsaeckt wort, welck verschil ick aldus vinde: De schijnbaer Son heeft inde dachtafels op de 6 daghen gheloopen 5 tr. 44 Ⓘ, maer de middelson deur het 3 voorstel 5 tr. 55 Ⓘ, die alleenelick 11 Ⓘ verschillende, en hier van gheender acht sijnde, ick latet by de voorgaende rekening blijven. Diet nauwer begheerde, soude dese 11 Ⓘ trecken vande Sonnens schijnbaer duysteraerlangde 88 tr. 20 Ⓘ, welcke sy hadde op den 10 Iunius, t'ghene datter blijft, te weten 88 tr. 9 Ⓘ, is voor plaets der middelson, om daer op te berekenen Saturnus teghestant die wat vrougher valt als vande ware Son, en om voort de rekeninghen te maken als boven.

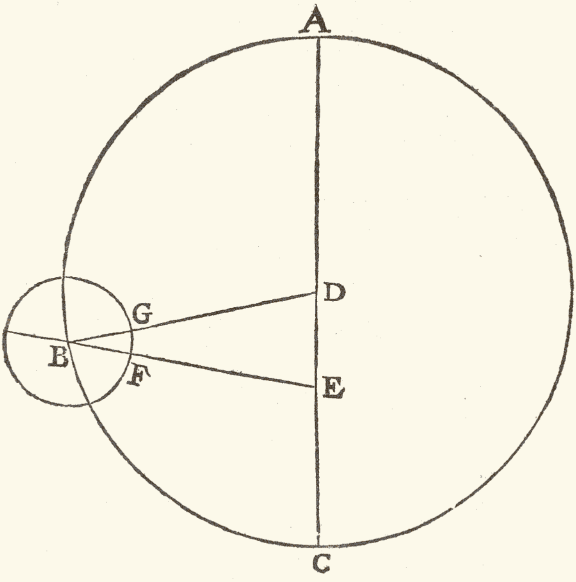
Van dese schijnbaer duysteraerlangde des verstepunts of naestepunts, mach voorder prouf ghedaen worden op ander even tijden dan 7 jaren: Als by voorbeelt, op 10 jaren voor en na den 10 Iunius 1576, vinde ick verschil alleenelick van 5 Ⓘ: En op 20 jaren alleenelick van 3 Ⓘ, waer uyt ick den boveschreven 268 tr. 20 Ⓘ houde voor t'begeerde.

Tot hier toe ghevonden sijnde onder den 268 tr. 20 Ⓘ het verste of naestepunt te wesen, daer rest noch te soucken wat elck van beyden is: Om daer toe te com-after it, it moves through equal apparent arcs. In order to find that, I open the book and I chance first, for example, upon the year 1569, where I find its opposition to the Sun to have occurred on 17th March, when Saturn was at 186°29′ (it is true that instead of the true Sun we ought to take the mean sun, for the above reasons, but it is time enough to operate therewith at the end, when we see that by taking the true Sun we come near enough to what is required). Herewith I examine how the above-mentioned two arcs correspond in equal times (for which times, for the reasons given in the Note to the 4th proposition, I choose 7 years, which is nearly the fourth part of a revolution taking about 30 years by the 14th proposition), the one before and the other after the opposition, and find that in 7 years previously (that is from 17th March 1562 at 88°14′ to 17th March 1569 at 186°29′) it has moved 98°15′. But in 7 years subsequently (that is from 17th March 1569 at 186°29′ to 17th March 1576 at 271°44′) I find it has moved only 85°15′; and since this differs much from the first arc of 98°15′, to wit: 13°, the apparent apogee or perigee is far away from the above-mentioned 186°29′, also from its opposite point, 6°29′. But only one of the two can at most be only 90° away therefrom; therefore the same thing can be examined between such known limits at the oppositions of the subsequent or preceding years, before or after the year 1569. And when I have come, for example, to the year 1572, I see that opposition then occurred on 23rd April, when Saturn was at 222°51′; if I there make the calculation as above, for 7 years before and after, I at last still find the difference between the two arcs to be 9°42′. Therefore I examine the same thing in another place, to wit, further forward, because the first arc was too large, for example, at the opposition that occurred in the year 1576, on 10th June, at 268°20′; and when the calculation is made there as above, for 7 years before and after, I at last find sufficient equality, to wit, a difference of only 4′, which is insignificant here.

This coming thus so near the opposition to the true Sun, I now find where the Mean Sun was at that time, in order to make a more accurate calculation on its opposition. To this end I take the apogee of the Sun's orbit, according to the 4th proposition, to be at 94°24′; and since the apparent Sun and the mean sun coincide there by the 16th definition, such on 16th June, they will differ as much as is caused by 6 days' motion, from 10th to 16th June, which difference I find as follows: In the ephemerides the apparent Sun in 6 days has moved 5°44′, but the Mean Sun by the 3rd proposition 5°55′, and since these values differ only 11′, which is insignificant here, I leave it at the preceding calculation. Who should require it more accurately, would have to subtract these 11′ from the Sun's apparent ecliptical longitude of 88°20′, which it had on 10th June; what remains, namely 88°9′, is the position of the Mean Sun, from which we may calculate Saturn's opposition, which falls slightly earlier than that of the true Sun, and further make the calculations as above.

With regard to this apparent ecliptical longitude of the apogee or perigee, further attempts may be made in equal times other than 7 years. Thus, for example, ten years before and after 10th June 1576 I find a difference of only 5′, and in 20 years of only 3′; from which I assume the above-mentioned 268°20′ to be the required value.

It having been found thus far that the apogee or perigee is at 268°20′, it remains to be found which of these has this value. To arrive at this, for the sake of a clearer exposition thereof let *ABC* be Saturn's deferent, its centre *D*, eccentric commen, laet tot opentlicker verclaring van diē A B C Saturnus inrontswech ſijn, diens middelpunt D, uytmiddelpuntichpunt, of den Eertcloot E, verſtepunt A, naeſtepunt C, en op B als middelpunt ſy beſchreven het inront F G, diens naeſtepunt F, en middelnaeſtepunt G. Dit ſoo weſende, t'blijct dat deur dien A het verſtepunt is, ſoo moet dē middelloop A D B int eerſte halfront A B C overal grooter ſijn dan dē ſchijnbaren A E B. En deur t'verkeerde van dien is openbaer, dat als des inronts middelpuntsloop of Saturnus middelloop, op een geſtelden tijt int eerſte halfront grooter bevonden wort dan de ſchijnbaer, ſoo moetet punt van daer de telling begint het verſtepunt weſen, maer t'verkeerde ghebeurende, alſdan het naeſtepunt. Dit ſoo ſijnde, ick ſouck inde dachtafels eenighe tegeſtant verre genouch van A of C, dats verre ghenouch vanden 268 tr. 20 ① of haer tegenoverpunt, en comt my te vooren, neem ick, de tegeſtant gebeurt int jaer 1584 den 15 September, weſende Saturnus onder den 2 tr. 44 ①, op welcke ſijn inronts middelpunts ſchijnbaer loop vanden 268 tr. 20 ① af, doet 94 tr. 24 ① voor den houck A E B: Maer des ſelven inronts middelpunts eyghenloop op dien tijt, te weten vandē 10 Iunius 1576 tot deſen 15 September 1584, geduerende 8 Egipſche jaren 99 daghen, doet deur het volghende 17 voorſtel 101 tr. 6 ① voor den houck A D B, welcke grooter ſijnde dan A E B 94 tr. 24 ①, ſoo moet om de voorgaende redenen A dats onder des duyſteraers 268 tr. 20 ① het verſtepunt ſijn, en C onder des duyſteraers 88 tr. 20 ① het naeſtepunt.



Angaende ymant nu mocht ſegghen dat de rechte lini die ten tijde der tegeſtant ſtrecte vanden Eertcloot E deur Saturnus F, niet nootſakelick des inronts middelpunt B en ontmoet, om bekende oirſaken, te weten de uytmiddelpuntichedén vanden inronts wech en Sonwech, over ſulcx twijffelende of t'boveſchreven beſluyt vaſt gaet: Hier op wort geantwoort, datmen daer af volcommen ſekerheyt can hebben in deſer voughen: T'ghetal des inrontboochs tuſſchen Saturnus en t'middelnaeſtepunt, even bevondē ſijnde mette naeſtepuntensbooch dieder ten tijde der tegheſtant behoort te weſen, ſoo moet Saturnus dan openbaerlick ant naeſtepunt weſen.

Om t'welck t'onderſoucken, ick treck den boveſchreven houck
A D B 101 tr. 6 ① van 180 tr. blijft den houck E D B 78 tr. 54 ①.
En den

point, or the Earth *E,* apogee *A,* perigee *C,* and about *B* as centre let there be described the epicycle *FG,* its perigee *F* and mean perigee *G.* This being so, it appears that because *A* is the apogee, the mean motion *ADB* in the first semi-circle *ABC* must always be larger than the apparent motion *AEB.* And conversely it is evident that if the motion of the epicycle's centre or Saturn's mean motion is found to be greater than the apparent motion in a given time in the first semi-circle, the point from which counting starts must be the apogee, but if the converse happens, then the perigee. This being so, I seek in the ephemerides some opposition far enough away from *A* or *C, i.e.* far enough away from 268°20′ or its opposite point, and I chance, for example, upon the opposition that occurred in the year 1584, on 15th September, when Saturn was at 2°44′, where the apparent motion of its epicycle's centre from 268°20′ makes 94°24′ for the angle *AEB.* But the proper motion of this epicycle's centre in that time, to wit, from 10th June 1576 to this 15th September 1584, taking 8 Egyptian years and 99 days, by the following 17th proposition makes 101°6′ for the angle *ADB,* and because this is greater than *AEB* (94°24′), for the above reasons *A, i.e.* at 268°20′ of the ecliptic, must be the apogee, and *C,* at 88°20′ of the ecliptic, the perigee.

If anyone now should say that the straight line which at the time of the opposition passed from the Earth *E* through Saturn *F* does not necessarily meet the epicycle's centre *B,* from known causes, to wit, the eccentricities of the deferent and the Sun's orbit, doubting whether the above-mentioned conclusion is quite certain, to this it is replied that we can have perfect certainty about this in the following manner: Since the amount of the epicycle's arc between Saturn and the mean perigee has been found equal to the perigee's arc that ought to be there at the time of opposition, Saturn must evidently be at the perigee.

En den houck A E B dats oock D E B doet als vooren 94 tr. 24 ①.

Dese twee houcken E D B, D E B, des driehoucx E D B, bedraghen t'samen 173 tr. 18 ①, welcke ghetrocken van 180 tr. blijft voor den houck D B E, t'welck oock is voor de naestepuntensbooch F G 6 tr. 42 ①.

Nu segh ick dat by aldien de inrontsbooch vant middelnaestepunt G tot Saturnus, ten tijde deser teghestant oock bevonden wierde van 6 tr. 42 ①, dat alsdan Saturnus an t'naestepunt F, en vervolghens inde rechte lini E B soude moeten gheweest hebben, maer sulcke booch wiert bevonden van 5 tr. 27. ①, want Saturnus heeft opde boveschreven 8 Egipsche jaren 99 daghen gheloopen int inront deur het volghende 18 voorstel 354 tr. 33 ①, welcke ghetrocken van 360 tr. blijft als boven 5 tr. 27 ①.

Die vande 6 tr. 42 ① alleenelick verschillen 1 tr. 15 ①.

Die naerder overeencoming begheert, soude dergheliicke wercking meugen ondersoucken met een tegestant voor of na de boveschreven tot dat hyse vonde. Als by voorbeelt, sulcx ghedaen mette tegestandt int jaer 1583 den 3 September, vinde daer me te luttel 1 tr. 13 ①. Maer mette tegestandt int jaer 1579 dē 15 Iulius, viel D E B van 33 tr. 55 ①, B D E 142 tr. 9 ①, en E B D 4 tr. 4 ①, welcke drie t'samen maken 180 tr. 8 ①, dat alleenelick 8 ① te veel is, en voor ghenouchsaem overcomming mach ghehouden worden. Inder voughen dat A het begheerde verstepunt blijft als boven onder den 268 tr. 20 ①, na ghenouch overcommende metten 268 tr. van *Rheinoldus* in *Prutenicis tabulis*.

Belanghende dat de volcommenheyt soude vereysschen met teghestant der Middelson te wercken, sulcx schijnt in dese anvangsche wijse der leering onnoodich, te meer dat wy tot so nauwe ondersoucking veel langdueriger dachtafels souden moeten hebben dan dese. TBESLVYT. Wy hebben dan deur ervarings dachtafels gevonden de schijnbaer duysteraerlangde van Saturnus inrontwechs verstepunt en naestepunt, na den eysch.

18 VOORSTEL.

Deur ervarings dachtafels Saturnus middelloop op een ghegheven tijt te vinden, en daeraf een tafel te beschrijven.

Het vinden des loops vant inronts middelpunt deur de ervarings dachtafels gaet aldus toe: Men siet dat Saturnus ontrent de 30 jaren een schijnbaer keer des duysteraers doet, daerom soucktmen in welck jaer en dach ontrent 30 jaren (of ontrent ettelicke 30 jaē als de dachtafels lang genouch duyrē) voor of nadē boveschrevē 10 Iunius 1576, dat ontrent dē 10 Iunius Saturnus onder dē 268 tr. 20 ① (wesende deur het 16 voorstel des inrontwechs verstepunt) in tegestant der Son is, en den keer of keeren alsdan gheschiet, sijn na ghenouch volcommen, om vande uytmiddelpuntickheden des inrontwechs en Sonwechs gheen hinderlicke dwaling te crijghen: Voort is den tijt van d'een tegestant tot d'ander bekent, waer deur mē oock vindt den loop eens dachs, en tafelen maect, die gelijck inde Sonloop geseyt is daer na op langer en langer tijden meer en meer verbetert worden. Maer om van t'ghene tot hier toe int ghemeen ghesceyt is, nu by

To investigate this, I subtract the above-mentioned angle *ADB* of 101°6′ from 180°; there remains the angle *EDB* of 78°54′

And the angle *AEB, i.e.* also *DEB,* as before makes 94°24′

These two angles *EDB, DEB* of the triangle *EDB* are together 173°18′, and when this is subtracted from 180°, there remains for the angle *DBE,* which is also the value of the perigees' arc *FG,* 6°42′

Now I say that if the epicycle's arc from the mean perigee *G* to Saturn at the time of this opposition were also found to be 6°42′, Saturn must then have been at the perigee *F,* and consequently on the straight line *EB;* but this arc was found to be 5°27′, for Saturn has moved, in the above-mentioned 8 Egyptian years and 99 days, by the following 18th proposition, 354°33′ on the epicycle, and if this is subtracted from 360°, there remains, as above, 5°27′

which differ from 6°42′ only 1°15′

Who should wish for closer agreement, might make a similar investigation with an opposition before or after the one described above until he found it. Thus, for example, when I do so with the opposition in the year 1583, on 3rd September, I find therewith 1°13′ short. But with the opposition in the year 1579, on 15th July, *DEB* was 33°55′, *BDE* 142°9′, and *EBD*, 4°4′, which three together make 180°8′, which is only 8′ too much and may be considered sufficient agreement; so that *A* remains the required apogee, as above, at 268°20′, which agrees substantially with the 268° of *Rheinoldus* in *Prutenicis tabulis* [1]).

As to the fact that perfection would require us to operate with opposition to the Mean Sun, this seems unnecessary in this elementary instruction, the more so because for such accurate investigations we should need ephemerides covering much longer periods than the present. CONCLUSION. We have thus found, by means of empirical ephemerides, the apparent ecliptical longitude of the apogee and perigee of Saturn's deferent; as required.

18th PROPOSITION.

To find, by means of empirical ephemerides, Saturn's mean motion in a given time, and to describe a table thereof.

The finding of the motion of the epicycle's centre, by means of the empirical ephemerides, takes place as follows. We see that Saturn performs in approximately 30 years one apparent revolution in the ecliptic; therefore we seek what year and day approximately 30 years (or several times 30 years, if the ephemerides cover a long enough period) before or after the above-mentioned 10th June 1576 Saturn is about 10th June at 268°20′ (which by the 16th proposition is the deferent's apogee) in opposition to the Sun, and the revolution or revolutions that have then taken place are sufficiently complete not to get any disturbing error due to the eccentricities of the deferent and the Sun's orbit. Further the time from one opposition to the other is known, by means of which we also find the motion of one day and make tables which, as has been said in the Sun's motion, are thereafter improved more and more in longer and longer times. But now to give an example of what has so far been said in general, I seek in the ephemerides

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[1]) See note p. 45.

nu by voorbeelt te ſpreken, ick ſoucke inde dachtafels ontrent 30 jaren (of ontrent ettellicke 30 jaren als de dachtafels langhe ghenouchduyren) voor of na de boveſchreven tegheſtant op den 10 Iunius 1576, een tegheſtant weſende Saturnus ten naeſten by den 268 tr. 20 ①, wort bevonden op den 4 Iunius 1605 onder den 263 tr. 35 ①, t'welck van d'ander 4 tr. 45 ① verſchilt, maer op ſoo cleenen boochſken en connen de uytmiddelpunticheden gheen hinderlick verſchil by brenghen, immers op een voorbeeltſche leering als deſe.

Dit ſoo weſende, ick ſie dat Saturnus, en vervolghens ſijn inronts middelpunt vanden 10 Iunius 1576, totten 4 Iunius 1605, doende 10586 daghen, gheloopen heeft 355 tr. 15 ①, te weten vanden 268 tr. 20 ①, totten 263 tr. 35 ①: Daerom ſegh ick, 10586 daghen, gheven 355 tr. 15 ①, wat 1 dach? Comt 2 ① 0 ② 49 ③: T'welck luttel ſchilt van *Copernicus* gheſtelde middelloop doende 2 ① 3 ② 36 ③, daer *Ptolemeus* voor nam by de 2 ① 0 ② 34 ③. En waer de boveſchreven tegheſtandt (die 3 tr. 18 ① van rechte tegheſtandt ſchilt) naerder rechte tegheſtandt gheweeſt (naerder ſoudemenſe connen vinden in dachtafels van langher jaren ghelijckmen in den Wijſentijt had) deſe loop eens dachs mocht oock naerder t'gheſtelde van *Copernicus* ghevallen hebben.

Nu dan de loop eens dach weſende als boven, t'is kennelick hoemen daer me ſoude connen maken, ghelijck vande Sonloop int 3 voorſtel ghedaen is, tafels om daer deur met lichticheyt te vinden Saturnus inronts middelpunts loop, anders gheſeyt Saturnus middelloop, op alle ghegheven tijt inde * daet *Praxi.*
ghemeenelick te vooren commende.

Maer angheſien wy *Ptolemeus* tafels voorbeeltſche wijſe ghebruycken ſullen, om de redenen van diergelijcke inde Sonloop en Maenloop verclaert, ſoo ſal ickſe ſtellen als volght:

about 30 years (or about several times 30 years, if the ephemerides cover a long enough period) before or after the above-mentioned opposition on 10th June 1576 an opposition with Saturn being nearest to 268°20′; this is found to be on 4th June 1605, at 263°35′, which differs 4°45′ from the other, but on such a small arc the eccentricities cannot cause any disturbing difference, especially in instruction by means of an example, such as the present.

This being so, I see that Saturn, and consequently its epicycle's centre, from 10th June 1576 to 4th June 1605, which makes 10,586 days, has moved 355°15′, to wit, from 268°20′ to 263°35′. I therefore say: 10,586 days give 355°15′; what does 1 day give? This gives 2;0,49 minutes, which differs little from the mean motion given by *Copernicus,* which is 2;3,36 minutes, for which *Ptolemy* took about 2;0,34 minutes. And if the above-said opposition (which differs 3°18′ from exact opposition) had been nearer to exact opposition (they could be found nearer in ephemerides covering more years, such as people had in the Age of the Sages), this motion of one day might also have been nearer the value given by *Copernicus.*

The motion of one day thus being as above, it is evident how one might make tables therewith, as has been done for the Sun's motion in the 3rd proposition, by means of which to find easily the motion of Saturn's epicycle's centre, in other words Saturn's mean motion, as it usually appears in practice in any given time.

But since we shall use *Ptolemy's* tables by way of example, for the reasons set forth for a similar matter in the Sun's motion and the Moon's motion, I will give them as follows.

uyren	tr.	①	②	③	④	⑤	⑥
1	0	0	5	1	23	48	42
2	0	0	10	2	46	37	24
3	0	0	15	4	11	26	6
4	0	0	20	5	35	14	48
5	0	0	25	6	59	3	31
6	0	0	30	8	22	52	13
7	0	0	35	9	46	40	55
8	0	0	40	11	10	29	37
9	0	0	45	12	34	18	19
10	0	0	50	13	58	7	1
11	0	0	55	15	21	55	43
12	0	1	0	16	45	44	25
13	0	1	5	18	9	33	8
14	0	1	10	19	33	21	50
15	0	1	15	20	57	10	32
16	0	1	20	22	20	59	14
17	0	1	25	23	44	47	55
18	0	1	30	25	8	36	38
19	0	1	35	26	32	25	20
20	0	1	40	27	56	14	2
21	0	1	45	29	20	2	45
22	0	1	50	30	43	51	27
23	0	1	55	32	7	40	9
Dagē							
1	0	2	0	33	31	28	51
2	0	4	1	7	2	57	42
3	0	6	1	40	34	26	33
4	0	8	2	14	5	55	24
5	0	10	2	47	37	24	15
6	0	12	3	21	8	53	6
7	0	14	3	54	40	21	57
8	0	16	4	28	11	50	48
9	0	18	5	1	43	19	39

Dagē.	tr.	①	②	③	④	⑤	⑥
10	0	20	5	35	14	48	30
11	0	22	6	8	46	17	21
12	0	24	6	42	17	46	12
13	0	26	7	15	49	15	3
14	0	28	7	49	20	43	54
15	0	30	8	22	52	12	45
16	0	32	8	56	23	41	36
17	0	34	9	29	55	10	27
18	0	36	10	3	26	39	18
19	0	38	10	36	58	8	9
20	0	40	11	10	29	37	0
21	0	42	11	44	1	5	51
22	0	44	12	17	32	34	42
23	0	46	12	51	4	3	33
24	0	48	13	24	35	32	24
25	0	50	13	58	7	1	15
26	0	52	14	31	38	30	6
27	0	54	15	5	9	58	57
28	0	56	15	38	41	27	48
29	0	58	16	12	12	56	39
30	1	0	16	45	44	25	30
60	2	0	33	31	28	51	0
90	3	0	50	17	13	16	30
120	4	1	7	2	57	42	0
150	5	1	23	48	42	7	30
180	6	1	40	34	26	33	0
210	7	1	57	20	10	58	30
240	8	2	14	5	55	24	0
270	9	2	30	51	39	49	30
300	10	2	47	37	24	15	0
330	11	3	4	23	8	40	30
360	12	3	21	8	53	6	0

Jaren.	tr.	(1)	(2)	(3)	(4)	(5)	(6)
1	12	13	23	56	30	30	15
2	24	26	47	53	1	0	30
3	36	40	11	49	31	30	45
4	48	53	35	46	2	1	0
5	61	6	59	42	32	31	15
6	73	20	23	39	3	1	30
7	85	33	47	35	33	31	45
8	97	47	11	32	4	2	0
9	110	0	35	28	34	32	15
10	122	13	59	25	5	2	30
11	134	27	23	21	35	32	45
12	146	40	47	18	6	3	0
13	158	54	11	14	36	33	15
14	171	7	35	11	7	3	30
15	183	20	59	7	37	33	45
16	195	34	23	4	8	4	0
17	207	47	47	0	38	34	15
18	220	1	10	57	9	4	30
36	80	2	21	54	18	9	0
54	300	3	32	51	27	13	30
72	160	4	43	48	36	18	0
90	20	5	54	45	45	22	30
108	240	7	5	42	54	27	0
126	100	8	16	40	3	31	30
144	320	9	27	37	12	36	0
162	180	10	38	34	21	40	30
180	40	11	49	31	30	45	0
198	260	13	0	28	39	49	30
216	120	14	11	25	48	54	0
234	340	15	22	22	57	58	30
252	200	16	33	20	7	3	0

Iaré.	tr.	(1)	(2)	(3)	(4)	(5)	(6)
270	60	17	44	17	16	7	30
288	280	18	55	14	25	12	0
306	140	20	6	11	34	16	30
324	0	21	17	8	43	21	0
342	220	22	28	5	52	25	30
360	80	23	39	3	1	30	0
378	300	24	50	0	10	34	30
396	160	26	0	57	19	39	0
414	20	27	11	54	28	43	30
432	240	28	22	51	37	48	0
450	100	29	33	8	46	52	30
468	320	30	44	45	55	57	0
486	180	31	55	43	5	1	30
504	40	33	6	40	14	6	0
522	260	34	17	37	23	10	30
540	120	35	28	34	32	15	0
558	340	36	39	31	41	19	30
576	200	37	50	28	50	24	0
594	60	39	1	25	59	28	30
612	280	40	12	23	8	33	0
630	40	41	23	20	17	37	30
648	0	42	34	17	26	42	0
666	220	43	45	14	35	46	30
684	80	44	56	11	44	51	0
702	300	46	7	8	53	55	30
720	160	47	18	6	3	0	0
738	20	48	29	3	12	4	30
756	240	49	40	0	21	9	0
774	100	50	50	57	30	13	30
792	320	52	1	54	39	18	0
810	180	53	12	51	48	22	30

TBESLVYT. Wy hebbẽ dan deur ervarings dachtafels Saturnus middelloop op een ghegheven tijt ghevonden, en daer af een tafel beſchreven, na den eyſch.

19 VOORSTEL.

Deur ervarings dachtafels Saturnus loop in ſijn inront op een ghegheven tijt te vinden, en daer af een tafel te beſchrijven.

TGHEGHEVEN. Laet te vinden ſijn Saturnus loop in ſijn inrondt op een dach.

MERCKT.

Nadien ick t'werck deſes voorſtels beſchreven hadde na de ghemeene wijſe, treckende Saturnus middelloop vande Sonnens middelloop, ghelijck hier onder blijcken ſal, ſo heeft ſijn VORSTELICKE GHENADE tot grontlicker kennis der oirſaken (ſoo wel van d'ander volghende Dwaelders, alwaer der ghelijcke ſal ghedaen worden, als van deſe) daer by noch begeert en vervought een werck op Saturnus eyghen gront ghebout, als volght.

1 WERCK.

Hy heeft hier toe inde dachtafels geſocht twee van Saturnus tegeſtandẽ met de Son, lange tijt van malcander gheſchiet, byna en onder een ſelve plaets des duyſteraers, want daer me moeten ſoo wel ſijn keeren int inront als de Sonkeeren volcommen weſen, ſonder van eenighe uytmiddelpunticheden hinder te crijghen, en oock onnoodich ſijnde rekening te maken mette Middelſon, om bekende reden. D'eerſte van ſulcke twee tegheſtanden nam hy dieder viel op den 10 Iunius 1576, d'ander den 4 Iunius 1605; tuſſchen welcke ſijn by de 29 jaren, doende 10586 daghen, waer op Saturnus int inront ſoo veel keeren ghedaen heeft alſſer tegheſtanden gebeurt ſijn, welcke bevonden wierden 28, elcke van 360 tr. maken 10080 tr. Hier me ſegghende, 10586 dagen geven 10080 tr. wat 1 dach? Comt voor t'begheerde 57① 7. 55. weynich verſchillende van *Copernicus* en *Ptolemeus* beſluyt int volghende.

2 WERCK.

Vande Sonnens middelloop eens dachs, doende deur het 3 voorſtel	0 tr. 59. 8. 17. 13. 12. 31.
Getrocken Saturnus middelloop eens dachs, doende deur het 17 voorſtel	0 tr. 2. 0. 33. 31. 28. 51.
Blijft voor begheerden Saturnusloop in ſijn inront op een dach	0 tr. 57. 7. 43. 41. 43. 40.

TBEWYS.

Angheſien Saturnus altijt ant inronts naeſtepunt is in ſijn tegheſtant der Middelſon deur t'voorgaende, ſoo moet nootſakelick dien inrontſloop even ſijn ande middelloop der Son, min ſoo veel als bedraecht des inronts middelpuntsloop.

Om

CONCLUSION. We have thus found, by means of empirical ephemerides, Saturn's mean motion in a given time, and described a table thereof; as required.

19th PROPOSITION.

To find, by means of empirical ephemerides, Saturn's motion on its epicycle in a given time, and to describe a table thereof.

WHAT IS REQUIRED [1]). Let Saturn's motion on its epicycle in one day have to be found.

NOTE.

After I had described the procedure of this proposition in the usual manner, subtracting Saturn's mean motion from the Sun's mean motion, as will appear below, for a more thorough knowledge of the causes (both of the other — following — Planets, where a similar operation will be carried out, and of this one) His PRINCELY GRACE required in addition and added a method based on data from Saturn's tables alone, as follows.

1st METHOD.

To this end he sought in the ephemerides two of Saturn's oppositions to the Sun, which had occurred a long time apart, nearly at the same place of the ecliptic, for therewith its revolutions on the epicycle as well as the Sun's revolutions must be complete, without being disturbed by any eccentricities, and it also being unnecessary to take account of the Mean Sun, for known reasons. For the first of these two oppositions he took the one that fell on 10th June 1576, for the other 4th June 1605, between which there are nearly 29 years, making 10,586 days, in which Saturn has performed so many revolutions on the epicycle as there have been oppositions, which were found to be 28, each of 360°, making 10,080°. Saying now: 10,586 days give 10,080°, what does 1 day give? The value required is 57; 7, 55 minutes. which differs little from *Copernicus*' and *Ptolemy's* results in the following.

2nd METHOD.

When from the Sun's mean motion in one day, making by the 3rd proposition	0°;59, 8,17,13,12,31
there be subtracted Saturn's mean motion in one day, making by the 18th proposition [2])	0°; 2, 0,33,31,28,51
there remains for Saturn's required motion on its epicycle in one day	0°;57, 7,43,41,43,40

PROOF.

Since Saturn by the above is always at the epicycle's perigee in its opposition to the Mean Sun, this epicyclic motion must necessarily be equal to the mean motion of the Sun, *minus* the amount of the motion of the epicycle's centre.

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[1]) Evident mistake in the Dutch text. Instead of *Tghegheven* read *Tbegheerde*.
[2]) In the Dutch text, read 18 instead of 17.

Om hier af met breeder reden te verclaren, ick segh aldus: Soo by voorbeelt het inronts middelpunt gheen loop en hadde, maer vast bleve, en dat Saturnus in saming dan evewel altijt in sijn inronts verstepunt waer, t'is kennelick dat Saturnusloop in sijn inront dan even soude moeten sijn ande Sonnens middelloop: Maer het inronts middelpunt heeft daerentusschen een loop gedaen, daerom soo veel die is, soo veel moet Saturnus inrontloop corter dueren dan de middelloop der Son, en vervolghens treckende Saturnus inronts middelpunts loop, vande Sonnens middelloop, de rest is de begheerde loop van Saturnus in sijn inront.

VERVOLGH.

De loop eens dachs wesende als boven, t'is kennelick hoemen daer me sal maken, ghelijck vande Sonloop int 3 voorstel ghedaen is, tafelen als de navolghende, om daer me met lichticheyt Saturnus loop in sijn inront te vinden op alle ghegheven tijt inde * daet ghemeenelick te vooren commende. *Prat.*

Merckt noch datmen tot prouf der voorschreven werckinghen mach sien soo dickwils alsment oirboir verstaet, of het laest ghevonden ghetal even is mettet ghetal daer me overcommende: Als by voorbeelt van t'ghetal des Sonloops eens jaers, ghetrocken t'ghetal van Saturnus inrontloop eens jaers, of die rest overcomt met soodanich ghetal anders ghevonden deur ervaring.

F 3 TAFEL

To set this forth more fully, I say as follows: If, for example, the epicycle's centre did not move, but remained fixed, and Saturn in conjunction were yet always in its epicycle's apogee, it is evident that Saturn's motion on its epicycle would then have to be equal to the Sun's mean motion. But the epicycle's centre has meanwhile moved on; therefore, as much as that is, by so much must the motion of Saturn's epicycle be less than the mean motion of the Sun [1]), and consequently, when the motion of Saturn's epicycle's centre is subtracted from the Sun's mean motion, the remainder is the required motion of Saturn on its epicycle.

SEQUEL.

The motion in one day being as above, it is evident how tables such as the following will be made therewith — as was done for the Sun's motion in the 3rd proposition — by means of which to find easily Saturn's motion on its epicycle as it usually appears in practice in any given time.

Note also that, to test the aforesaid methods, we may ascertain as often as we deem suitable whether the value last found is equal to the value corresponding thereto. Thus, for example, when from the value of the Sun's motion in one year there is subtracted the value of Saturn's motion on its epicycle in one year, whether the remainder corresponds to a similar value found in another manner, by experience.

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[1]) The words *corter dueren* in the Dutch text should be *corter sijn*, which means just the reverse.

uyren	tr.	①	②	③	④	⑤	⑥
1	0	2	22	49	19	14	19
2	0	4	45	38	28	28	38
3	0	7	8	27	57	42	57
4	0	9	31	17	16	57	17
5	0	11	54	6	36	11	36
6	0	14	16	55	55	25	55
7	0	16	39	45	14	40	14
8	0	19	2	34	33	54	33
9	0	21	25	23	53	8	52
10	0	23	48	13	12	23	12
11	0	26	11	2	31	37	31
12	0	28	33	51	50	51	50
13	0	30	56	41	10	6	9
14	0	33	19	30	29	20	28
15	0	35	42	19	48	34	47
16	0	38	5	9	7	49	7
17	0	40	27	58	27	3	26
18	0	42	50	47	46	17	45
19	0	45	13	37	5	32	4
20	0	47	36	26	24	46	23
21	0	49	59	15	44	0	42
22	0	52	22	5	3	15	2
23	0	54	44	54	22	29	21
Dagē							
1	0	57	7	43	41	43	40
2	1	54	15	27	23	27	20
3	2	51	23	11	5	11	0
4	3	48	30	54	46	54	40
5	4	45	38	38	28	38	20
6	5	42	46	22	10	22	0
7	6	39	54	5	52	5	40
8	7	37	1	49	33	49	20
9	8	34	9	33	15	33	0

Dagē	tr.	①	②	○	④	⑤	⑥
10	9	31	17	16	57	16	40
11	10	28	25	0	39	0	20
12	11	25	32	44	20	44	0
13	12	22	40	28	2	27	40
14	13	19	48	11	44	11	20
15	14	16	55	55	25	55	0
16	15	14	3	39	7	38	40
17	16	11	11	22	49	22	20
18	17	8	19	6	31	6	0
19	18	5	26	50	12	49	40
20	19	2	34	33	54	33	20
21	19	59	42	17	36	17	0
22	20	56	50	1	18	0	40
23	21	53	57	44	59	44	20
24	22	51	5	28	41	28	0
25	23	48	13	12	23	11	40
26	24	45	20	56	4	55	20
27	25	42	28	39	46	39	0
28	26	39	36	23	28	22	40
29	27	36	44	7	10	6	20
30	28	33	51	50	51	50	0
60	57	7	43	41	43	40	0
90	85	41	35	32	35	30	0
120	114	15	27	23	27	20	0
150	142	49	19	14	19	10	0
180	171	23	11	5	11	0	0
210	199	57	2	56	2	50	0
240	228	30	54	46	54	40	0
270	257	4	46	37	46	30	0
300	285	38	38	28	38	20	0
330	314	12	30	19	30	10	0
360	342	46	22	10	22	0	0

Iaren.	tr	①	②	③	④	⑤	⑥
1	347	32	0	48	50	38	20
2	335	4	1	37	41	16	40
3	322	36	2	26	31	55	0
4	310	8	3	15	22	33	20
5	297	40	4	4	13	11	40
6	285	12	4	53	3	50	0
7	272	44	5	41	54	28	20
8	260	16	6	30	45	6	40
9	247	48	7	19	35	45	0
10	235	20	8	8	26	23	20
11	222	52	8	57	17	1	40
12	210	24	9	46	7	40	0
13	197	56	10	34	58	18	20
14	185	28	11	23	48	56	40
15	173	0	12	12	39	35	0
16	160	32	13	1	30	13	20
17	148	4	13	50	20	51	40
18	135	36	14	39	11	30	0
36	271	12	29	18	23	0	0
54	46	48	43	57	34	30	0
72	182	24	58	36	46	0	0
90	318	1	13	15	57	30	0
108	93	37	27	55	9	0	0
126	229	13	42	34	20	30	0
144	4	49	57	13	32	0	0
162	140	26	11	52	43	30	0
180	276	2	26	31	55	0	0
198	51	38	41	11	6	30	0
216	187	14	55	50	18	0	0
234	322	51	10	92	29	30	0
252	98	27	25	8	41	0	0

Iaré.	tr.	①	②	③	④	⑤	⑥
270	234	3	39	47	52	30	0
288	9	39	54	27	4	0	0
306	145	16	9	6	15	30	0
324	280	52	23	45	27	0	0
342	56	28	38	24	38	30	0
360	192	4	53	3	50	0	0
378	327	41	7	43	1	30	0
396	103	17	22	22	13	0	0
414	238	53	37	1	24	30	0
432	14	29	51	40	36	0	0
450	150	6	6	19	47	30	0
468	285	42	20	58	59	0	0
486	61	18	35	38	10	30	0
504	196	54	50	17	22	0	0
522	332	31	4	56	33	30	0
540	108	7	19	35	45	0	0
558	243	43	34	14	56	30	0
576	19	19	48	54	8	0	0
594	154	56	3	33	19	30	0
612	290	32	18	12	31	0	0
630	66	8	3	51	42	30	0
648	201	44	4	30	54	0	0
666	337	21	2	10	5	30	0
684	112	57	16	49	17	0	0
702	248	33	31	28	28	30	0
720	24	9	46	7	40	0	0
738	159	46	0	46	51	30	0
756	295	22	15	26	3	0	0
774	70	58	30	5	14	30	0
792	206	34	44	44	26	0	0
810	342	10	59	23	37	30	0

TBESLVYT. Wy hebben dan deur ervarings dachtafels gevonden Saturnus loop in sijn inront op een gheghen tijt, en daer af een tafel beschreven, na den eysch.

MERCKT.

By aldien dese dachtafels van *Stadius* soo lang duerden, en soo veel Saturnus keeren hadden, datmen daer in mocht vinden twee sijnder teghestanden mette Son onder een selve duysteraerlangde, wy souden dan hier meughen een voorstel beschrijven, gelijck inde Sonloop en Maenloop gedaen is, te weten: Deur ervarings dachtafels te maken berekende dachtafels van Saturnus loop op toecommende tijden: Maer angesien hy daer in alleenelick een volcommen keer ghedaen heeft, soo en wil sulcke vinding hier niet vallen: Doch hoemen deur wisconstige rekeningen vindt van hoe langen tijt de dachtafels souden moeten beschreven sijn om sulcken selfheyt te crijgen, dat sal elders verclaert worden.

20 VOORSTEL.

Deur ervarings dachtafels den loop van Saturnus inront vvechs verstepunt te vinden.

Angesien de dachtafels van *Stadius* (die wy nemen al offe deur ervaringhen bevonden waren, om de redenen verclaert int 1 voorstel) te cort sijn om daer deur bequamelick t'begheerde te vinden, soo sal ick tot hulp nemen de ervaringhen van *Ptolemeus*, welcke int 5 hoofstick sijns 11 boucx Saturnus inrondt wechs verstepunt bevondē heeft onder des duysteraers 233 tr. maer int 16 voorstel bevintment onder den 268 tr. 20 ①, T'welck 35 tr. 20 ① voorder sijnde, het verstepunt is na dese rekening soo veel verloopen op dien tijt, bedraghende ontrent 1450 jaren, te weten vant jaer 127 tottet jaer deser ervaring 1576. En hier me wort bekent des verstepunts loop op alle ghegheven tijt: Als by voorbeelt, om die te hebben op een jaer, ick segh 1450 jaren gheven 35 tr. 20 ①, wat 1 jaer? Comt 1 ① 28 ②. TBESLVYT. Wy hebben dan deur ervarings dachtafels gevonden den loop van Saturnus inront wechs verstepunt, na den eysch.

21 VOORSTEL.

Deur ervarings dachtafels Saturnus inronts middelpunts loop in sijn vvech te vinden.

Den loop van Saturnus inrontweechs verstepunt wort op een jaer bevonden deur het 18 voorstel van	1 ① 28 ②.
Die ghetrocken van Saturnus middelloop oock op een jaer, te weten een Egips doende deur het 17 voorstel	12 tr. 13 ①.
Blijft voor begheerde loop des inronts middelpunts in sijn wech op een Egips jaer	12 tr. 12 ①.

En is openbaer dat alsoo ghevonden sal worden den loop van alle voorghestelde tijt: Waer af t'bewijs deur t'werck openbaer is. TBESLVYT. Wy hebben dan deur ervarings dachtafels Saturnus inronts middelpunts loop in sijn wech ghevonden, na den eysch.

VIERDE

CONCLUSION. We have thus found, by means of empirical ephemerides, Saturn's motion on its epicycle in a given time, and described a table thereof; as required.

NOTE.

If these ephemerides of *Stadius* covered so long a time and contained so many revolutions of Saturn that we might find therein two of its oppositions to the Sun at identical ecliptical longitude, we could here describe a proposition as has been done in the Sun's motion and the Moon's motion, to wit: To make, by means of empirical ephemerides, calculated ephemerides of Saturn's motion in future times. But since it has performed only one complete revolution therein, such finding cannot here take place. But it will be set forth elsewhere how it is found by mathematical calculations what length of time the ephemerides would have to cover in order to obtain such identity.

20th PROPOSITION.

To find, by means of empirical ephemerides, the motion of the apogee of Saturn's deferent.

Since the ephemerides of *Stadius* (which we use as if they had been found by experience, for the reasons set forth in the 1st proposition) cover too short a period to find therewith easily what is required, I will have recourse to the experiences of *Ptolemy,* who in the 5th chapter of his 11th book has found the apogee of Saturn's deferent at 233° of the ecliptic; but in the 16th proposition we find it at 268°20′. This being 35°20′ ahead, according to this calculation the apogee has moved this amount in that time, being about 1,450 years, to wit, from the year 127 to the year of the present observation, 1576. And in this way the motion of the apogee in any given time becomes known. Thus, for example, to have it in one year, I say: 1,450 years give 35°20′, what does one year give? This gives 1′28″. CONCLUSION. We have thus found, by means of empirical ephemerides, the motion of the apogee of Saturn's deferent; as required.

21st PROPOSITION.

To find, by means of empirical ephemerides, the motion of the centre of Saturn's epicycle on its deferent.

The motion of the apogee of Saturn's deferent is found, by the 18th proposition, to be in one year 1′28″

This being subtracted from Saturn's mean motion in one year, to wit, an Egyptian year, making by the 17th proposition 12°13′

there remains for the required motion of the epicycle's centre on its deferent in an Egyptian year 12°12′

And it is evident that thus the motion in any suggested time can be found, the proof of which is evident from the procedure. CONCLUSION. We have thus found, by means of empirical ephemerides, the motion of the centre of Saturn's epicycle on its deferent; as required.

[*Chapters 4 up to and including 7, dealing with Jupiter, Mars, Venus, and Mercury, have not been reproduced here*]

ACHTSTE ONDERSCHEYT DES EERSTEN BOVCX, VANDE VINding des loops der vaſte Sterren.

50 VOORSTEL.

Te verclaren hoemen deur ervarings dachtafels merct de vaſte Sterren in haer hemel vaſt te vveſen, en den hemel te drayen op den as des duyſteraers.

Angheſien dit roerſel ſoo traech is, datmen daer af weynich beſcheyts can mercken op den tijt deſer dachtafels van *Stadius* geduerende 52 jaren, ſoo ſullen wy hier en int volgende 51 voorſtel tot hulp gebruycken *Ptolemeus* nagelaten ſchriften vande plaetſen der vaſte ſterren, nemende voorbeeltſche wijſe al of die met deſe t'ſamen ons ervarings dachtafels maeckten.

Dit ſoo weſende, angeſien de vaſte ſterren nu in *Stadius* dachtafels met ſulcken verheyt van malcander ghevonden worden als ten tijde van *Ptolemeus*, na luyt des 5 hoofdſticx ſijns 7 boucx, en dat die doen in rechte linien ſtonden, nu daer in noch ſijn, niet teghenſtaende t'ſelve over de 1400 jaren gheleden is, ſoo beſluytmen daer uyt haer vaſticheyt.

Maer want haer duyſteraerbreeden de ſelve blijven, en haer evenaerbreeden met duyſteraerlangden veranderen, ſoo beſluytmen den heelen hemel te moeten draeyen op den as des duyſteraers. Als by voorbeelt de * are des Maechts *Spica virginis.*
diens Zuyderſche duyſteraerbreede *Ptolemeus* ſtelt op 2 tr. 10 ①, wort alſoo oock in *Stadius* dachtafels beſchreven: Maer de duyſteraerlangde die *Ptolemeus* ſtelde op 176 tr. 40 ①, is by *Stadius* van 197 tr. 38 ①: En haer evenaerbreede welcke *Ptolemeus* int 3 hoofdſtick ſijns 7 boucx ſtelt op 30 ① na het Zuyden, die comt, volghende *Stadius* beſchrijving 8 tr. 56 ① Zuytwaert, want hoewelſe in ſijn dachtafels niet en ſtaet, ſoo volght ſulcx uyt de boveſchreven duyſteraerlangde en breede deur het 9 wercſtuck der Hemelclootſche werckſtucken: En want dit roerſel gheſchiet na t'vervolgh der trappen, ſoo moetet ſijn van Weſten na Ooſten. TBESLVYT. Wy hebben dan verclaert hoemen deur ervarings dachtafels merckt de vaſte Sterren in haer hemel vaſt te weſen, en den hemel te drayen op den as des duyſteraers, na den eyſch.

EIGHTH CHAPTER

OF THE FIRST BOOK,

Of the Finding of the Motion of the Fixed Stars[1])

50th PROPOSITION.

To explain how it is found by means of empirical ephemerides that the fixed Stars are fixed in their heaven and that this heaven rotates on the axis of the ecliptic.

Since this motion is so slow that little information can be obtained about it in the time of these ephemerides of *Stadius,* which cover 52 years, here and in the subsequent 51st proposition we will use as aids the writings left by *Ptolemy* on the positions of the fixed stars, assuming by way of example that those together with these constitute empirical ephemerides to us.

This being so, since now in *Stadius'* ephemerides the fixed stars are found to be at the same distances from one another as at the time of *Ptolemy,* according to the 5th chapter of his 7th book, and that those which then were in straight lines now still are, notwithstanding the fact that this was more than 1400 years ago, it is concluded from this that they are fixed.

But because their ecliptical latitudes remain the same and their equatorial latitudes vary with the ecliptical longitudes, it is concluded that the whole heaven must rotate on the axis of the ecliptic. Thus, for example, *Spica Virginis,* whose Southerly ecliptical latitude *Ptolemy*[2]) puts at 2°10′, is also described thus in *Stadius'* ephemerides. But the ecliptical longitude, which *Ptolemy* put at 176°40′, is 197°38′ in *Stadius.* And its equatorial latitude, which *Ptolemy* in the 3rd chapter of his 7th book puts at 30′ to the South, according to *Stadius'* description comes at 8°56′ to the South, for though it does not occur in his ephemerides, this follows from the above-mentioned ecliptical longitude and latitude by the 9th problem of the problems on Heavenly Spheres. And because this motion takes place in the order of the degrees, it must be from West to East.

CONCLUSION. We have thus explained how it is found by means of empirical ephemerides that the fixed Stars are fixed in their heaven and that this heaven rotates on the axis of the ecliptic; as required.

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[1]) By a printing error in the original text the preceding page 114 has been called 104.
[2]) *Syntaxis* VII, 5 (Manitius II, p. 48).

51 VOORSTEL.

Deur ervarings dachtafels te vinden den eyghen loop der vaste Sterren.

Anghesien de Are des Maechts ten tijde van *Ptolemeus* int jaer 139 was in des duysteraers 176 tr. 40 ①, maer ten tijde van *Stadius* int jaer 1554 inden 197 tr. 38 ①, dats 20 tr. 58 ① voorder, soo heeftse de selve booch geloopen op den tijt tusschen beyden bedraghende 1415 jaren, daerom segh ick, 1415 jaren gheven 20 tr. 58 ①, wat 100 jaren? Comt 1 tr. 29 ①. En sghelijcx can openbaerlick ghevonden worden den loop op alle ghegheven tijt, waer af t'bewijs openbaer is. T'BESLVYT. Wy hebben dan deur ervarings dachtafels ghevonden den eyghen loop der vaste Sterren, na den eysch.

DES EERSTEN BOVCX

EINDE.

51st PROPOSITION

To find, by means of empirical ephemerides, the proper motion of the fixed Stars.

Since at the time of *Ptolemy* in the year 139 *Spica Virginis* was at 176°40′ of the ecliptic, but at the time of *Stadius* in the year 1554 at 197°38′, *i.e.* 20°58′ farther, it has passed over this arc in the time between these two, which is 1415 years. I therefore say: 1415 years give 20°58′; what do 100 years give? This gives 1°29′. And in the same way the motion in any given time can clearly be found, the proof of which is clear. CONCLUSION. We have thus found, by means of empirical ephemerides, the proper motion of the fixed Stars; as required.

END OF THE FIRST BOOK.

TWEEDE
BOVCK DES
HEMELLOOPS
VANDE
DVVAELDERLOOP
DEVR WISCONSTIGHE
vverckingghegront op de oney-
ghen ſtelling eens vaſten
Eertcloots

SECOND BOOK OF THE HEAVENLY MOTIONS

OF THE MOTIONS OF THE PLANETS

by Means of Mathematical Operations,
Based on the Untrue Theory[1]) of a Fixed Earth

[1]) The word *stelling*, as used by Stevin, corresponds literally to *supposition*, *assumption*. In the following we have translated it by *theory*, because of the vast scope of the Ptolemaic or Copernican conceptions, their numerous consequences, and the great amount of work which has been devoted to their development. In treatises on the history of Astronomy, the expression "planetary theories" is commonly used.

CORTBEGRYP
deses tvveeden Boucx.

INt eerste bouck des Hemelloops deur ervaringhen uyt den rouwen bemerckt sijnde, de Dwaelders in uytmiddelpuntighe ronden en inronden te loopen, met ander omstandighen dies angaende, waer me ons ghedacht een gront heeft, om deur wisconstighe werckinghen veel nauwer en sekerder t'ondersoucken oft het oock volcommen ronden sijn, en ofse daer in altijt eenvaerdich everas looopen, voort wat reden haer halfmiddellijnen en uytmiddelpunticheytlijnen tot malcander hebben, tot wat plaetsen sy in toecommende tijden sijn sullen, boven dien om te gheraken tot kennis der oneven daghen, der stilstanden en deysinghen, der grootheyt vande verduysterde deelen die Son en Maen in haer duysteringhen crijghen, der verheden daerse vanden Eertcloot in sijn, der grootheden diese hebben teghen den Eertcloot verleken, met meer ander dierghelijcke daer an clevende, soo sullen wy nu tottet tweede bouck commẽ, inhoudende der selve wisconstige werckinghen, en dat noch al op de ghemiste stelling eens vast en Eertcloots, gelijck de beschrijving van dien Ptolemeus *eerst ter handt quam, te weten sonder daer in vermengt te wesen sijn verdochte vonden der onbekende roersels (als de tweede oneventheyt der Maen, mette derde oneventheden van Saturnus, Iupiter, Mars, Venus en Mercurius, metsgaders het onbekent breederoersel der vijf laetste, welcke men int ghemeen der Dwaelders onbekende loop noemt) die ick daer uyt scheyden sal, en daer na besonderlick beschrijven, op dat elck alsoo claerlicker siende watter deur stelling eens vasten Eertcloots ghebreeckt, te bequamelicker na ander beter vonden trachten mach. En sal hier af seven onderscheytsels maken.*

Het 1 vande wisconstighe uytmiddelpunticheyts handel int ghemeen.

Het tweede vande Sonloop.

Het 3 vande Maenloop.

Het 4 van Saturnus, Iupiters, Mars, Venus en Mercurius loop.

Het 5 van der Dwaelders saminghen, teghenstanden, en duysteringhen.

Het 6 van Ptolemeus *verdochte tweede oneventhedẽ der Maen, en derde oneventheden van Saturnus, Iupiter, Mars, Venus en Mercurius.*

Het 7 van Ptolemeus *verdochte breedeloop der vijf Dwaelders, Saturnus, Iupiter, Mars, Venus en Mercurius.*

MERCKT.

Ick noem dese wercking wisconstich, tot onderscheyt der wercking ghetrocken uyt ervaringhen int eerste bouck, en hoewelse dickwils deur rekeningen met tafels in geen heele volcommenheyt der getalen en bestaet, gelijck in volcommen wisconstighe wercking vereyscht wort, nochtans anghesien daer in is een voet van oneindelicke naerdering, mettet wisconstich groote ghemeenschap hebbende, en de bewijsen oock wisconstich sijnde, soo schijnet datmense om t'boveschreven onderscheyts wille wisconstich noemen mach.

SUMMARY OF THE SECOND BOOK

As it has been roughly perceived in the first book of the Heavenly Motions by experience that the Planets move in eccentric circles and epicycles, with other circumstances relating thereto, so that our thoughts have a basis on which to examine much more accurately and surely, by means of mathematical operations, whether the circles are perfect circles and whether they always move therein with uniform velocity, further what proportion their semi-diameters and lines of eccentricity have to one another, in what places they will be in future times, and further to acquire knowledge of the unequal days, of the stations and retrogradations, of the magnitude of the eclipsed parts of Sun and Moon during their eclipses, of their distances from the Earth, of their magnitudes as compared with the Earth, with more such related phenomena, we shall now come to the second book, containing their mathematical treatment, such all on the incorrect theory of a fixed Earth, as the description thereof first came into the hands of *Ptolemy,* to wit, without being mingled with his supposed discoveries of the unknown motions (such as the second inequality of the Moon, with the third inequalities of Saturn, Jupiter, Mars, Venus, and Mercury, as also the unknown motion in latitude of the five last-mentioned planets, which in general is called the unknown motion of the Planets), which I will remove therefrom and describe thereafter separately, so that everybody, thus seeing all the more clearly what defect is involved in the theory of a fixed Earth, may strive the more easily to make other and better discoveries. And I will make thereof seven chapters.

The first of the mathematical treatment of eccentricity in general.

The second of the Sun's motion.

The third of the Moon's motion.

The fourth of the motions of Saturn, Jupiter, Mars, Venus, and Mercury.

The fifth of the Planets' conjunctions, oppositions, and eclipses.

The sixth of *Ptolemy's* supposed second inequalities of the Moon, and third inequalities of Saturn, Jupiter, Mars, Venus, and Mercury.

The seventh of *Ptolemy's* devised motion in latitude of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury.

NOTE.

I call this operation mathematical to distinguish it from the procedure based on experiences, in the first book; although it often operates, through computations with tables, with numbers which are lacking the perfect accuracy required in perfect mathematical operations; nevertheless, since there is in it a basis of infinite approximation, having great similarity to the mathematical, and since the proofs are also mathematical, it seems that because of the above distinction it may be called mathematical.

[*For reasons, set forth in the Introduction, the Second Book has not been reproduced, with the exception of the last paragraph only*] ſeyt is, ſoude connen verſtrecken tot ghemeene reghel van al d'ander, overmidts men mette bekende winſt van d'een Dwaelder boven d'ander ſoo ſoude wercken, als vooren gedaen is: Sulcx datter niet en gebreeckt dan ſekerder kennis van der Dwaelders loop in langde en breede.

2 MERCK.

Mijn voornemen was int beſchrijven des Cortbegrijps deſes 2 boucx als blijckt, hier noch by te voughen een ſeſte Onderſcheyt van *Ptolemeus* verdochte tweede oneventheden der Maen, en derde oneventheden van Saturnus, Iupiter, Mars, Venus, en Mercurius in langdeloop: Metſgaders een ſevende Onderſcheyt van *Ptolemeus* verdochte breedeloop der vijf Dwaelders, Saturnus, Iupiter, Mars, Venus, en Mercurius: Maer nadien ick volghens mijn voornemen, int eerſte en tweede bouck beſchreven hadde den Hemelloop met ſtelling eens vaſten Eertcloots, ſonder daer in te vermenghen *Ptolemeus* vonden der boveſchreven onbekende roerſels, die ick beſonderlick alleen gheſtelt hadde: Sghelijcx oock gedaen hebbende met *Copernicus* beſchrijving eens roerenden Eertcloots, die ſijn eyghen vonden der ſelve onbekende roerſels daer oock in vermengde, welcke ick mede daer uyt liet, en alleen beſchreef, om daer deur de ſoucking des onbekenden handels voor yghelick claerder en verſtaenlicker te maken, en na beter ſpiegeling te meugen trachten: Soo iſt gebeurt dat t'ghene ick aldus voor anderē bereyt hadde, my ſelf tot inleyding verſtrecte, om tot ſpiegeling te geraken die my beter docht, want alſoo ick eens quam te overſien mijn geſchreven derde bouck na *Copernicus* wijſe (dat een tijt lang ſtil gelegen hadde) om dat inden druck te brengen, ick quam tot ander kennis des breedeloops der Dwaelders, Saturnus, Iupiter, Mars, Venus, en Mercurius, ſulcx dat my docht de ſelve gheen onbekende roerſels meer en behooren te heeten, en vervolghens oock onnoodich te weſen het voornoemde ſevende Onderſcheyt te beſchrijven, want alle ramingen der menſchē int ſoucken der Dwaelders loop, ſeer nauwe te willen deurgronden, het ſchijnt datmē den tijt beter ſoude connen beſteden met ghewiſſe dinghen te leeren. Nu dan angeſien de breede loop om de voorgaende reden comen ſal int derde bouck met ſtelling eens roerenden Eertcloots, ſoo is dit d'oirſaeck waer deur ick het boveſchreven ſevende Onderſcheyt hier uyt laet. Angaende het 6 Onderſcheyt, dat heb ick oirboir verſtaen inden Anhang des Hemelloops te brengen, om de redenen welcke aldaer van dies ſullen verclaert worden.

Noch is te ghedencken, dat ſoo ymant int voorgaende of volghende deſer wiſconſtige ghedachteniſſe, quaem t'ontmoeten woorden of redenen in welcke de boveſchrevē roerſels der breede onbekent genoemt worden, die ick nochtans nu ſegh bekent te ſijn, d'oirſaeck daer af te weſen dat het volghende derde bouck t'laetſte was, datter gedruckt wiert, hoewel ander ſtoffen t'bouck gebonden ſijnde daer achter volghen, en dat ick als gheſeyt is, int overſien des ſelven derden boucx eerſt tot breeder kennis gherocht.

DES TWEEDEN BOVCX
EYNDE.

FIFTH CHAPTER

OF THE SECOND BOOK

63rd PROPOSITION 2nd NOTE

When writing the Summary of this 2nd book, I meant to add — as appears — a sixth Chapter on *Ptolemy's* devised second inequalities of the Moon and third inequalities of Saturn, Jupiter, Mars, Venus, and Mercury in the motion in longitude; as also a seventh Chapter on *Ptolemy's* imagined motion in latitude of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury. But after, according to my intention, in the first and the second book I had described the Heavenly Motions on the theory of a fixed Earth, without including therein *Ptolemy's* discoveries of the above-mentioned unknown motions, which I had placed apart, while I had also done similarly with *Copernicus'* description of a moving Earth, who also included therein his own findings of the said unknown motions, which I also left out and described separately, in order thus to make the search for the unknown phenomena clearer and more intelligible for everyone and to be able to strive after a better theory, it so happened that what I had thus prepared for others, served as a preliminary for myself to arrive at a theory which seemed better to me. For when I went through my third book as written after the manner of *Copernicus* (which had been laid aside for some time) in order to prepare it for publication, I arrived at other knowledge of the motion in latitude of the Planets Saturn, Jupiter, Mars, Venus, and Mercury, so that it appeared to me that they ought no longer to be called unknown motions, and that consequently it was also unnecessary to describe the aforesaid seventh Chapter, because it seems that we might spend our time better teaching things that are certain rather than wish to fathom exactly all men's estimates in the search after the Planets' motions. Since therefore the motion in latitude for the aforesaid reason is to come in the third book, on the theory of a moving Earth, this is the cause why I here omit the above-mentioned seventh Chapter. As to the 6th Chapter, I thought it appropriate to put this in the Supplement of the Heavenly Motions, for the reasons that will there be given for it.

It is further to be borne in mind that if in the preceding or following parts of these mathematical memoirs anyone should come across words or arguments in which the above-mentioned motions in latitude are called unknown, which nevertheless I now say to be known, the cause of this is that the subsequent third book was the last to be printed, though, after the book was bound, other subjects follow thereafter, and that, as has been said, I arrived at greater knowledge while going through the said third book.

END OF THE SECOND BOOK.

DERDE
BOVCK DES
HEMELLOOPS,
VANDE
VINDING DER DVVAEL-
DERLOOPEN, DEVR WIS-

constighe vvercking ghegront op de
wesentlicke stelling des roe-
renden Eertcloots.

THIRD BOOK
OF THE HEAVENLY MOTIONS

OF THE FINDING OF THE MOTIONS OF THE PLANETS

by Means of Mathematical Operations,
Based on the True Theory of the Moving Earth

CORT BEGRYP DESES DERDEN BOVCX.

OM de ſomme van deſen te verclaren, ſoo ſtaet te ghedencken dat de Dvvaelders tvveederley loop hebben, d'een in langde, de ander in breede: Angaende de langdeloop, die ſal hier vviſconſtelick bevveſen vvorden met ſtelling eens roerenden Eertcloots t'ſelve beſluyt te crijgen, dat ſe met ſtelling eens vaſten heeft, alleenelick daer in verſchillende, dat t'ghene mette ſtelling eens vaſten voor vreemt en met vervvondering angheſien vvort, deur d'ander ſonder vvonder is, als gegront ſijnde op t'gene vveſentlick inde natuer beſtaet. Vyt het voornomde bevvijs, te vveten dat d'een en d'ander ſtelling een ſelve beſluyt voortbrengt, ſal deſe beſchrijving ſeer cort vallẽ, vvant ick op deſe roerende ſtelling geen nieuvve vvercſtucken maken en ſal van t'vinden der Hemelloopſche dinghen, maer voor gemeene regel nemen, dat al t'gene ons vande langdeloop inde daet voorvalt te berekenen, gedaen ſal vvorden deur de vverckſtucken gegront op ſtelling eens vaſten Eertcloots, en beſchreven inde voorgaende tvvee eerſte boucken: Ick ſal oock in een beſonder voorſtel verclaren mijn gevoelen, vvaerom ick ſulcke vvercking gedaen op een oneygen ſtelling, voor bequamer houde dan die op de eygen vvare gegront is. Na de langdeloop ſal de breedeloop volgen, die ick oock met d'een en d'ander ſtelling een ſelve beſluyt voortbrenghende beſchrijven ſal.

Dit inhoudt int gemeẽ ſoodanich ſijnde ſal vijf onderſcheytſels hebben. Het eerſte van der Dvvaelders Hemelẽ gedaente, ſo veel noodich ſchijnt tot verclaring haers loops met ſtelling eens roerenden Eertcloots. Daer na ſullen volgen drie onderſcheytſels vande langdeloop der Dvvaelders met ſtelling eens roerendẽ Eertcloots: Te vveten het tvveede vandẽ Eertcloot: Het derde vande Maẽ: Het vierde van Saturnus, Iupiter, Mars, Venus, en Mercurius. Daer na ſal volgen t'laetſte onderſcheyt des breedeloops met ſtelling eens roerenden Eertcloots.

EER-

SUMMARY OF THIS THIRD BOOK

To expound all this, it should be borne in mind that the Planets have two kinds of motion, one in longitude and the other in latitude. As regards the motion in longitude, this will here be proved mathematically to lead to the same conclusion on the theory of a moving Earth as on the theory of a fixed Earth, with the only difference that what is considered strange and gives rise to astonishment in the theory of a fixed Earth does not give rise to astonishment in the other theory, because it is based on what actually happens in nature. On account of the aforesaid proof, to wit, that the one as well as the other theory leads to the same conclusion, this description will be very short, for on this theory of a moving Earth I will not make any new problems of the finding of things connected with the Heavenly Motions, but I will make it a general rule that all that we have to compute in practice with regard to the motion in longitude must be done by means of the propositions based on the theory of a fixed Earth and described in the preceding first two books. In a special proposition I will also set forth my opinion why I regard such a procedure based on an untrue theory as more convenient than that which is based on the true theory. The motion in longitude is to be followed by the motion in latitude, which I will also describe as leading to the same conclusion with one theory and with the other.

The contents of this book, which are in general as given above, are divided into five chapters. The first of the figure of the Planets' Heavens, as much as seems necessary to explain their motion on the theory of a moving Earth. This is to be followed by three chapters on the motion in longitude of the Planets, on the theory of a moving Earth: to wit, the second of the Earth, the third of the Moon, the fourth of Saturn, Jupiter, Mars, Venus, and Mercury. This is to be followed by the last chapter, of the motion in latitude on the theory of a moving Earth.

EERSTE ONDERSCHEYT
DES DERDEN BOVCX VAN der Dvvaelders Hemelen gedaente, soo veel noodich schijnt tot verclaring haers loops met stelling eens roerenden Eertcloots.

CORTBEGRYP DESES EERSTEN ONDERSCHEYTS.

NAdien t'voornemen is hier te beschrijven der Dvvaelders loop met stelling eens roerenden Eertcloots, soo schijnet oirboir vant vvesen en gedaente des vverelts, als gront daer t'voorgaende op ghebout vvort, soo veel te verclaren als vvy daer af vveten of vermoeden, en totte kennis des voorghenomen loops behulpich is: Tot dien einde sal ick in dit eerste Onderscheyt ses voorstellen beschrijven.

Het eerste van d'oirden der hemelen vande Dvvaelders, met stelling eens roerenden Eertcloots.

Het tvveede van des Eertcloots roersel in plaets, en haer seylsteenighe stilstandt.

Het derde vande seylsteenighe stilstant der Dvvaeldervveghen en haer hemelen.

Het vierde vande plaets des crachts die den Eertcloot en vveghen der Dvvaelders in haer seylsteenighe stilstandt houdt.

Het vijfde dattet niet nootsakelicken blijckt de Son middelpunt te vvesen vanden vastesterrens hemel, maer met goede reden daer toe vercoren vvort.

Het seste vande vervvonderinghen sonder vvonder der ghene die een vasten Eertcloot stellen.

I VOORSTEL.

Te beschrijven d'oirden der hemelen vande Dvvaelders met stelling eens roerenden Eertcloots.

De Menschen eertijts deur stelling eens vasten Eertcloots gecommen sijnde tot stelling van sommighe inronden, waer in der Dwaelders keering in tijt

FIRST CHAPTER

OF THE THIRD BOOK

of the Figure of the Planets' Heavens, as Much as Seems Necessary to Expound Their Motion on the Theory of a Moving Earth

SUMMARY OF THIS FIRST CHAPTER

Since the object is here to describe the Planets' motions on the theory of a moving Earth, it seems appropriate to set forth as much about the nature and figure of the world — being the basis on which the foregoing is built — as we know or surmise and is conducive to the knowledge of the motion in view. To this end I will describe six propositions in this first Chapter.

The first of the arrangement of the Heavens of the Planets, on the theory of a moving Earth.

The second of the motion of the Earth in its place and its magnetic rest.

The third of the magnetic rest of the Planets' orbits and their Heavens.

The fourth of the place of the force keeping the Earth and the orbits of the Planets in their magnetic rest.

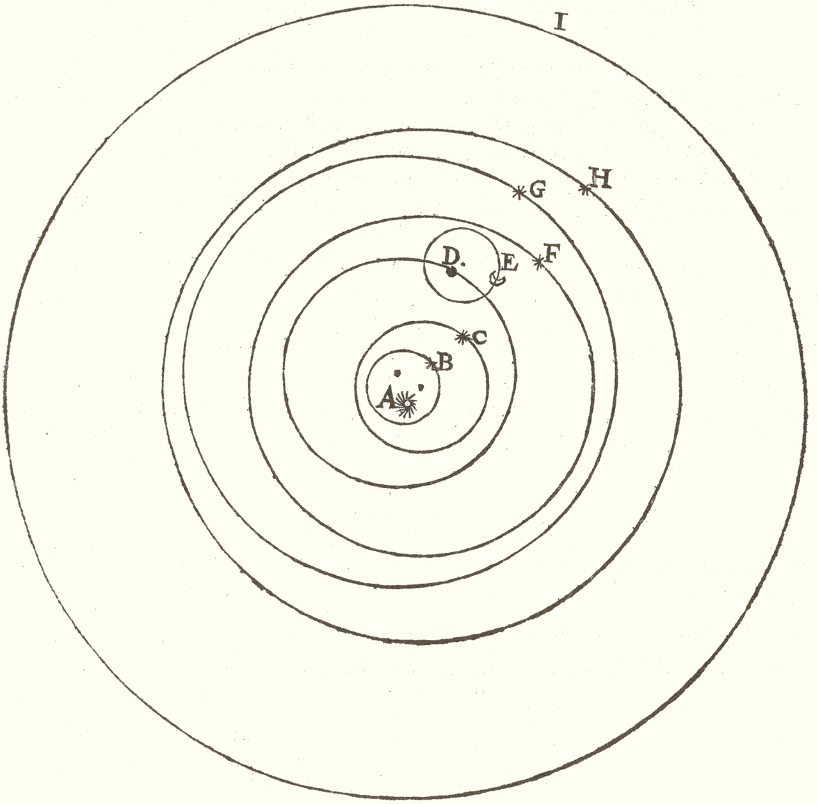
The fifth that it appears not to be necessary that the Sun should be the centre of the Heaven of the fixed stars, but that it is chosen for this with good reason.

The sixth of the wondering at what is no wonder, of those who assume a fixed Earth.

1st PROPOSITION.

To describe the arrangement of the Heavens of the Planets on the theory of a moving Earth.

Whereas formerly, on the theory of a fixed Earth, man came to assume some epicycles on which the revolution of the Planets corresponded exactly in time effen over quam mettet overſchot des Sonloops boven den loop vant inronts middelpunt in haer wegen, als van Saturnus, Iupiter en Mars: Voort ander inrond:n diens middelpuntens loop effen mette Sonloop overquam, ghelijck int eerſte bouck gheſeyt is, ſulcx en docht ander menſchen die deſe ſaken grondelicker begoſten t'onderſoucken, alſoo inde natuer niet te beſtaen: Overdenckende daer na oſmen die inronden met haer verſierde roerſels niet en ſoude meugen weeren, ſtellende den Eertcloot in een rondt te loopen, bevonden dat ja, ghelijck by ettelicke ouden betuycht wort eertijts ſmenſchen ghevoelen van een loopenden Eertcloot gheweeſt te hebben, maer de eyghentlicke ſorm en ghedaente van hun beſchrijving en is na den onderganck des Wijſentijts ter handt van *Ptolemeus* noch ymannt anders ghecommen, datmen weet, dan met veel ander dinghen verloren bleven. Doch is ten laetſten *Nicolaus Copernicus* verſchenen, welcke de ſelve ſtelling, of een ander die groote gemeenſchap daer me ſchijnt te hebben, weerom int licht gebrocht heeft, welcke ick in dit voorſtel beſchrijven ſal, en tot dien einde aldus ſegghen: Binnen den Hemel der vaſte ſterren by *Copernicus* onbeweeghlick geſtelt, ſijn oirdentlick vervolghende de hemelen van Saturnus, Iupiter, en Mars, daer na des Eertcloots opt natuerlick jaer een keer volbrenghende, met noch twee roerſels in plaets, waer af



de verclaring int 2 Hooftſtick ghedaen ſal worden weſende de middellijn deſes hemels ſonder gevoelelicke reden tegen de halſmiddellijn des Hemels der vaſte ſterren to the surplus of the Sun's motion over the motion of the epicycle's centre in their orbits, as of Saturn, Jupiter, and Mars — while further the motion of the centres of other epicycles (Venus, Mercury) corresponded exactly to the Sun's motion, as has been said in the first book — it appeared to other men, who began to inquire into these matters more thoroughly, that this did not exist in nature. Considering thereafter whether those epicycles with their imagined motions could not be eliminated, assuming the Earth to move in a circle, they found that indeed, as is stated by several ancient writers, man formerly held the opinion that the Earth moved; but the true form and appearance of their description did not, after the end of the Age of the Sages, come into the hands of either *Ptolemy* or anyone else, as far as we know, but they were lost together with many other things. But at last *Nicolaus Copernicus* appeared, who has brought to light again this theory, or one that seems to resemble it closely, which I shall describe in this proposition, and to this end I say: Inside the Heaven of the fixed stars, assumed by *Copernicus* to be immovable, there are arranged in due order the Heavens of Saturn, Jupiter, and Mars, thereafter that of the Earth, performing one revolution in the natural year, with two more motions in its place, the exposition of which will be given in the 2nd Chapter, the diameter of this Heaven having no perceptible ratio to the semi-diameter of the Heaven of the fixed ſterren: Rontom den Eertcloot draeyt de Maen in een uytmiddelpuntichront, daer na volghen Venus en Mercurius, loopende in uytmiddelpuntighe ronden om de Son, die als weerelts middelpunt onbeweeghlick blijft. De loopen der boveſchrevē ſes Dwaelders ende des Eertcloots ſijn altemael na t'vervolgh der trappen, dat is van Weſten int Ooſten.

Om t'boveſchreven deur een form opentlicker te verclaren, laet den tip A de vaſtſtaende Son beteyckenen, rontom welcke beſchreven ſijn ſes uytmiddelpuntighe ronden: Int eerſte van dien gaet Mercurius an B: Int tweede Venus an C: Int derde den Eertcloot an D, en daerom in een uytmiddelpuntichront de Maen an E: Int vierde vijfde en ſeſte volghen oirdentlick Mars, Iupiter, Saturnus gheteyckent F, G, H, altemael ſonder inront, I beduyt den Hemel der vaſte ſterren beſchreven opt middelpunt A, t'welck als vooren de Son is.

De boveſchreven oirden aldus eenvoudelick int corte gheſeyt ſijnde ſonder eenich bewijs, ſoo ſullen wy nu daer af wat breeder verclaring doen. De reden waer uyt men merckt Venus en Mercurius binnen den Eertclootwech te loopen, is, datſe nummermeer tot tegheſtant der Son en gheraken: Maer Mercurius loop binnen Venus wech begrepen te weſen, blijckt an ſijn cleender afwijckinghen daer t'ſijnder plaets int eerſte bouck deur ervaringen afgheſeyt is.

Belanghende dat ymant twijffelen mocht en ſegghen, Venus en Mercurius wegen binnen den Eertcloot te connen weſen, ſonder nochtans de Son te vervanghen, dat en mach ſoo niet ſijn, om dat haer meeſte verheden vanden Eertcloot veel langher bevonden worden dan des Eertclootwechs halfmiddellijn, na t'inhoudt der rekening die daer af int volghende ghedaen ſal worden. Angaende Saturnus, Iupiter en Mars, ghemerckt die tot tegheſtant der Son geraken, ſoo moeten haer weghen den Eertclootwech vervanghen, anders waert onmeughelick. De reden waer uyt men merckt Saturnus de verſte te weſen, daer na Iupiter, en Mars de naeſte, can tweederley ſijn: Ten eerſten dat den Dwaelder der twee die in ſaming bedeckt wort, de verſte moet ſijn, doch dat ghebeurt ſeer ſelden, oock ſijnder inde werelt te luttel gaſlaghers dieder op paſſen. Ten anderen dat ſulcx deur ſtelling eens loopenden Eertcloots (anders dan deur ſtelling eens vaſten Eertcloots, waer me dit onbekent blijft) dadelick ghemeten can worden, ghelijckmen opt landt meet welcke ſichtbaer torre wijtſt van ons is, want de verheyt van twee ſtanden die den Eertcloot tot verſcheyden plaetſen haers wechs heeft, verſtreckt ons voor driehoucx * gront, hebbende teghen haer ſijden ſeer ghevoelelicke reden, ghelijck daer af t'ſijnder plaets eyghentlicker gheſeyt ſal worden. Angaende de reden waer uyt men beſluyt de Maenloop te moeten weſen ghelijck de voorgaende form anwijſt, is duſdanich: By aldienſe liepe ghelijck een van d'ander Dwaelders, dat ſoude moeten ſijn buyten den Eertclootwech, ghelijck Saturnus, Iupiter en Mars, of daer binnen ghelijck Venus en Mercurius. Buyten en cant niet weſen, om datter alſdan nummermeer Sonduyſtering vallen en ſoude. Binnen en macht oock niet ſijn om tweederley reden: Ten eerſten datſe nummermeer ghelijck Venus en Mercurius, in tegheſtandt der Son en ſoude gheraken. Ten anderen datſe ſoude connen commen achter de Son, ſulcx dat al waren dan haer beyder middelpunten en het oogh des ſienders, alle drie in een rechte lini, ſoo en ſouder nochtans gheen Sonduyſtering weſen, ſtrijdende teghen d'ervaring.

* Baſis.

De redenen waerom ghelooft wort de ſtelling des loopenden Eertcloots lijckformich te weſen mettet ghene inde natuere beſtaet, en niet de ſtelling des vaſten Eertcloots, ſijn duſdanich: Ten eerſten datmen daer deur beſluyt, ghelijck int volghende breeder blijcken ſal, de Dwaelders eenvoudelick te drayen

 in ronstars. Round the Earth moves the Moon in an eccentric circle; after this follow Venus and Mercury, moving in eccentric circles round the Sun, which remains immobile, as being the centre of the world. The motions of the above-mentioned six Planets and of the Earth are all in the order of the degrees, *i.e.* from West to East.

To explain the above more clearly by means of a figure, let the dot *A* denote the fixed Sun, about which have been described six eccentric circles. In the first of these moves Mercury at *B,* in the second Venus at *C,* in the third the Earth at *D,* and round this in an eccentric circle the Moon at *E.* In the fourth, fifth, and sixth follow successively Mars, Jupiter, Saturn, drawn at *F, G, H,* all without any epicycle; *I* denotes the Heaven of the fixed stars, described about the centre *A,* which, as said above, is the Sun.

After the above-mentioned arrangement has thus been simply stated in brief, without any proof, we shall now give a more detailed exposition of it. The reason from which it is perceived that Venus and Mercury move within the Earth's orbit is that they never come in opposition to the Sun. But the fact that Mercury's motion is contained within Venus' orbit becomes apparent from its smaller deviations, which have been referred to in their place in the first book by means of experiences.

As for the case that someone should doubt and say that Venus' and Mercury's orbits may be within that of the Earth, but without containing the Sun, this cannot be so because their greatest distances from the Earth are found to be much greater than the semi-diameter of the Earth's orbit, according to the contents of the computation that is to be made thereof in the following. As to Saturn, Jupiter, and Mars, since they come in opposition to the Sun, their orbits must contain the Earth's orbit, otherwise it would be impossible. The reason from which it is perceived that Saturn is the most distant, then Jupiter, and Mars the nearest may be twofold. Firstly, that that Planet of the two that is covered in conjunction must be the most distant, but this happens very seldom, and there are also too few observers in the world who attend to it. Secondly, that on the theory of a moving Earth (unlike on the theory of a fixed Earth, when this remains unknown) this can be measured in practice, as we measure on the land which visible tower is most distant from us, for the distance of two positions which the Earth has in different places of its orbit serves us as base of a triangle, whose ratio to the sides is very perceptible, as will be described more truly in its place. As for the reason from which it is concluded that the Moon's motion must be as denoted in the foregoing figure, this is as follows. If it moved like one of the other Planets, this would have to be outside the Earth's orbit, like Saturn, Jupiter, and Mars, or within it, like Venus and Mercury. It cannot be outside, because there would then never occur a Solar Eclipse. It cannot be within either, for two reasons. Firstly, that it would never, like Venus and Mercury, come in opposition to the Sun. Secondly, that it might come behind the Sun, so that, even if the centres of both and the eye of the observer were all three in a straight line, there would yet be no Solar Eclipse, which is contrary to experience.

The reasons why it is believed that the theory of the moving Earth is in accordance with what happens in nature, and not the theory of the fixed Earth, are as follows. Firstly, that it is concluded therefrom, as will become more fully apparent in the following, that the Planets simply revolve in circles, without in ronden, ſonder ſtelling van inronden met haer verſierde loopen inde ſel-ve, welcke daer deur al verlaten worden.

Ten tweeden, angeſiẽ het inde natuer ſoo veroirdent is, dat de Dwaelders die inde grootſte ronden of hemels draeyen, ſlappelicxt ommeloopen, ſoo ſtrijdet tegen deſe gemeene oirden alſmen den aldergrootſten Hemel der vaſte ſterren ſtelt alderſnelſt te draeyen, te weten alle daghe een keer: Ende is daerom de natuerlicke reden lijckformiger te gelooven en te ſtellen ſulck alderſnelſte roerſel het cleenſte rondt toe te commen, te weten t'rondt des Eertcloots in ſijn plaets.

Ten derden, na dient int weſen ſoo vervought is, dat al de loopẽ der Dwaelders ſijn van Weſten in Ooſten, ſoo ſtrijdet tegen deſe gemeene oirden datmen dẽ loop der vaſte ſterren verkeerdelick ſtelt van Ooſtẽ in Weſten, en is daerom de natuerlicke reden lijckformiger te ghelooven en te ſtellen ſulcke loop den Eertcloot toe te commen, oock van Ooſten na Weſten, gelijck van al d'ander.

Angaende mẽ de ſwaerheyt des Eertcloots houdt voor oirſaeck van haer onbeweeghlicheyt, men mocht daer op aldus antwoordẽ: Angeſien dẽ Eertcloot openbaerlick eẽ Hemels licht is, ontfangende vande Son haer claerheyt gelijck de Maen, ten is niet buyten natuerlicke reden, toe te ſtaen dat de ſtof deſer twee lichten, en oock van al d'ander ſterren, deur gelijcke genegentheyt by malcander gehouden worden, te weten ghelijck de ſtoffen daer den Eertcloot uyt beſtaet, gheneycht ſijn na heur middelpunt te ſtrecken, en derwaert te vallen ſoo lange tot datſe niet voorder en connẽ, en dã geduerlick derwaert druckẽ, dattet alſoo oock toegaet mette ſtoffen daer d'ander voorſchrevẽ lichten uyt beſtaen, welcke ghemeene ghenegentheyt om natuerlicke redenen oock nootſakelick ſchijnt, want by aldient ſoo niet en waer, het ſandt, water, en ander Eertſche ſtoffen, ſoudẽ van malcander ſcheyden ſonder een clootſche form te blijvẽ, genouchſaem als eẽ ſichtbaer hoop aſſchẽ die deur de wint int wilde wech vlieght, verſpreyt en onſichtbaer wort: Nu dan de natuerlicke reden willende datmen toelate, ſoodanighe ghelijcke lichten ghelijcke eyghenſchappen te hebben, en datmen voor ſeker houdt de ſwaricheyt der Dwaelders hemlien gheen onbeweeglickheyt te veroirſaken, men behoort derghelijcke vande ſwaricheyt des Eertcloots oock te oirdeelen, en ſulcx ſchijnt oock t'ghevoelen van *Copernicus* int 9 Hoofſtick ſijns eerſten boucx: Maer om hier af by voorbeelt noch wat claerder te ſpreken, ghenomen dat eenighe menſchen verre vanden Eertcloot waren, en ſiende die blincken als een Dwaelder tuſſchen d'ander hemelſche lichten (welcke claerheyt voornamelick ſijn moet ter plaets daer t'water vande Son beſchenen is) dat ymant van hemlien ſeyde die ſterre boven d'ander wonderlick ſwaer te weſen, t'is kennelick dat d'ander menſchen (ſulcx ſonder bewijs gheſeyt weſende, en gheen teycken van meerder ſwaricheyt int een licht dan int ander ſiende) dien eenen niet ghelooven en ſouden: Ende alſoo en behooren wy die op den Eertcloot ſijn, hem noch al de ghene die hier alſoo ſonder bewijs ſpreken, oock niet te ghelooven.

Belangende t'ghene *Ptolemeus* teghen de ſtelling eens roerenden Eertcloots voorwent, te weten by aldien ſy draeyde dat torren en gheſtichten ſouden omvallen, van weghen de gheweldighe ſtrijcking dieſe teghen de locht ſouden doen: Voort dat yet opſpringhende, niet ter ſelve plaets neer en ſoude vallen, maer ſoo verre van daer als den Eertcloot daerentuſſchen verloopen waer: Al dit en mach niet beſtaen uyt oirſaeck dat na ſulck gheſtelde de ſelve ongevallen ſouden moeten volgen met ſtelling eens vaſten Eertcloots, want nadiẽ mẽ hier neemt dat de locht of den hemel die dẽ Eertcloot vervangt, niet en draeyt metten Eertcloot, maer d'een alleen ſonder d'ander, ſoo ſal de locht om den vaſten

Eert-the assumption of epicycles with their imagined motions therein, which are thus all eliminated.

Secondly, since it is so arranged in nature that the Planets revolving in the largest circles or heavens revolve slowest, it is contrary to this general arrangement if the greatest Heaven of all, that of the fixed stars, is assumed to move fastest of all, to wit, every day one revolution. And therefore it is more in accordance with natural reason to believe and to assume that this fastest motion of all is to be assigned to the smallest circle, to wit, the circle of the Earth in its place.

Thirdly, since matters are so arranged that all the motions of the Planets are from West to East, it is contrary to this general arrangement that the motion of the fixed stars should inversely be assumed to be from East to West, and therefore it is more in accordance with natural reason to believe and to assume that this motion is to be assigned to the Earth, also from West to East [1]), like all the others.

If the heaviness of the Earth is held to be the cause of its immobility, to this the following answer may be given: Since the Earth is evidently a Heavenly luminary, receiving from the Sun its light, like the Moon, it is not beyond natural reason to admit that the matter of these two luminaries, and also of all the other stars, is kept together by a similar affinity, to wit, as the substances of which the Earth consists tend towards its centre and to fall in that direction until they cannot get any further and then continually press in that direction; and that it also happens thus with the substances of which the other aforesaid luminaries consist, which general affinity also seems necessary for natural reasons; for if it were not so, the sand, water, and other Earthy substances would fall apart without the spherical form being preserved, somewhat like a visible heap of ashes which flies away at random on the wind, is dispersed, and becomes invisible. Natural reason therefore requiring it to be admitted that such similar luminaries have similar properties, and it being considered certain that the heaviness of the Planets does not cause their immobility, it should also be judged similarly of the heaviness of the Earth, and this also seems to be the opinion of *Copernicus* in the 9th Chapter of his first book. But to speak a little more clearly about this by means of an example: let us assume that some people were far from the Earth, and saw it shine like a Planet among the other heavenly luminaries (which brightness must be mainly in the places where the water is lit up by the Sun), and that one among them should say that this star was singularly heavy, more so than the others, it is evident that the other people (since this was said without any proof and they saw no sign of greater heaviness in one luminary than in another) would not believe that one man. And in the same way, we who are on the Earth ought not to believe him nor all those who here thus speak without any proof.

As to what *Ptolemy* [2]) advances against the theory of a moving Earth, to wit, that if it moved, towers and buildings would collapse, on account of their enormous friction against the air; further, that something which leaps up would not fall down in the same place, but so far thence as the Earth had meanwhile moved — all this cannot be so, because on this supposition the same accidents would have to happen on the theory of a fixed Earth; for since it is here assumed that the

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[1]) For *van Oosten na Westen* in the Dutch text read *van Westen na Oosten.*
[2]) *Syntaxis* I, 7 (Manitius p. 19).

Eertcloot vlieghende even soo stijf teghen de gestichten strijcken, als den loopenden Eertcloot teghen de stilstaende locht, want ghenomen dat een stock overeinde ghesteken sy in een loopende rivier, en een ander stock deur een stilstaende water over einde voort ghedronghen worde, soo ras als t'water vande rivier loopt, t'is toe te laten dattet water teghen d'een en d'ander stock evestijf sal drucken: En alsoo soudet oock openbaerlick toegaen mette locht teghen de ghestichten, of de ghestichten teghen de locht. Vyt het ghene voorseyt is soude volghen, dat yet op een vasten Eertcloot opspringhende, niet ter selver plaets weerom neer en soude vallen, maer soo verre van daer als hem de locht daerentusschen wech dronghe. Doch sulcx niet ghebeurende, soo ist nootsakelick dat de lochtcloot en den Eertcloot van hem vervanghen, t'samen een cloot maken, die mette stelling eens vasten Eertcloots int gheheel stil staet, of mette stelling eens draeyenden Eertcloots int gheheel draeyt: En wijder die sulcke vereening des cloots uyt aerde en locht, seggen soo te wesen alsmen van een vasten Eertcloot spreect, maer soo niet te sijn als t'verschil van een loopende is, met hemlien die alsoo haer selven tegenspreken, soudet ander swaer vallen te overcommen. T'BESLVYT. Wy hebben dan beschreven d'oirden der hemelen vande Dwaelders met stelling eens roerende Eertcloots, na dē eysch.

2 VOORSTEL.

Te verclaren des Eertcloots roersel in plaets, en haer seylsteenighe stilstandt.

Benevens den loop des Eertcloots van plaets tot plaets in haer wech, doende alle jare een keer uyt Westen na Oosten, daer int 1 voorstel afghesyt is, soo heeftser noch twee in plaets: D'een loop is den daghelickschen keer op haer as van Westen in Oosten, maer om dit roersel in plaets by voorbeelt wat breeder te verclaren, men mocht segghen dattet is ghelijck een draeyenden slijpsteen in een varende schip, welcke deur t'schip een roersel ontfangt van plaets tot plaets, maer heur draeying op den as blijft daerentusschen int schip op een selve plaets, en alsoo metten Eertcloot nieuwelini. D'ander loop (na dē sin van *Copernicus* int 11 Hoofstick sijns 1 boucx) is dusdanich: Te wijle den Eertcloot haer jaerlickschen keer doet van Westen na Oosten daer int 1 Hoofstick af gheseyt is, soo doetse daerentusschen op de selve tijt teghen den voorschreven loop een keer in plaets van Oosten na Westen, sulcx dat hier deur den as gheduerlick na een selven oirt streckt. Maer om dit roersel en strecking vanden as gheduerlick na een selven oirt, te verclaren mette bequaemste gelijckenis die my nu te vooren comt, ick segh aldus: Ghenomen dat ymant opt middelpunt des ront papiers van een seecompas stack een stroyken, streckende evewijdich metten as des Eertcloots, dat seecompas staende in een schip, varende neem ick in een ronde gracht van een Slot of Schans, t'is kennelick dattet selve schip een keer ghedaen hebbende van plaets tot plaets na d'een sijde, soo sal daerentusschen t'compas in sijn plaets oock eens ghekeert sijn na d'ander sijde, dats teghen den keer vant schip, en sulcken ghedeelte eens keers t'selve schip ghedaen heeft na d'een sijde, soodanich ghedeelte sal oock het seecompas ghedaen hebben na de ander sijde, blijvende het boveschreven stroyken gheduerlick evewijdich metten as des Eertcloots: En alsoo salmen derghelijcke oock verstaen vanden loop des Eertcloots in haer wech, welcke te wijle sy daer een keer doet, soo draeytse een keer in haer plaets teghen den voorgaenden keer, en blijvende den as altijt air or the heaven that contains the Earth does not move with the Earth, but the one alone without the other, the air flying about the fixed Earth will press just as closely against the buildings as the moving Earth against the stationary air; for if we assume that a stick be put up vertically in a flowing river, and another stick be drawn vertically through stagnant water as fast as the water of the river flows, it must be admitted that the water will press equally against one stick and the other. And the same would evidently happen with the air against the buildings or the buildings against the air. From what has been said before it would follow that something leaping up on a fixed Earth would not fall down again in the same place, but so far thence as the air drew it away meanwhile. But since this does not happen, it is necessary that the sphere of air and the Earth contained by it together form one sphere which on the theory of a fixed Earth stands still in its entirety or on the theory of a rotating Earth rotates in its entirety. And further, as for those who say that this composition of the sphere of earth and air is thus when a fixed Earth is spoken of, but not when there is the difference that it moves, it would be difficult for others to agree with those who thus contradict themselves. CONCLUSION. We have thus described the arrangement of the Heavens of the Planets on the theory of a moving Earth; as required.

2nd PROPOSITION.

To expound the motion of the Earth in its place, and its magnetic rest.

In addition to the motion of the Earth from place to place in its orbit, performing every year one revolution from West to East, as has been described in the 1st proposition, it has two more motions in its place: one motion is the daily rotation on its axis from West to East, but to explain this motion in its place somewhat more fully by means of an example, it might be said to be like a turning grindstone in a vessel sailing, which from the ship receives a motion from place to place, but its rotation on the axis meanwhile remains in the ship in the same place; and the same is the case with the Earth. The other motion (according to *Copernicus* in the 11th Chapter of his 1st book) is as follows: While the Earth performs its annual revolution from West to East, which has been described in the 1st Chapter, in the same time it meanwhile performs, against the aforesaid motion, a rotation in its place from East to West, in such a way that the axis constantly tends in the same direction. But to explain this motion and constant tendency of the axis in the same direction by means of the most convenient comparison that now occurs to me, I say as follows; if a man were to put in the centre of the round paper of a mariner's compass a straw parallel to the axis of the Earth, this compass standing in a ship sailing, I assume. in a circular moat of a Castle or Entrenchment, it is evident that when this ship has made one turning from place to place towards one side, in the mean time the compass in its place will also have made one turning towards the other side, *i.e.* against the turning of the ship; and such part of a turning as the ship shall have made towards one side, the same part the compass will also have made towards the other side, the aforesaid straw constantly remaining parallel to the axis of the Earth. And it must be similarly understood also with regard to the motion of the Earth in its orbit: while it performs one revolution, it rotates once in its place against the former revolution, the axis always tending in the same direction. This is, I think, na een selven oirt streckende. Dit meyn ick te wesen den rechtē sin vant roersel des Eertcloots deur *Copernicus* beschreven, en met een figuere verclaert int 11 Hoofstick sijns eersten boucx. Maer want dit roersel van hem aldus eenvoudelick gheseyt wort sonder eenighe natuerlicke reden of bewijs, soo heeft my dese stelling langhen tijt int ghedacht ghequollen, overmidts alle Hemelsche roersels diemen everas versiert en op malcander doet passen, ghelijckmen de rayers van een uyrwerck doet over een commen, my niet en bevallen, als niet schijnende inde natuer te bestaen: Nochtans moest dit roersel soo toeghelaten sijn, om al d'ander natuerlicke overeencomminghen die uyt stelling des roerenden Eertcloots volghen een seker gront te gheven: Doch is daer na int licht ghecommen het bouck vanden grooten * Eertclootschen seylsteen, beschreven deur *Guilielmus Gilbertus*, waer in de natuerlicke oirsaeck deses roersels, mijns beduninckens ghetroffen en gheopenbaert is, waer af ick hier de somme int corte stellen sal: Men bevint inden Eertcloot soo groote menichte van seylsteen, en ander stoffen met seylsteensche cracht (als ysergheberchten die overal seer menichvuldich sijn, en altemael dien aert hebbē) datse als een grooten seylsteen in heur de eyghenschappen heeft diemen vint inde cleene clootkens van seylsteen gemaeckt, want sulck een met corck clootsche wijse soo beset wesende, dattet int water hangt gelijck dē Eertcloot inde locht, alsdan en sal den noortschen aspunt niet alleen na t'Noorden wijsen, gelijckt mette bestreken seylnaeldē toegaet, maer den heelen as stelt heur evewijdich met des Eertcloots as, te weten den Noorderschen aspunt des steens na t'Noordē, den Zuyderschen na t'Zuyden: Voort soomen opt seylsteenclootken stelt seylnaeldekens, ghelijck ghemeenelick inde dragelicke Sonwijserkens sijn, men bevintse daer op sulcke rijsinghen, afwijckinghen, beweginghen, en eyghenschappen te hebben, als op den grooten Eertcloot. Noch heeft dese groote gemeenschap gheleert d'oirsaeck vande ongeregelde wijckinghen der seylnaelde vant Noorden, daer veel menschen al verwonderende hun gedachten dus langhe me becommert hebben, want alsmen neemt een cloot van seylsteen met putten daer in, en datmen een seylnaeldeken stelt niet boven t'middel des puts daert rechte Noortwijsing hebben can, maer by de kant na t'Oosten, het sal Oostelicken, maer ghestelt sijnde by de cant na Westen, sal Westelicken, nu want den Eertcloot oock sulcken seylsteencloot is met diepe puttē der zeen, in wiens beweghende water de boveschreven seylsteenschen aert niet sijn en can, soo bevintmen wel rechte Noortwijsing ontrent het middel der groote zeen, als in de Oceane tusschen America en Europa, maer op de oostsijde na Europa commende de naelde oostelickt, en westwaert na America sy westelickt: De selve reden is oock voor groote uytstekende punten van Landen, als Cabo de Bona Esperanca en meer ander, waer af den Schrijver inde laetste Hoofsticken sijns vierden boucx verscheyden voorbeelden stelt.

De magno magnete tellure.

Nu dan den Eertcloot in heur hebbende desen seylsteenschen aert, soo moetet boveschreven roersel (te weten eens jaerlickschē keers in plaets van Oosten na Westen, sulcx dat den as gheduerlick na een selven oirt streckt) alsoo nootsakelick wesen.

Tot hier toe is dese seylsteenigen aert des Eertcloots genoemt een draeyende roersel, om mijn voornemen beter te verclaren, maer acht nemende op t'gene ick voorder segghen wil, vinde bequamer dat te heeten seylsteenighe stilstandt, om dese reden: Ick heb hier vooren gheseyt dat hemelsche roersels diemen everas versiert, en op malcander doet passen, my niet en gevallen, als niet schijnende inde natuer te bestaen, nochtans, mocht ymant segghen, sulcx nu te blijcthe correct meaning of the motion of the Earth described by *Copernicus* and explained by means of a drawing in the 11th Chapter of his first book. But since this motion is thus simply described by him, without any natural argument or proof, this supposition long troubled me in my mind, since the notion that all the Heavenly motions are imagined equally fast and made to fit into one another, as the wheels of a timepiece are made to fit together, does not satisfy me, as not seeming to happen in nature. Nevertheless this motion had to be admitted in order to give a sure basis for all the other natural correspondences that follow from the theory of a moving Earth. But thereafter there was published the book about the great terrestrial magnet, described by *Guilelmus Gilbertus* [1]), in which the natural cause of this motion in my opinion is hit off and revealed, which I will here summarize as follows. In the Earth there is found such a large amount of loadstone and other substances with magnetic force (such as mountains containing iron, which are of frequent occurrence everywhere and all have that property) that, like a big loadstone, it has in itself the properties that are found in small spheres made of loadstone: for if such a small sphere is covered with cork in the form of a sphere, so that it hangs in the water like the Earth in the air, not only will the northerly pole point towards the North, as happens with magnetized needles, but the whole of the axis takes up a position parallel to the axis of the Earth, to wit, the Northerly pole of the stone pointing towards the North, the Southerly towards the South. Further, if on the sphere of loadstone we put magnetic needles, as they are generally present in portable Sun-dials, they are there found to have the same elevations, deviations, movements, and properties as on the big Earth. This great similarity has also taught the cause of the irregular deflections of magnetic needles from the North, about which people have wondered and puzzled so long. For if we take a sphere of loadstone with pits therein and we put a magnetic needle, not over the centre of the pit, where it can point straight towards the North, but sideways towards the East, it will deviate towards the East, but if it is put sideways towards the West, it will deviate towards the West. Now since the Earth is also such a sphere of loadstone with deep pits, namely, the seas, in whose moving water the above-mentioned magnetic character cannot be present, straight Northward pointing indeed is found about the middle of the great seas, such as in the Ocean between America and Europe, but when we get to the East, towards Europe, the needle deviates towards the East, and when we get to the West, towards America, it deviates towards the West. The same reason also holds for large protruding points of land, such as Cabo de Bona Esperança, and others, of which the Author gives several examples in the last Chapters of his fourth book [2]).

The Earth therefore having this magnetic character, the motion described above (to wit, that of an annual rotation in its place from East to West, such that the axis constantly tends in the same direction) must be necessary.

Up to this point this magnetic character of the Earth has been called a rotary motion, in order to set forth my intention the better, but with a view to what I further intend to say, I find it more suitable to call it magnetic rest, for the following reason. Above I have said that the notion that heavenly motions are

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[1]) William Gilbert (Colchester 1544—London 1603), physician to Queen Elizabeth in London, author of *De Magnete, Magneticisque Corporibus, et de Magno Magnete Tellure; Physiologia Nova* (London, 1600).

[2]) This refers probably to *The Haven-Finding Art* (Work XI, i 25).

te blijcken met des Eertcloots bovefchreven foodanige twee even roerfels. Ick antwoorde hier op datmen dit twee even roerfels noemen mach die op malcander paffen, om, als voorfeyt is, de voorghestelde faeck bequamelicker me te verclaren, maer om eyghentlicker te fpreken foo foudement beter des Eertcloots feylsteenighe stilstant heeten, fulcx datfe heur de keering haers wechs niet an en treckt: Als by voorbeelt, de caffe daer een feylnaelde in staet, een keer omghedraeyt fijnde na de rechterfijde, datmen foude feggen de feylnaelde daerentuffchen een keer gedaen te hebben na de flijncker fijde, ten waer niet foo eyghentlick ghefproken als te fegghen datfe stil staet, fonder heur de keering der caffe an te treckē: En alfoo oock metten Eertcloot in haer wech: Sulcx dat t'gene ick tot hier toe genoemt hebbe des Eertcloots tweede loop in plaets, dat heet ick na het inhoudt vant opfchrift defes voorstels feylsteenige stilstant, als wefende een woort dat ghenouch fchijnt uyt te beelden de manier van stilstant dieder int voorgaende ghemeent is, en int volghende ghemerckt fal worden. Merckt noch dat alfmen defe feylsteenighe stilstandt roerfel noemde, foo foudet een roerfel fijn teghen t'vervolgh der trappen, t'welck deur fich felfs bestaende ick niet en fie in eenighe hemelen te ghebeuren.

T'BESLVYT. Wy hebben dan verclaert des Eertcloots roerfel in plaets, en haer feylsteenighe stilstant, na den eyfch.

3 VOORSTEL.

Te verclaren de feylsteenighe stilstant der Dvvaelders vveghen en haer hemelen.

Defe eyghenfchap van feylsteenighe stilstandt en is niet alleen inden Eertcloot als vooren, maer oock (dienende t'een tot breeder bevesting vant ander) in haer wech, als opentlick blijckt deur t'verstepunt des felfden, t'welck om dē loop van Mars hemels wille daer den Eertcloot af vervangen is, en in gedregen wort, te twee jaren een keer foude moeten doen, dat nochtans niet en ghebeurt foo de ervaring leert, deur welcke men bevint datter veel jaren verloopen eert 1 tr. voordert, en is bycans al oft stil stonde, waer uytmen befluyten mach niet alleen den Eertcloot, maer oock haer heelen wech den feylsteenfchen aert te hebben: Ia dattet om derghelijcke reden alfoo toegaet met elcken wech van d'ander Dwaelders, diens verstepunten geen roerfel en crijgen vande hemelen haer omvangende, fo merckelicxt blijckt in Mercurius wech, diens verstepunt uyt oirfaeck van Venus wechs keering feer fnellick foude moeten ommedraeyen, te 225 daghen een keer doende, en noch foo veel rafscher als dan veroirfaeckt wierde deur al d'ander Dwaelders hemelen die daer boven d'een d'ander begrijpen, al t'welck niet en ghebeurt, want Mercurius wechs verstepunt foo flappen voortganck heeft als al d'ander.

Noch is te weten defen aert der feylsteenighe stilstandt niet alleen te fijn in der Dwaelders platte weghen als boven, maer oock inde heele hemelfche clooten daerfe in ghedreghen worden, fulcx dat haer affen (gelijck vooren int tweede voorstel vanden Eertcloots as ghefeyt is) geduerlick na een felven oirt strecken, want fooder die feylsteenighe stilstandt niet en waer, de twee afpunten des Hemelcloots die den Eertcloot draecht, en fouden niet op een felve plaets blijven, maer te twee jaren fulcken rondt moetē befchrijvē, als deur Maenswechs afwijcking vanden duysteraer veroirfaeckt wierde: Of anders ghefeyt, den Eertcloot en foude niet gheduerlick fonder breede blijven int plat datmen voor imagined equally fast and made to fit into one another does not satisfy me, as not seeming to happen in nature; yet someone might say that this now is evident from the above-mentioned two equal motions of the Earth. To this I reply that we may call them two equal motions which fit together, in order — as has been said before — to set forth the object in view more properly therewith; but to speak more truly it would be better to call it the magnetic rest of the Earth, meaning that it does not take account of the revolution in its orbit. For example, if, the box in which a magnetic needle is contained having performed one revolution to the right, it should be said that the magnetic needle had meanwhile performed a revolution to the left, this would not be as true a statement as saying that it stands still without taking account of the revolution of the box. It is the same with the Earth in its orbit, so that what I have hitherto called the second motion of the Earth in its place, according to the wording of the heading of this proposition I call magnetic rest, this being a word that seems to denote sufficiently the kind of rest that is meant in the foregoing and is to be noted in the following. Note also that if this magnetic rest were called motion, it would be a motion contrary to the order of the degrees, which as existing in itself I do not see happening in any heavens.

CONCLUSION. We have thus expounded the motion of the Earth in its place and its magnetic rest; as required.

3rd PROPOSITION.

To expound the magnetic rest of the Planets' orbits and their heavens.

This property of magnetic rest resides not only in the Earth, as said above, but also (the one serving to confirm the other more fully) in its orbit, as is evident from the apogee of the latter, which — because of the motion of Mars' heaven, within which the Earth is contained and carried — would have to perform one revolution in two years, which nevertheless does not happen, as experience teaches, by which it is found that many years elapse before there is a progress of 1°; and it is almost as if it stood still; from which it may be concluded that not only the Earth, but also its whole orbit has a magnetic character, nay, that for similar reasons the same holds for any orbit of the other Planets, whose farthest points do not acquire a motion from the heavens containing them, as is most apparent in Mercury's orbit, whose farthest point on account of Venus' orbital revolution would have to revolve very fast, performing one revolution in 225 days, and even so much more fast as would be caused by the heavens of all the other Planets which are above and contain one another — all of which does not happen, for the farthest point of Mercury's orbit has just as slow a progress as all the others.

It should also be known that this character of magnetic rest resides not only in the plane orbits of the Planets, as said above, but also in the entire heavenly spheres in which they are carried, such that their axes (as has been said above in the second proposition of the Earth) constantly tend in the same direction. For if this magnetic rest were not present, the two poles of the Heavenly Sphere carrying the Earth would not remain in the same place, but in two years would have to describe a circle such as would be caused by the deviation of Mars' [1]) orbit from the ecliptic. Or in other words, the Earth would not constantly remain

[1]) For *Maenswechs* in the Dutch text read *Marswechs*.

duysteraer houdt, maer somwijlen daer af soo groote afwijckinghen hebben als Marswech vanden duysteraer heeft. Ende sulcx als hier gheseyt is vanden Hemelscloot die den Eertcloot draecht, salmē oock verstaen op de Hemelclooten van al d'ander Dwaelders, onder welcke dese seylsteenighe stilstandt inde Manens hemel seer merckelick is, overmidts het verstepunt hem dē jaerlickschen keer die den hemel int volgen of leyden des Eertcloots niet an en treckt. Oock is te gedencken dat inde breedeloop vande onderste Dwaelders sulcken ongheregheltheyt soude moeten wesen als veroirsaeckt wierde uyt de mengheling van al de verscheyden afwijckinghen der Dwaelderweghen die boven hemlien sijn: Al t'welck niet ghebeurende (ghelijck grondelicker sal blijcken deur de volghende beschrijving vande eenvoudighe oirden diese in breedeloop houden) soo valter uyt te besluyten, desen ghemeenen aert der seylsteenighe stilstandt niet alleen te wesen in der Dwaelders weghen, maer oock geheelick in haer Hemelclooten.

Merckt noch dit: Alsmen mette beschrijving der Dwaelders Hemelen sulcken oirden wilde volghen als *Copernicus* int 11 Hoofstick sijns eersten boucx mette beschrijving des Eertcloots ghedaen heeft, welcke hier verhaelt wiert in het 2 voorstel, men soude moeten aldus segghen: Te wijle Iupiters hemel heur dertichjarige keer doet van Westen na Oosten diese van Sarurnus hemel ontfangt, soo doetse daerentusschen op de selve tijt teghen den voorschreven loop een keer in plaets van Oosten na Westen, sulcx dat hier deur den as geduerlick na een selven oirt streckt. Maer my dunckt om de voorgaende redenen verstaenlicker en natuerlicker, dit in plaets van soodanich roersel te noemen seylsteenighe stilstandt, te meer dat opt roersel van d'onderste Dwaelders als neem ick van Mercurius, meer soude moeten geseyt sijn dan van Venus loop, ghemerckt het soude moeten wesen de somme der loopen van al de Dwaelders dieder boven sijn.

Noch moet ick seggen dat ick over een tijt van desen handel onbesloten gedachten hadde, houdende ter eender sijde voor ghemeene reghel, dat alle begrepen die wech henen moet daer hem sijn begrijpende draecht, waer uyt volghen soude dat elck Dwaelder een roersel moest hebben ghemengt uyt al de roersels der Dwaelders die boven hem sijn: Ter ander sijde sach ick metter daet t'verkeerde ghebeuren: Dit dede my dencken oft soude meughen sijn dat de Dwaelders niet en waren in Hemelen ghehecht, maer deur de locht vloghen ghelijck de voghelen om een torre, sonder het roersel van d'een, ant roersel van d'ander eenige beweeghnis te veroirsaken, waer tegen ander redenen my weerom anders deden vermoeden: Maer gecommen sijnde ter kennis vande voorgaende eyghenschap die ick seylsteenighe stilstandt noem, die twijffelachtighe ghedachten namen daer me een einde. T'BESLVYT. Wy hebben dan verclaert de seylsteenighe stilstandt der Dwaelders weghen en haer Hemelen, na den eysch.

4 VOORSTEL.

Te seggen vande plaets der crachten die den Eertcloot, vveghen, en Hemelen der Dvvaelders in haer seylsteenighe stilstandt houden.

De crachten die den Eertcloot, weghen, en Hemels der Dwaelders in haer seylsteenighe stilstandt houden (welcke crachten men by verstaenlicke gelijckenis with latitude zero in the plane that is taken for the ecliptic, but would sometimes have as great deviations therefrom as Mars' orbit has from the ecliptic. And the same that has here been said of the Heavenly Sphere carrying the Earth is also to be understood for the Heavenly Spheres of all the other Planets, among which this magnetic rest is very considerable in the Moon's heaven, since the apogee does not take account of the annual revolution which the heaven performs [1]) in following or leading the Earth. It should also be remembered that in the motion in latitude of the lowermost Planets there would have to be such irregularity as would be caused by the combination of all the different deviations of the Planets' orbits that are above them. Since all this does not happen (as will become more fully apparent from the following description of the simple order they keep in their motion in latitude), it may be concluded therefrom that this general character of magnetic rest resides not only in the Planets' orbits, but also altogether in their Heavenly Spheres.

Note also the following. If in the description of the Planets' Heavens we wished to follow the same order as *Copernicus* has done in the 11th Chapter of his first book with the description of the Earth, which has here been related in the 2nd proposition, we should have to say as follows: While Jupiter's Heaven performs its thirty years' revolution from West to East, which it receives from Saturn's Heaven, it performs meanwhile in the same time, contrary to the aforesaid motion, a rotation in its place from East to West, so that the axis thus tends constantly in the same direction. But for the above reasons it seems to me more intelligible and more natural to call this, instead of such a motion, magnetic rest; the more so because with regard to the motion of the lowermost Planets, such as, for example, Mercury, more would have to be said than on Venus' motion, because it would have to be the sum total of the motions of all the Planets that are above it.

I also have to say that for some time I was undecided in my mind about this matter, holding it on the one hand a general rule that all bodies contained by other bodies must take the course in which their containing bodies carry them, from which it would follow that every Planet must have a motion consisting of a combination of all the motions of the Planets that are above it. On the other hand, in practice I saw the reverse happening. This caused me to think whether it could be possible that the Planets were not attached to Heavens, but were flying through the air like birds about a tower, without the motion of the one causing any change in the motion of the other; but other reasons again made me think differently. But when I had gained knowledge of the foregoing property, which I call magnetic rest, my doubts were resolved. CONCLUSION. We have thus expounded the magnetic rest of the Planets' orbits and their Heavens; as required.

4th PROPOSITION.

To speak of the place of the forces which keep the Earth, the orbits and the Heavens of the Planets in their magnetic rest.

The forces which keep the Earth, the orbits and the Heavens of the Planets in their magnetic rest (which forces, by an intelligible comparison, might be

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[1]) This or some such word must have been omitted in the Dutch text.

kenis yders ſeylſteen mocht noemen) ſchijnen altemael, uytghenomen vande Maenwech, te moeten weſen buyten de Hemelen der Dwaelders. Om van t'welck by voorbeelt te ſpreken, ſoo ymant een ſeylſteen (als treckende cracht der ſeylnaelde) leyde inde caſſe daer een ſeylnaelde op de pinne in ruſt, de ſelve caſſe ghekeert weſende, ſoo en ſoude de naelde niet gheduerlick na een ſelven oirt wijſen, maer altijt na den ſeylſteen gheneycht ſijn, om dat de treckende cracht ſelf me draeyt: Ende alſoo oock by aldien de treckende crachten die den Eertcloot en haer heelen Hemel in die ſtant houden, waren binnen haer caſſe, dats Mars Hemel, en daer in me voortghedregen wierden, des Eertclootwechs verſtepunt ſoude den loop van Marswech crijghen te twee laren eens ommeloopende. Doch want ſulcx niet en ghebeurt, ſoo iſt daer voor te houden dat die treckende cracht in Mars Hemel niet en is, noch om dergheliȷcke redenen in den Hemel van eenighe van d'ander Dwaelders: Maer angheſien haer verſtepunt ſoo veel men deur ervaring bemerckt, altijt ſtreckt na een ſelve plaets tuſſchen de vaſte ſterren, volghende den tragen loop der ſelve, ſoo machment daer voor houden die treckende cracht inde caſſe of Hemel der vaſte ſterren te weſen, en dit niet alleen vandẽ Eertcloot en haer Hemel, maer oock vande Hemelen van al d'ander Dwaelders, uytghenomen ſoo gheſeyt is de Maen, diens wechs verſtepunt ontrent de negen jaren eens ommeloopende deur het 9 voorſtel des 1 boucx, haer treckende cracht ſoude in een ander caſſe of leeger hemel moeten draeyen, en dat ſoot ſchijnt tuſſchen Mars en Iupiter. Maer te wijle ick an deſe ſtof ghecommen ben, ſal daer af met eene noch dit ſegghen: Alſoo ick bevonden hadde de Maenwechs eyghentlicke verſtepunts loop niet te weſen van 6① 41 ② ſdaechs na t'vervolgh der trappen, gelijckmen deur ſtelling eens vaſten Eertcloots beſluyt, maer als uyt ſtelling eens roerenden Eertcloots volght van 52 ① 27 ② ſdaechs teghen t'vervolgh der trappen, ghelijck t'ſijnder plaets beweſen ſal worden, ſoo gaſt my vreemt de eyghentlicke ſtelling des roerenden Eertcloots me te brenghen, yet ghevonden te worden dat teghen t'vervolgh der trappen liep: Maer overdenckende daer na dat haer treckende cracht in eenighen anderen hemel ontrent de neghen jaren als gheſeyt is een keer dede na t'vervolgh der trappen, ſoo docht my dat hier niet teghen de regel en ginck, maer eyghentlick alles van Weſten na Ooſten te loopen, en ſulck ſchijnſel ſijn bekende oirſaken te hebben; Dergheliȷcke oock vermoedende vande duyſteringſne loop die niet ſdaechs 3 ① 11 ② volghens de ſtelling eens vaſten Eertcloots, maer eyghentlick als uyt ſtelling eens roerenden Eertcloots volght ſdaechs 1 tr. 2. 19. tegen t'vervolgh der trappen, wiens treckende cracht ſchijnt ghedreghen te ſijn in een hoogher tragher hemel dan de voorgaende, doende ontrent de 18 jaren een keer na t'vervolgh der trappen.

T'BESLVYT. Wy hebben dan gheſeyt vande plaets der crachten die den Eertcloot, weghen, en Hemelen der Dwaelders in haer ſeylſteenige ſtilſtandt houden, na den eyſch.

5 VOORSTEL.

Te verclaren dattet niet nootſakelick en blijckt de Son middelpunt te vveſen vanden vaſte ſterrens hemel, maer met goede reden daer toe vercoren vvort.

Ghelijck int Eertclootſchrift oirboir is op den Eertcloot eenich halfmiddachront te verkieſen, dat by al de ghene die vande ſelve ſtof handelen int ghemeen called the loadstone of each) all seem to have to be outside the Heavens of the Planets, with the exception of the Moon's orbit. To give an example of this, if someone laid a magnet (as attractive force of the magnetic needle) in the box in which a magnetic needle rests on the pin, then, if the box were turned, the needle would not constantly tend in the same direction, but would always incline towards the magnet, because the attractive force itself takes part in the revolution. And in the same way, if the attractive forces which keep the Earth and its entire Heaven in that position were within its box, *i.e.* Mars' Heaven, and were carried along in it, the apogee of the Earth's orbit would receive the motion of Mars' orbit, revolving once in two years. But because this does not happen, it is to be concluded that this attractive force does not reside in Mars' Heaven, nor for similar reasons in the Heaven of any of the other Planets. But since its apogee, as far as is noted by experience, always tends in the same direction among the fixed stars, following the slow motion of the latter, it may be assumed that this attractive force resides in the box or Heaven of the fixed stars, such not only for the Earth and its Heaven, but also for the Heavens of all the other Planets, except, as has been said, for the Moon; since the apogee of its orbit revolves once in about nine years, by the 9th proposition of the 1st book, its attractive force would have to revolve in another box or lower Heaven, such apparently between Mars and Jupiter. But now that I have arrived at this subject, I will at the same time also say the following about it. As I had found the true apogee's motion of the Moon's orbit not to be 6′41″ a day, in the order of the degrees, as is concluded from the theory of a fixed Earth, but, as follows from the theory of a moving Earth, 52′27″ a day, against the order of the degrees, as will be proved in its place, it seemed strange to me that the true theory of the moving Earth should involve that something was found that moved against the order of the degrees. But reflecting thereafter that its attractive force in any other heaven, as has been said, would perform one revolution in about nine years according to the order of the degrees, it seemed to me that this was not contrary to the rule, but that in reality everything moved from West to East, and that this apparent state of affairs has its known causes. And I suspected the same for the motion of the line of nodes, which is not 3′11″ a day, as according to the theory of a fixed Earth, but in reality, as follows from the theory of a moving Earth, 1°2′19″ a day, against the order of the degrees, its attractive force seeming to reside in a higher, slower Heaven than the preceding, performing one revolution in about 18 years according to the order of the degrees [1]).

CONCLUSION. We have thus spoken of the place of the forces which keep the Earth, the orbits and the Heavens of the Planets in their magnetic rest; as required.

5th PROPOSITION.

To expound that it appears not to be necessary that the Sun should be the centre of the Heaven of the fixed stars, but that it is chosen for this with good reason.

Just as it is expedient in Geography to choose on the Earth some half-meridian which is generally looked upon as the starting point by all those who deal with the

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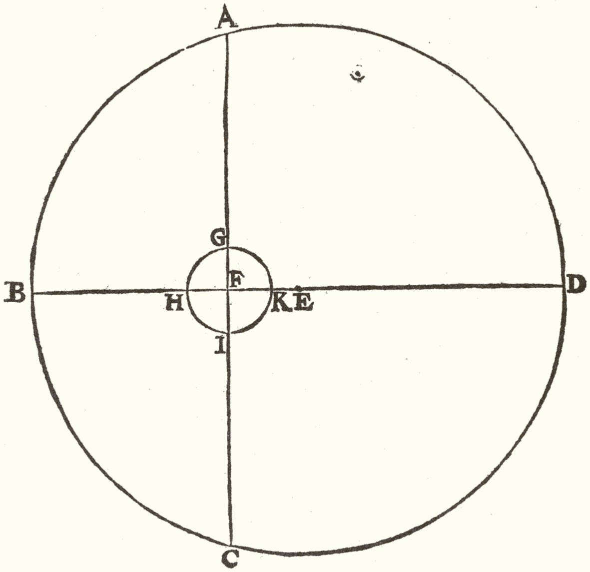
[1]) This statement is wrong, though it is not a clerical or a printer's error.

meen voor begin gehouden wort, daer eyghentlicker af ghesfeyt is inde 4 bepaling vant 1 bouck des Eertclootschrifts, alsoo ist int Hemelloopschrift oirboir inde werelt eenich punt te verkiesen, dat al de gene die hemlien inde selve stof oeffenen voor haer middelpunt annemen: Hier toe wort met stelling eens vasten Eertcloots billichlick den Eertcloot vercoren, want alsmen besluyt den hemel der vaste sterren op haer middelpunt te draeyen, soo moet den Eertcloot daer an sijn, of anders en soude d'een helft des ghesteerenden hemels niet boven den sichteinder noch d'ander helft daer onder sijn, strijdende teghen d'ervaring, ghelijck *Ptolemeus* dat verclaert int 5 Hoofstick sijns 1 boucx.

Angaende des werelts middelpunt met stelling eens roerenden Eertcloots, daer wort billichlick de Son toe ghenomen, om datse ghenouchsaem het middelpunt is vande ronden beschreven deur de verstepunten der Dwaelderswegen, maer t'middelpunt vanden hemel der vaste sterren te wesen machmen vermoeden, dan men cant, soo ick meen, niet volcommelick bewijsen. Laet tot verclaring van dien A B C D den ghesternden hemel beteyckenen, diens



middelpunt E, waer deur ghetrocken is de middellijn B E D, en A C rechthouckich op B D, sniende de selve in F buyten t'middelpunt E, en op F als middelpunt sy beschreven den Eertcloot wech G H I K. Dit soo sijnde t'is kennelick dat hoewel de booch A B C cleender is dan C D A, nochtans anghesien het rondt G H I K altijt tot die plaets blijft, en het rondt A B C D stil staet, soo schijnt d'een en d'ander booch uyt des Eertclootwechs middelpunt F ghesien, altijt een halfront te wesen, ghelijck A B of den houck A F B altijt een vierendeel schijnt: T'selve heeft hem oock alsoo ghesien uyt den Eertcloot tot yder plaets des omtrecx G H I K om dat de heele middellijn H K gheen ghevoelicke reden en heeft teghen de halfmiddellijn B E of teghen B H: Men mocht oock aldus segghen; nadien den heelen Eertcloots hemel verleken by den hemel der vaste sterren, maer en is als een punt, soo en connen wy niet bewijsen de Son meer middelpunt des vastesterrens hemel te wesen als het verstepunt des

Eert-same subject matter, as is described more properly in the 4th definition of the 1st book of Geography, so it is expedient in Astronomy to choose in the world some point which all those who pursue this subject take for its centre. For this, on the theory of a fixed Earth, the Earth is justly chosen, for if it is decided that the Heaven of the fixed stars rotates about its centre, the Earth must be there; otherwise one half of the starry Heaven would not be above the horizon, nor the other half below it, which is contrary to experience, as *Ptolemy* sets it forth in the 5th Chapter of his 1st book.

As to the world's centre on the theory of a moving Earth, for that the Sun is justly taken, because it is sufficiently near the centre of the circles described through the apogees of the Planets' orbits; but that it is the centre of the Heaven of the fixed stars can be surmised, but in my opinion it cannot be fully proved. By way of explanation let *ABCD* denote the starry Heaven, whose centre be *E,* through which is drawn the diameter *BED,* and *AC* at right angles to *BD,* intersecting the latter in *F* outside the centre *E;* and about *F* as centre let there be described the Earth's orbit *GHIK.* This being so, it is evident that, though the arc *ABC* is smaller than *CDA,* yet since the circle *GHIK* always remains in that place and the circle *ABCD* stands still, one arc as well as the other, seen from the centre of the Earth's orbit *F,* seems always to be a semi-circle, just as *AB* or the angle *AFB* always seems to be a quarter circle. The same is also the case, when seen from the Earth in any place on the circumference *GHIK,* because the entire diameter *HK* has no perceptible ratio to the semi-diameter *BE* or to *BH.* We might also say as follows: Since the entire Heaven of the Earth is but as a point in comparison with the Heaven of the fixed stars, we cannot prove that the Sun rather than the apogee of the Earth's orbit or any other point contained therein is the centre of the heaven of the fixed stars. Nay, it clootwechs, of eenich ander punt daer in begrepen: Ia men macht daer voor houden, dat self Saturnus hemel verleken by den hemel der vaste sterren maer en is als een punt, om dese reden: Haer halfmiddellijn is ontrent de negen mael soo lanck als des Eertcloot hemels halfmiddellijn, ghelijck int voorstel deses 3 boucx blijcken sal: Hier uyt volght dat de vaste sterren ghesien uyt Saturnus wech, een verscheensicht ofte voor of achtring crijghen, negen mael soo groot als hemlien verscheensicht uyt den Eertclootwech ghesien, dats neghen mael een onbemerckelicke sake, welcke oock of onbemerckelick is, of immers seer cleen moet sijn: Nu dan Saturnus hemel als middelpunt schijnende des hemels der sterren, soo sal elck punt in Saturnus hemel begrepen, meughen ghenomen worden voor middelpunt des vaste sterrens hemel, sonder uytmiddelpunticheyt te connen bemerckt worden, en vervolghens soo en schijnet niet bewijselick de Son meer haer eygentlick middelpunt te wesen dan t'verstepunt van Saturnus wech, of eenich ander in sijn hemel begrepen.

Angaende *Copernicus* int 10 Hoofstick sijns 1 boucx vraeght, wie in dese schoonste kercke die lampe in een ander beter plaets soude stellen dan int middel, van daer sijt over al t'samen mach lichten? t'sijn wel beweeghlicke natuerlicke redenen, maer op gheen Meetconstich bewijs ghegront. Soo veel isser af, by aldienmen eenich ander puut dan de Son, ick neem des Eertclootwechs middelpunt, wilde houden voor werelts middelpunt, stellende de Son daer rontom te draeyen, met een halfmiddellijn even an des Eertclootwechs uytmiddelpunticheytlijn, men soude daer op een beschrijving des Hemelloops connen doen sonder dwaling, maer de Son voor werelts middelpunt te nemen valt gherievigher, soo wel om bequamelick te leeren de overeencomminghen der stellinghen eens vasten en roerenden Eertcloots, daer hier na afgheschreven sal worden, als om meer ander ontmoetende saken die aldus lichter en verstaenlicker sijn. T'BESLVYT. Wy hebbē dan verclaert dattet niet nootsakelick en blijckt de Son middelpunt te wesen vanden vastesterrens hemel, maer met goede reden daer toe vercoren wort, na den eysch.

6 VOORSTEL.

Te segghen vande vervvonderinghen sonder vvonder der ghene die een vasten Eertcloot stellen.

Ettelicke der gene die *Ptolemeus* beschrijving der Dwaelderloopen met een vasten Eertcloot verstaen, en voor recht houden, verwonderen hun in sommighe eyghenschappen dieser in mercken: Ten eersten dat Saturnus, Iupiter en Mars in teghestant der Son altijt ten naesten by den Eertcloot commen, maer in saming ten versten. Ten tweeden dat haer loop int inront altijt effen overcomt mettet overschot des Sonloops boven den loop van haer inronts middelpunten. Ten derden dat Venus en Mercurius t'verkeerde ghebeurt, want haer loop int inront en heeft mette Son sulcke overcomming niet, maer den loop van haer inronts middelpunt isser even me: Dit houden sy voor een teycken van besonderheyt des weerdichsten Dwaelders de Son, na wiens roersel d'ander als na een Koninck opsicht nemen en haer loop vervoughen: Doch men macht houden voor ghedwaelde * spieghelinghen, volghende uyt ghemiste stelling eens vasten Eertcloots. Maer want dit groote ghelijckheyt heeft met luyden die het scheepvaren onghewoon sijnde, ghemeenlick het roersel van haer schip ander schepen toeschrijven, als wanneer sy die teghencommen en beneen Theorijs.

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may be assumed that even Saturn's Heaven, in comparison with the Heaven of the fixed stars, is but as a point, for the following reason. Its semi-diameter is about nine times the length of the semi-diameter of the Earth's Heaven, as will appear in the [13th] [1]) proposition of this book. From this it follows that the fixed stars, seen from Saturn's orbit, receive a parallax or advance-or-lag nine times greater than their parallax when seen from the Earth's orbit, that is nine times an imperceptible amount, which is also either imperceptible or at any rate must be very small. If therefore Saturn's Heaven appears as centre of the Heaven of the fixed stars, any point contained in Saturn's Heaven can be taken for centre of the Heaven of the fixed stars, without eccentricity being perceptible, and consequently it does not seem possible to prove that the Sun is its true centre rather than the apogee of Saturn's orbit, or any other point contained in its Heaven.

As to the fact that *Copernicus* asks in the 10th Chapter of his 1st book who would in this beautiful church place that lamp in any other, better place than in the middle, from where it can illuminate everything: these are moving, natural reasons indeed, but not based on a geometrical proof. So much is true that if one wished to take any point other than the Sun, for example the centre of the Earth's orbit, for the centre of the world, assuming the Sun to revolve about it, with a semi-diameter equal to the line of eccentricity of the Earth's orbit, one might base on this a description of the Heavenly Motions without any error; but it is more convenient to take the Sun for the centre of the world, both in order to learn properly the correspondences of the theories of a fixed and a moving Earth, which are to be described hereinafter, and on account of other matters that may arise, which are easier and more intelligible in this way. CONCLUSION. We have thus expounded that it appears not to be necessary that the Sun should be the centre of the Heaven of the fixed stars, but that it is chosen for this with good reason; as required.

6th PROPOSITION.

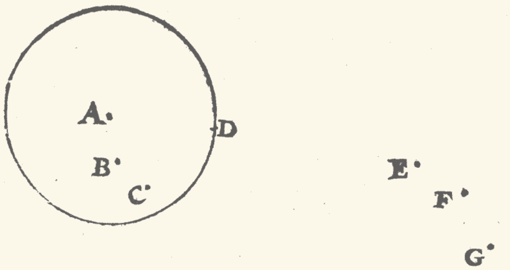
To speak of the wondering at what is no wonder, of those who assume a fixed Earth.

Some of those who understand *Ptolemy's* description of the Planetary Motions based on a fixed Earth and consider it correct, are astonished at some properties they perceive therein. Firstly, that Saturn, Jupiter, and Mars, when in opposition to the Sun, always come nearest to the Earth, but when in conjunction, farthest from it. Secondly, that their motion on the epicycle always corresponds exactly to the surplus of the Sun's motion over the motion of the centres of their epicycles. Thirdly, that with Venus and Mercury the converse takes place, for their motion on the epicycle has no such correspondence to the Sun's motion, but the motion of the centre of their epicycle is equal to it. They take this for a sign of the special character of the worthiest Planet, the Sun, from whose motion the others take their guidance as from a King and move accordingly. But these may be considered erroneous speculations, resulting from the incorrect theory of a fixed Earth. But since this is greatly similar to people who, not being used to sailing, generally ascribe the motion of their ship to other ships, — such as,

[1]) Apparently this number has been inadvertently omitted in the original text.

beneen boort ligghen, sonder water of landt te sien, segghen, hoe ras vaert dat schip buyten t'onse. Of hun schip een keer doende, segghen t'ander dat misschien stil light rontom het haer te draeyen, soo sal ick dit als voorbeelt ghebruycken tot verclaring deser stof.

Laet dese seven punten A, B, C, D, E, F, G, seven schepen in zee beteyckenen waer af A den Admirael sijnde stil light: Maer t'schip D vaert gheduerlick in een rondt, daer de drie schepen A, B, C, binnen sijn, en de drie E, F, G, buytē. Dit soo sijnde, en ymant in het schip D wesende, meynt na de boveschreven ghemeene wijse dattet stille light, en d'ander al rondtom hem ongchereghelt draeyen, En volghende sulck gestelde neemt acht op de ghedaente des loops, en seght met een verwonderen aldus: Wat een vreemde saeck ist, dat telckemael als een der drie schepen E, F, G, comt in een rechte lini van hem over ons totten Admirael A, soo is dan elck van dien ons altijt ten naesten: En ten versten, wesende in de selve rechte lini over d'ander sijde vanden Admirael, hoe ongchereghelt oock hun gheduerighe vaert is: Hier uyt besluyt hy elck dier drie schepen noch te draeyen in een cleender rondt, daer deur sy naerderen en afwijcken, hem verwonderende waerom sulcken keer in tijt seker overcomming heeft metten keer vanden Admirael: Sghelijcx voor een vreemdicheyt houdende waerom de twee schepen C B, oock een reghel houden metten Admirael, doch verkeert vande voorgaende, te weten dat den keer des groote ronts diese doen om t'schip D draeyende, in tijt effen overcomt met een keer des Admiraels, seght voort dat sulcx is een teycken van eerbieding die den Admirael van d'ander schepen anghedaen wort.



Dit soo sijnde, ghenomen nu dat een ervaren Schipper wetende hoet mette saeck ghestelt is, tegen sulck een aldus seyde: Ghy breeckt u hooft met voor wonder te houden daer geen wonder en is, want ons schip t'welck ghy meent stil te legghen, vaert gheduerlick rondtom de drie schepen A, B, C, waer uyt nootsakelick volght, dat soo dickwils wy sijn tusschen den Admirael A en een der drie E, F, G, soo moet ons dan elck van dien ten naesten sijn, en ten versten als A tusschen ons en een van hemlien is: Inder voughen dat die schepen niet en varen in ronden, met soodanighen versierden loop, die hun doet naerderen en afwijcken, noch oock de twee schepen B, C, in sulcke ronden, effen overcommende metten loop van A, soo ghy meent: Maer men mochtet voor onnatuerlick houden dat t'ghene voor de onervarenen alsoo schijnt, eyghentlick niet anders en waer.

Ende even eens soude een ervaren Hemelmeter tot een onervaren meughen segghen: Ghy breeckt u hooft met voor wonder te houden daer gheen wonder en is, want ons weereltlicht dats den Eerteloot die ghy meent stil te ligghen, draeyt gheduerlick rondtom de drie Dwaelders, Son, Venus, Mercurius, waer uyt nootsakelick volght dat soo dickwils wy sijn tusschen de Son en een when they meet them while lying below deck without seeing water or land, they say: how fast that ship outside ours sails, or, if their ship makes a turning, say that the other, which perhaps lies still, moves round theirs — I will use this as an example to illustrate this subject matter.

Let these seven points *A, B, C, D, E, F, G* denote seven ships at sea, of which *A,* being the Admiral, lies still. But the ship *D* continually sails in a circle, within which are the three ships *A, B, C,* while the three *E, F, G* are on the outside. This being so, a man being in the ship *D* is of opinion, according to the above, that it lies still and all the others move irregularly about it. And according to this supposition he observes the nature of the motion and, wondering, says as follows: How strange it is that whenever one of the three ships *E, F, G* comes in a straight line from it *via* us to the Admiral *A,* each of them is always nearest to us; and farthest, when it is in this straight line to the other side of the Admiral, however irregular their continual sailing may be! From this he concludes that each of those three ships also moves in a smaller circle, in consequence of which they approach and withdraw, wondering why this turning has an exact correspondence in time to the turning of the Admiral. Likewise, considering it strange why the two ships *C, B* also have a connection with the Admiral, but contrary to the foregoing, to wit, that the motion of the large circle they perform in moving round the ship *D* corresponds exactly in time to one turning of the Admiral, he says further that this is a sign of homage paid to the Admiral by the other ships.

This being so, let us assume that a skilled Skipper, knowing what is the matter, said to such a person as follows. You rack your brains in wondering where there is no wonder, for our ship, which you think is lying still, is continually sailing round the three ships *A, B, C,* from which it follows necessarily that whenever we are between the Admiral *A* and one of the three *E, F, G,* each of those must be nearest to us, and farthest from us when *A* is between us and one of them; so that those ships do not sail in circles, with such an imagined motion as makes them approach and withdraw, nor do the two ships *B, C* sail in such circles, corresponding exactly to the motion of *A,* as you think. But it might be considered unnatural that what seems thus to the inexperienced should not in reality be otherwise.

And in the same way an experienced Astronomer might say to an inexperienced one: You rack your brains in wondering where there is no wonder, for our luminary, *i.e.* the Earth, which you think lies still, travels continually round the three Planets Sun, Venus, Mercury, from which it follows necessarily that whenever we are between the Sun and one of the three: Saturn, Jupiter, Mars, een der drie Saturnus, Iupiter, Mars, ſoo moet ons dan elck van dien ten naeſten ſijn, en ten verſten als de Son tuſſchen ons en een van hemlien is: Inder voughen dat die drie Dwaelders niet en draeyen in ronden met ſoodanighen verſierden loop die hun doet naerderen en afwijcken, noch oock de twee Venus en Mercurius in ſulcke ronden, effen overcommende mette Sonloop ſoo ghy meent, maer men mochtet voor onnatuerlick houden dat t'ghene voor d'onervaernen ſoo ſchijnt niet eyghentlick anders en waer.

Noch iſſer by veelen een verwonderen vande ſeltſaem haſpeling des breedeloops der Dwaelders Saturnus, Iupiter, Mars, Venus, en Mercurius, ghegront op ſtelling eens vaſten Eertcloots: Maer volghens de ſtelling eens roerenden Eertcloots, ſoo en iſſer gheen wonder, dan ſijn eenvoudelicke weghen afwijckende vanden duyſteraer, ghelijck de Maenwech, waer uyt rekeninghen der breede volghen met kennis der oirſaken, als t'ſijnder plaets blijcken ſal.

T'BESLVYT. Wy hebben dan gheſeyt vande verwonderinghen ſonder wonder der ghene die een vaſten Eertcloot ſtellen.

Dit eerſte Onderſcheyt van der Dwaelders Hemelen ghedaente ten einde ſijnde, ick ſal nu tottet beſchrijven des loops commen, en eerſt vande langdeloop.

each of them must be nearest to us, and farthest from us when the Sun is between us and one of them; so that those three Planets do not move in circles with such an imagined motion as makes them approach and withdraw, nor do the two, Venus and Mercury, move in such circles, corresponding exactly to the Sun's motion, as you think. But it might be considered unnatural that what seems thus to the inexperienced should not in reality be otherwise.

Many people also wonder at the curious jumbling of the motion in latitude of the Planets Saturn, Jupiter, Mars, Venus, and Mercury, based on the theory of a fixed Earth. But according to the theory of a moving Earth there is nothing astonishing, but they are simple orbits deviating from the ecliptic, like the Moon's orbit, from which follow computations of the latitudes with knowledge of the causes, as will appear in its place.

CONCLUSION. We have thus spoken of the wondering at what is no wonder, of those who assume a fixed Earth.

This first Chapter of the figure of the Planets' Heavens being at an end, I am now coming to the description of their motion, and first of their motion in longitude.

VANDE LANG-DE LOOP.

TWEEDE ONDERSCHEYT DES DERDEN BOUCX VAN des Eertcloots loop en de Sonnens schijnbaer roersel.

CORTBEGRYP DESES TWEEDEN ONDERSCHEYTS.

NA vier bepalinghen sullen volghen vier voorstellen: T'eerste, wesende in d'oirden het 7 vanden loop des Eertcloots in haer wech.

Het tweede, wesende in d'oirden het 8, dat de Son met stelling eens roerenden Eertcloots, de selve schijnbaer duysteraerlangde, verheyt vanden Eertcloot, en voorofachtring ontfangt, diese heeft met stelling eens vasten Eertcloots.

Het derde, wesende in d'oirden het 9, dat des Eertclootwechs naestepunt onder de selve duysteraerlangde is, daer de Sonwechs verstepunt met stelling eens vasten Eertcloots onder is.

Het vierde, t'welck in d'oirden is het 10, dat wesende de Son met stelling eens vasten Eertcloots in haer wechs verstepunt of eerste halfront, den Eertcloot met haer roerende stelling oock in haer wechs verstepunt of eerste halfront is, en sulcken langde en voorofachtring de Son in haer wech heeft, derghelijcke langde en voorofachtring oock den Eertcloot inde hare te hebben.

1 BEPALING.

Wesende de Son ghenomen vast te staen als weerelts middelpunt, en den Eertcloot daer rontom te draeyen, in een rondt en met een roersel even ant rondt en roersel datmen de Son met stelling eens vasten Eertcloots toeschrijft: Sulcx heet stelling eens roerenden Eertcloots.

OF THE MOTION IN LONGITUDE

SECOND CHAPTER

OF THE THIRD BOOK

Of the Motion of the Earth and the Sun's Apparent Motion

SUMMARY OF THIS SECOND CHAPTER

After four definitions there are to follow four propositions: The first, which in the sequence is the 7th, of the motion of the Earth in its orbit.

The second, which in the sequence is the 8th, that on the theory of a moving Earth the Sun acquires the same apparent ecliptical longitude, distance from the Earth, and advance-or-lag which it has on the theory of a fixed Earth.

The third, which in the sequence is the 9th, that the nearest point (perihelion) of the Earth's orbit is at the same ecliptical longitude where the farthest point (apogee) of the Sun's orbit is on the theory of a fixed Earth.

The fourth, which in the sequence is the 10th, that when, on the theory of a fixed Earth, the Sun is at the farthest point (apogee) of its orbit or its first semi-circle, the Earth, when assumed to be moving, is also at the farthest point (aphelion) of its orbit or its first semi-circle, and that such longitude [1]) and advance-or-lag as the Sun has in its orbit, the same longitude and advance-or-lag the Earth also has in its orbit.

1st DEFINITION.

When it is assumed that the Sun is fixed as the world's centre and that the Earth moves round it, in a circle and with a motion equal to the circle and the motion ascribed to the Sun on the theory of a fixed Earth, this is called the theory of a moving Earth.

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[1]) The longitude "in its orbit" is reckoned from the apogee.

2 BEPALING.

Wesende de Maen ghestelt te draeyen rontom den roerendē Eertcloot, gelijckmēse andersins neemt te draeyen rontom een vasten: Sulcx heet Maenloop met stelling eens roerenden Eertcloots.

3 BEPALING.

Wesende gestelt den Eertcloot te loopen in een vvech even ant inrondt van Saturnus, Iupiter of Mars, en de selve Dvvaelders niet in haer inrondt als met stelling eens vasten Eertcloots, maer ter plaets van des inronts middelpunt: Sulcx heet haer loop met stelling eens roerenden Eertcloots.

4 BEPALING.

Wesende gestelt den Eertcloot te loopen in een vvech even an den inrontvvech van Venus of Mercurius, en de selve Dvvaelders in haer inrondt binnen den Eertclootvvech: Sulcx heet haer loop met stelling eens roerenden Eertcloots.

7 VOORSTEL.

Te beschrijven den loop des Eertcloots in haer vvech op een ghegheven tijt.

De boucken des Hemelloops *Ptolemeus* ter handt ghecommen, betuyghen dat de Ouden voor hem een ghebruyck hadden, inde beschrijving van yder Dwaelder te beginnen met sijn daghelicksche loop, of anders gheseyt met sijn loop op een bekenden tijt deur ervaring bevonden, welcke wijse in reden ghegront schijnende, ick salse in dit derde bouck met stelling eens roerenden Eertcloots soo navolghen, ghelijck ick int eerste mette stelling eens vasten gedaen heb. Het is dan te weten dat yder Dwaelder een ander wesentlicke loop heeft dan men hem deur de stelling eens vasten Eertcloots toeschrijft: T'ghene daer af vanden Eertcloot te segghen valt, t'is dat hy in hem heeft den wesentlicken loop diemen in d'ander stelling de Son-toeschrijft, welcke int 3 voorstel des 1 boucx berekent is sdaechs op 59 ① 8. 17. 13. 12. 31. en int natuerlick jaer een keer te doen. T'BESLVYT. Wy hebben dan beschreven den loop des Eertcloots in haer wech op een ghegheven tijt, na den eysch.

2nd DEFINITION.

When the Moon is assumed to move round the moving Earth, as it is otherwise taken to move round a fixed Earth, this is called motion of the Moon on the theory of a moving Earth.

3rd DEFINITION.

When it is assumed that the Earth moves in an orbit equal to the epicycle of Saturn, Jupiter or Mars, and the said Planets do not move on their epicycles, as on the theory of a fixed Earth, but are at the place of the epicycle's centre, this is called their motion on the theory of a moving Earth.

4th DEFINITION.

When it is assumed that the Earth moves in an orbit equal to the deferent of Venus or Mercury, and the said Planets move on their epicycles within the Earth's orbit, this is called their motion on the theory of a moving Earth.

7th PROPOSITION.

To describe the motion of the Earth in its orbit in a given time.

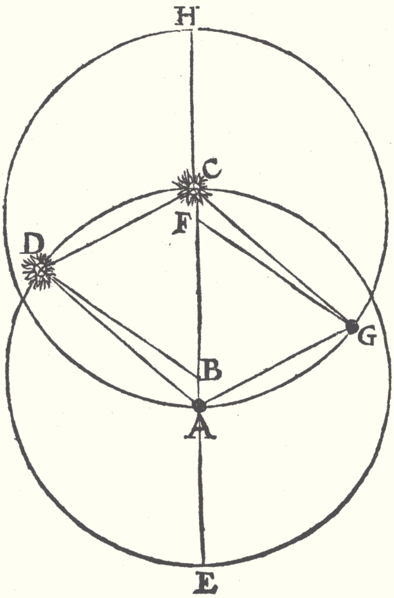
The books on the Heavenly Motions that came into *Ptolemy's* hands declare that the Ancients before him were accustomed to begin in the description of each Planet with its daily motion, in other words: with its motion in a given time found by experience, which method, seeming to be based on good reasons, I will follow in this third book on the theory of a moving Earth, as I have done in the first on the theory of a fixed Earth. It should be known that each planet has a true motion different from the one ascribed to it on the theory of a fixed Earth. What can be said in this respect of the Earth is that it has in it the true motion which is ascribed to the Sun on the other theory, which in the 3rd proposition of the 1st book has been computed to move 59;8,17,13,12,31 minutes a day and to perform one revolution in the natural year. CONCLUSION. We have thus described the motion of the Earth in its orbit in a given time; as required.

VERTOCCH. 8 VOORSTEL.

De Son ontfangt met ſtelling eens roerenden Eertcloots, de ſelve ſchijnbaer duyſteraerlangde, verheyt vanden Eertcloot, en voorofachtring, dieſe heeft met ſtelling eens vaſten Eertcloots.

T'GHEGHEVEN. Laet voor eerſte ſtelling ghenomen worden t'punt A een vaſten Eertcloot te beteyckenen, van A tot B ſy de Sonwechs uytmiddelpunticheytlijn, doende na *Ptolemeus* rekening ſulcke 417, alſſer de halfmiddellijn die B C ſy 10000 doet, mette ſelve B C ſy op B als middelpunt beſchreven de Sonwech C D E, waer in C A voortgetrocken tot E, ſoo is E t'naeſtepunt, C t'verſtepunt, waer an ick voor t'eerſte neem de Son te weſen, welcke van daer ghecommen ſy tot D.

Laet voor tweede ſtelling ghenomen worden den Eertcloot A te loopen, en de Son C vaſt te ſtaē: Tot deſen einde teycken ick in C A t'punt F, alſoo dat de uytmiddelpunticheytlijn C F evē ſy an A B, en beſchrijf op F als middelpunt, mette halfmiddellijn F A, die evē moet ſijn mette halfmiddellijn B C, het ront A G H als Eertclootwech, diens naeſtepunt H, verſtepunt A, waer in ick neem dē Eertcloot van A gecommen te ſijn tot G, ſulcx dat de booch A G even is met de booch C D: Laet daer na ghetrocken worden de ſes rechte linien A D, G C, C D, G A, B D, F G.



T'BEGEERDE. Wy moeten bewijſen dat de vaſte Son an C, geſien uyt den loopenden Eertcloot an G, de ſelve ſchijnbaer duyſteraerlangde en verheyt heeft der loopende Son an D, geſien uyt den vaſten Eertcloot an A.

T'BEWYS. Ten eerſten ſegh ick dat weſende de Son an C, en den Eertcloot an A, t'ſy datmen neemt den Eertcloot A vaſt te ſtaen, en de Son C loopende, of den Eertcloot A loopende, en de Son C vaſt, dat gheeft openbaerlick int punt des tijts datſe alſoo elck an des anders verſtepunt ſijn een ſelve ſchijnbaer plaets, en verheyt der Son vanden Eertcloot: Maer om te bewijſen dat ſulcx overal gheſchiet, ick ſegh aldus: Angheſien de booch A G even en ghelijck is mette booch C D, ſoo is den * evebeenighen driehouck B C D, even en ghelijck metten evebeenigen F A G, en daerom den houck B C D, even mettē houck F A G, dats oock A C D met C A G, waer deur de twee even rechte linien D C, A G evewijdeghe ſijn, en de twee rechte A D, G C daer tuſſchen ghetroc-

Iſoſceles.

THEOREM. 8th PROPOSITION.

On the theory of a moving Earth the Sun acquires the same apparent ecliptical longitude, distance from the Earth, and advance-or-lag which it has on the theory of a fixed Earth.

SUPPOSITION. Let it be assumed first that the point *A* denotes a fixed Earth; let the line from *A* to *B* be the line of eccentricity of the Sun's orbit, which according to *Ptolemy's* computation makes 417 if the semi-diameter, which shall be *BC*, makes 10,000. With this *BC* let there be described about *B* as centre the Sun's orbit *CDE*, in which, if *CA* is produced to *E*, *E* is the perigee, *C* the apogee, at which I first take the Sun to be, which shall have travelled thence to *D*.

Let it be assumed secondly that the Earth *A* moves and the Sun *C* is fixed. To this end I mark on *CA* the point *F* so that the line of eccentricity *CF* be equal to *AB*, and I describe about *F* as centre, with the semi-diameter *FA*, which must be equal to the semi-diameter *BC*, the circle *AGH* as the Earth's orbit, its perihelion being *H*, its aphelion *A*, in which I take the Earth to have travelled from *A* to *G*, so that the arc *AG* is equal to the arc *CD*. Thereafter let there be drawn the six straight lines *AD*, *GC*, *CD*, *GA*, *BD*, *FG*.

WHAT IS REQUIRED. We have to prove that the fixed Sun at *C*, seen from the moving Earth at *G*, has the same apparent ecliptical longitude and distance as the moving Sun at *D*, seen from the fixed Earth at *A*.

PROOF. Firstly I say that when the Sun is at *C* and the Earth at *A*, whether the Earth *A* is taken to be fixed and the Sun *C* moving or the Earth *A* moving and the Sun *C* fixed, this gives evidently as regards time that thus each is at the other's farthest point at the same apparent place and distance of the Sun from the Earth. But to prove that this happens always, I say as follows: Since the arc *AG* is equal and similar to the arc *CD*, the isosceles triangle *BCD* is equal and similar to the isosceles triangle *FAG*, and therefore the angle *BCD* is equal to the angle *FAG*, *i.e.* also *ACD* to *CAG*, in consequence of which the two equal straight lines *DC*, *AG* are parallel, and the two straight lines *AD*, *GC* drawn between ghetrocken moeten oock even en evewijdeghe wesen : Maer A D evewijdege sijnde met G C, soo moet de vaste Son C ghesien uyt den loopenden Eertcloot an G, op de selve plaets schijnen der loopende Son an D ghesien uyt den vasten Eertcloot an A, om dat A G noch oock de heele middellijn des Eertclootwechs gheen ghevoelelicke reden en heeft totte halfmiddellijn des vaste sterrecloots. Oock ist openbaer de verheyt G C even te sijn mette verheyt A D, ghemerckt het twee evewijdeghe sijn tusschen de twee evewijdege D C, A G. Angaende het derde punt, te weten dat de voorofachtring van d'een en d'ander stelling de selve is, blijckt aldus : De roerende Son an D wort uyt den vasten Eertcloot A, soo veel in den duysteraer schijnbaerlick meer achterwaert ghesien dan de middelson (die anghewesen is mette lini welcke vande Sonwechs middelpunt B deur D na den duysteraer streckt) als den houck A D B bedraecht : Maer soo veel wort de vaste Son uyt den roerenden Eertcloot G oock inden duysteraer schijnbaerlick meer achterwaert ghesien dan de Middelson (die anghewesen is mette lini welcke vanden Eertcloot G deur des Eertclootwechs middelpunt F na den duysteraer streckt) om dat A D evewijdege is met G C, en B D met G F, en vervolghens den houck A D B even metten houck C G F, waer deur de voorofachtring, t'welck hier achtring valt, van d'eē en d'ander stelling de selve is. T'BESLVYT. De Son dan ontfangt met stelling eens roerenden Eertcloots, de selve schijnbaer duysteraerlangde, verheyt vanden Eertcloot, en voorofachtring, diese heeft met stelling eens vasten Eertcloots, t'welck wy bewijsen moesten.

9 VOORSTEL.

Des Eertclootvvechs naestepunt onder de selve duysteraerlangde te vvesen daer des Sonvvechs verstepunt met stelling eens vasten Eertcloots onder is.

T'GHEGHEVEN. Laet de form des 8 voorstels andermael voor t'ghegeven ghenomen worden, alwaer blijckt H naestepunt des Eertclootwechs onder de selve duysteraerlangde te wesen, daer de Sonwechs verstepunt C met stelling eens vasten Eertcloots onder is, want ghetrocken van C (t'welck is de Son als weerelts middelpunt met stelling eens roerenden Eertcloots) een rechte lini deur haer naestepunt H, sy wijst inden duysteraer t'selve punt der langde die anghewesen wort mette rechte lini ghetrocken van A (t'welck is den Eertcloot als weerelts middelpunt met stelling eens vasten Eertcloots) deur C verstepunt des Sonwechs. T'BESLVYT. Des Eertclootwechs naestepunt dan is onder de selve duysteraerlangde daer des Sonwechs verstepunt met stelling eens vasten Eertcloots onder is, t'welck wy bewijsen moesten.

VERTOOCH. 10 VOORSTEL.

Wesende de Son met stelling eens vasten Eertcloots in haers vvechs verstepunt, of eerste halfrondt, den Eertcloot is met haer roerende stelling oock in haer vvechs verstepunt, of eerste halfront, en sulcken langde en voor-

them must also be equal and parallel. But when *AD* is parallel to *GC*, the fixed Sun *C*, seen from the moving Earth at *G*, must appear to be in the same place as the moving Sun at *D*, seen from the fixed Earth at *A*, because neither *AG* nor the entire diameter of the Earth's orbit has any perceptible ratio to the semi-diameter of the sphere of the fixed stars. It is also evident that the distance *GC* is equal to the distance *AD*, since they are two parallel lines between the two parallel lines *DC*, *AG*. As to the third point, to wit, that the advance-or-lag is the same for one theory and for the other, this becomes apparent as follows: The moving Sun at *D* is seen from the fixed Earth *A* as much apparently more backwards in the ecliptic than the mean sun (which is denoted by the line which extends from the centre of the Sun's orbit *B* through *D* to the ecliptic) as is the amount of the angle *ADB*. But this amount also the fixed Sun is seen from the moving Earth *G* apparently more backwards in the ecliptic than the Mean Sun (which is denoted by the line which extends from the Earth *G* through the centre of the Earth's orbit *F* to the ecliptic) because *AD* is parallel to *GC*, and *BD* to *GF*, and consequently the angle *ADB* is equal to the angle *CGF*, in consequence of which the advance-or-lag, which is lag in this case, is the same for one theory and for the other. CONCLUSION. The Sun therefore on the theory of a moving Earth acquires the same apparent ecliptical longitude, distance from the Earth, and advance-or-lag which it has on the theory of a fixed Earth; which we had to prove.

9th PROPOSITION.

That the perihelion of the Earth's orbit is at the same ecliptical longitude where the apogee of the Sun's orbit is on the theory of a fixed Earth.

SUPPOSITION. Let the figure of the 8th proposition be taken once more for the supposition, where it appears that *H*, the perihelion of the Earth's orbit, is at the same ecliptical longitude where the apogee of the Sun's orbit *C* is on the theory of a fixed Earth; for when from *C* (which is the Sun as the world's centre on the theory of a moving Earth) a straight line is drawn through its perihelion *H*, it indicates in the ecliptic the same point of longitude that is indicated by the straight line drawn from *A* (which is the Earth as the world's centre on the theory of a fixed Earth) through *C*, the apogee of the Sun's orbit. CONCLUSION. The perihelion of the Earth's orbit is therefore at the same ecliptical longitude where the apogee of the Sun's orbit is on the theory of a fixed Earth; which we had to prove.

THEOREM. 10th PROPOSITION.

When the Sun, on the theory of a fixed Earth, is at the apogee of its orbit, or its first semi-circle, the Earth, when assumed to be moving, is also at the farthest point (aphelion) of its orbit, or its first semi-circle, and such longitude ofachtring de Son in haer vvech heeft, dergelijcke langde en voorofachtring heeft oock den Eertcloot inde hare.

Laet de form des 8 voorstels andermael voor t'ghegheven verstrecken, waer me ick aldus segh: Wesende de Son met stelling eens vasten Eertcloots in haer wechs C D E verstepunt C, t'is openbaer dat den Eertcloot dan met haer roerende stelling oock is in haer wechs A G H verstepunt A.

Maer wesende de Son met stelling eens vasten Eertcloots in haer wechs C D E eerste halfrondt C D E an D, t'is openbaer dat den Eertcloot dan met haer roerende stelling oock is in haer wechs A G H eerste halfront A G H.

Voort sulcken langde de Son an D heeft in haer wech C D E, te weten de booch C D, derghelijcke langde heeft oock den Eertcloot an G in haer wech A G H, te weten den booch A G, want die even is met C D deur t'ghestelde.

Ten laetsten sulcken achtring A D B de Son D, heeft in haer wech C D E, derghelijcke achtring heeft oock den Eertcloot G, in haer wech A G H, want den houck der achtring A D B, is even metten houck C G F.

T'BESLVYT. Wesende dan de Son met stelling eens vasten Eertcloots in haer wechs verstepunt, of eerste halfront, den Eertcloot is met haer roerende stelling oock in haer wechs verstepunt of eerste halfront, en sulcken langde en voorofachtring de Son in haer wech heeft, derghelijcke langde en voorofachtring heeft oock den Eertcloot inde hare, t'welck wy bewijsen moesten.

VERVOLGH.

Anghesien den Eertcloot mette roerende stelling altijt tot sulcken plaets haers wechs is, als de Son met stelling eens vasten Eertcloots inde hare, soo volght daer uyt dat tot sulcke drie plaetsen alsmen de Son in haer wech neemt te wesen om daer deur de uytmiddelpunticheyt te berekenen, tot sulcke drie plaetsen moet oock den Eertcloot wesentlick sijn, sulcx datmen met haer loopende stelling soude moghen berekenen de selve uytmiddelpunticheyt, metsgaders de effening der daghen, en alles watter int tweede bouck deur stelling eens vasten Eertcloots berekent wort, maer sich in sulcke rekeninghen een vasten Eertcloot int ghedacht te prenten valt gherievigher, om de redenen die t'haerder plaets breeder verclaert sullen worden.

DERDE

and advance-or-lag as the Sun has in its orbit, the same longitude and advance-or-lag the Earth also has in its orbit.

Let the figure of the 8th proposition serve once more for the supposition, so that I say as follows. When the Sun, on the theory of a fixed Earth, is at the farthest point (apogee) *C* of its orbit *CDE,* it is evident that the Earth, when assumed to be moving, is also at the farthest point (aphelion) *A* of its orbit *AGH*.

But when the Sun, on the theory of a fixed Earth, is in the first semi-circle *CDE* of its orbit *CDE* at *D,* it is evident that the Earth, when assumed to be moving, is also in the first semi-circle *AGH* of its orbit *AGH.*

Further, such longitude as the Sun has at *D* in its orbit *CDE,* to wit, the arc *CD,* the same longitude the Earth also has at *G* in its orbit *AGH,* to wit, the arc *AG,* because this is equal to *CD* by the supposition.

Lastly, such lag *ADB* as the Sun *D* has in its orbit *CDE,* the same lag the Earth *G* also has in its orbit *AGH,* because the angle of the lag *ADB* is equal to the angle *CGF.*

CONCLUSION. When the Sun therefore, on the theory of a fixed Earth, is at the apogee of its orbit, or its first semi-circle, the Earth, when assumed to be moving, is at the aphelion of its orbit, or its first semi-circle, and such longitude and advance-or-lag as the Sun has in its orbit, the same longitude and advance-or-lag the Earth also has in its orbit; which we had to prove.

SEQUEL.

Since the Earth, when assumed to be moving, is always in the same place of its orbit as the Sun in its orbit on the theory of a fixed Earth, it follows therefrom that in such three places as the Sun is taken to be in its orbit in order to compute the eccentricity therefrom, in the same three places the Earth also must be in reality, so that when it is assumed to be moving, this eccentricity might be computed, as well as the equation of time, and all that is computed in the second book on the theory of a fixed Earth; but it is more convenient in such computations to impress on one's mind a fixed Earth, for the reasons to be set forth in their place.

DERDE ONDERSCHEYT DES DERDEN BOVCX VAN de Manens langdeloop met ſtelling eens roerenden Eertcloots.

CORTBEGRYP DESES DERDEN ONDERSCHEYTS.

Dit derde Onderſcheyt ſal tvvee voorſtellen hebben: T'eerſte, vveſende in d'oirden het 11, om op een gheghevem tijt den loop des Maenvvechs verſtepunts en der duyſteringſne te vinden, deur vviſconſtighe vverckinq ghegront op ſtelling eens roerenden Eertcloots.

Het tvveede, vveſende in d'oirden het 12, dat de Maen met ſtelling eens roerenden Eertcloots de ſelve ſchijnbaer duyſteraerlangde en verheyt vanden Eertcloot ontfangt, dieſe heeft met ſtelling eens vaſten Eertcloots.

11 VOORSTEL.

Te vinden op een gegeven tijt den loop van des Maenvvechs verſtepūt, en der duyſteringſne, deur vviſconſtige vverckinq gegront op ſtelling eens roerendē Eertcloots.

1 Voorbeelt van t'vinden des Maenvvechs verſtepunts middelloop.

T'GHEGHEVEN. Het is den tijt eens dachs. T'BEGHEERDE. Men wil daer op ghevonden hebben des Maenwechs verſtepunts middelloop in ſchijnbaer duyſteraerlangde, ghegront op ſtelling eens roerenden Eertcloots.

T'WERCK.

Des Eertcloots middelloop doet deur het 3 voorſtel des 1 boucx (welverſtaende dat de getalē des Sonloops aldaer beſchrevē hier om bekende reden voor Eertcloots middelloop ghenomen worden) ſdaechs	otr.59. 8.17.13.12.31.
Daer af ghetrocken de middelloop der voordering diemen des Maenwechs verſtepunt met ſtelling eens vaſten Eertcloots bevint te voorderen in ſchijnbaer duyſteraerlangde op 1 dach, bedraghende deur het 11 voorſtel des 1 boucx	otr. 6.41. 2.15.38.31.

 Blijft

THIRD CHAPTER

OF THE THIRD BOOK

Of the Moon's Motion in Longitude on the Theory of a Moving Earth

SUMMARY OF THIS THIRD CHAPTER

This third Chapter is to contain two propositions: the first, which in the sequence is the 11th, to find in a given time the motion of the apogee of the Moon's orbit and of the nodes, by mathematical operations based on the theory of a moving Earth.

The second, which in the sequence is the 12th, that on the theory of a moving Earth the Moon acquires the same apparent ecliptical longitude and distance from the Earth that it has on the theory of a fixed Earth.

11th PROPOSITION.

To find in a given time the motion of the apogee of the Moon's orbit and of the nodes, by mathematical operations based on the theory of a moving Earth.

1st Example, of the Finding of the Mean Motion of the Apogee of the Moon's Orbit.

SUPPOSITION. Let the time be one day. WHAT IS REQUIRED. It is required to find in this time the mean motion of the apogee of the Moon's orbit in apparent ecliptical longitude, based on the theory of a moving Earth.

PROCEDURE.

By the 3rd proposition of the 1st book the mean motion of the Earth (it being understood that for known reasons the figures of the Sun's motion there described are taken for the mean motion of the Earth) in one day is	0°;59, 8,17,13,12,31
When from this is subtracted the mean amount of the advance which the apogee of the Moon's orbit is found to make, on the theory of a fixed Earth, in apparent ecliptical longitude in 1 day, which by the 11th proposition of the 1st book is	0°; 6,41, 2,15,38,31

Blijft des Maenwechs verstepunts begheerde middelloop teghen t'vervolgh der trappen op een dach 0 tr. 52. 27. 14. 57. 34. 8.

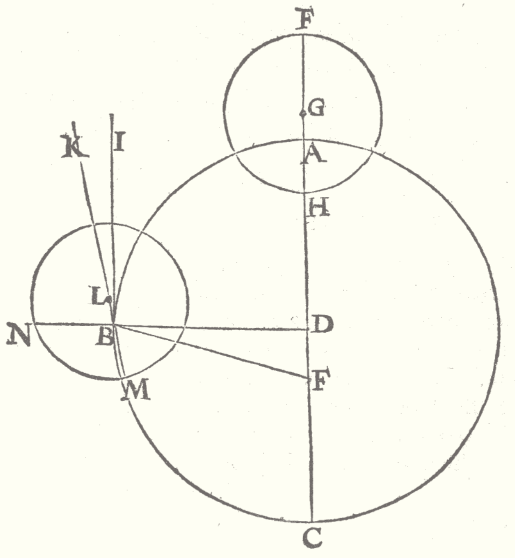
Tot hier toe is voorbeelt ghegheven op den loop eens dachs, waer deur men sekerder sien can t'groot verschil deses eyghen loops, teghen den oneygen met stelling eens vasten Eertcloots, dan deur langhe tijden daer heele ronden in commen diemen verlaet: Maer want de boochskens eens dachs seer cleen sijn, sulcx dattet volghende bewijs daer deur soo claer niet vallen en soude als op meerder, soo sal ick tot dien einde andermael nemen den tijt van 90 daghen.

Hier op doet den Eertcloots middelloop deur het 3 voorstel des 1 boucx 88 tr. 42.

Daer af getrocken de middelloop der voordering diemen des Maenwechs verstepunt met stelling eens vasten Eertcloots bevint te voorderen in schijnbaer duysteraerlangde op 90 daghen, bedraghende deur het 11 voorstel des 1 boucx 10 tr. 2.

Blijft des Maenwechs verstepunts begeerde middelloop tegen t'vervolgh der trappen op 90 daghen 78 tr. 40.

BEREYTSEL VAN T'BEWYS. Om van dit bewijs wat int ghemeen te segghen eer ick totte besonder verclaring comme, soo is voor al kennelick dat des Maenwechs verstepunts loop die wy schijnbaerlick inden duysteraer mercken, ghemengt is van haer eygen mette ghene diese vanden loop des Eertcloots ontfangt, welcke om tot rechte spiegeling te geraken nootsakelick moeten onderscheyden sijn, ghelijckt int wesen toegaet, want by aldienmen sonder daer opacht te nemen, des Maenwechs verstepunt met een roerenden Eertcloot sdaechs voordering gave van 6 ① 41 ②, sulcke teyckening en rekening en soude mette sake niet overeencommen. Dit verstaen sijnde soo laet A B C den



Eertclootwech beteyckenẽ, diẽs middelpunt D, de Son E, en deur de twee punten D, E, ghetrocken sijnde de rechte lini A D E C, so beteyckent A het verstepunt, an het welck ick ten eerstẽ neem dẽ Eertcloot te wesen, daer na D A voort getrockẽ sijnde tot F, ick teyckẽ inde lini A F het punt G, beschrijf daer op als middelpunt het rondt F H bediedẽde dẽ Maenwech, diens verstepunt ick neem te wesen F, daer na sy den Eertcloot op de boveschreven 90 dagen gecommen van A tot B, waer op haer middelloop doet deur t'werck 88 tr. 42 ① voor den houck A D B: Ick treck daer na de lini B I evewijdege met D A, eñ B K alsoo dat den houck I B K

doe de

There remains for the required mean motion of the apogee of the Moon's orbit, against the order of the degrees, in one day 0°;52,27,14,57,34, 8

Hitherto an example has been given of the motion of one day, from which the great difference between the true motion and the untrue motion on the theory of a fixed Earth can be seen with greater certainty than from long times, in which there are whole circles, which are discarded. But since the arcs of one day are very small, such that the following proof would not be as clear as for a longer time, for this purpose I will take next the time of 90 days.

In this time, by the 3rd proposition of the 1st book, the mean motion of the Earth is 88°42′

From this is subtracted the mean amount of the advance which the apogee of the Moon's orbit is found to make on the theory of a fixed Earth in apparent ecliptical longitude in 90 days, which by the 11th proposition of the 1st book is 10° 2′

There remains for the required mean motion of the apogee of the Moon's orbit, against the order of the degrees, in 90 days 78°40′

PRELIMINARY TO THE PROOF. To make a general statement about this proof before I come to the particular explanation, it is especially evident that the motion of the apogee of the Moon's orbit, which we apparently observe in the ecliptic, is a combination of its own motion with that which it receives from the motion of the Earth, which must necessarily be separated to reach a right theory, as happens in reality; for if, without heeding this, one should give the advance in one day of the apogee of the Moon's orbit, on the theory of a moving Earth, as 6′41″, neither the figure nor the computation would be in agreement with the true state of affairs. This being understood, let *ABC* denote the Earth's orbit, its centre being *D,* the Sun *E;* then, the straight line *ADEC* being drawn through the two points *D* and *E, A* denotes the apogee, where I first assume the Earth to be. Thereafter, *DA* being produced to *F,* I mark on the line *AF* the point *G* and describe about this as centre the circle *FH,* which denotes the Moon's orbit, whose apogee I assume to be *F.* Thereafter let the Earth have moved in the above-mentioned 90 days from *A* to *B,* in which its mean motion, according to the procedure, is 88°42′ for the angle *ADB.* Thereafter I draw the line *BI* parallel to *DA,* and *BK* so that the angle *IBK* is the doe de voordering diemen de middelloop van des Maenwechs verstepunt op de 90 daghen bevint ghevoordert te sijn in schijnbaer duysteraerlangde, bedraghende deur t'werck 10 tr. 2 ①: Ick stel daer na in B K t'punt L, sulcx dat B L sy de uytmiddelpunticheytlijn, en beschrijf op L als middelpunt de Maenwech K M, diens verstepunt K, en naestepunt M, treck oock D B voorwaert tot inden omtreck an N. T'BEWYS. By aldien het verstepunt als F gheen eyghen roersel ghehadt en hadde, t'soude, den Eertcloot ghecommen wesende an B, dan sijn inde voortghetrocken D B deur N, maer het is van daer achterwaert ghecommen tot K, als hebbende tot die plaets onder den duysteraer bevonden gheweest deur t'werck, daerom moeten wy bewijsen den houck N B K te doen de boveschreven 78 tr. 40 ①, t'welck aldus toegaet: Anghesien B I evewijdege is met D A, sulcx datmen van B deur I t'selve punt des duysteraers siet, datmen uyt D deur A sach, soo moet den houck I B N even sijn metten houck A D B, en doen als die 81 tr. 42 ①, daer af ghetrocken den houck K B I doende deur t'werck 10 tr. 2 ①, blijft voor den houck N B K 78 tr. 40 ①: Daerom op de 90 daghen in welcke den Eertcloot ghecommen is van A tot B na t'vervolgh der trappen, heeft het verstepunt in sijn eyghen roersel geloopen tegen t'vervolgh der trappen den houck N B K, doende gelijck wy bewijsen moestē 78 tr. 40 ①.

2 *Voorbeelt van t'vinden des duystering snees loop.*

T'GHEGHEVEN. Het is den tijt eens dachs. T'BEGHEERDE. Men wil daer op gevonden hebben des duysteringsnees middelloop in schijnbaer duysteraerlangde, ghegront op stelling eens roerenden Eertcloots.

T'WERCK.

Des Eertcloots middelloop doet deur het 3 voorstel des 1 boucx sdaechs	0 tr. 59.	8.17.13.12.31.
Daer toe vergaert de middelloop der achtring diemen de duysteringsne met stelling eens vasten Eertcloots bevint te verachteren in schijnbaer duysteraerlangde op 1 dach, bedraghende deur het 11 voorstel des 1 boucx	0 tr.	3.10.41.15.26. 7.
Comt de duysteringsnees begheerde middelloop tegen t'vervolgh der trappen op een dach	1 tr.	2.18.58.28.38.38.

Waer af t'bewijs deur t'voorgaende bewijs des 1 voorbeelts als daer groote ghelijckheyt me hebbende kennelick ghenouch is. T'BESLVYT. Wy hebben dan ghevonden op een gegeven tijt den loop van des Maenwechs verstepunt, en der duysteringsne, deur wisconstighe wercking ghegront op stelling eens roerenden eertcloots, na den eysch.

VERVOLGH.

Anghesien deur het 31 voorstel des 2 boucx bekent is des Maenwechs verstepunts schijnbaer duysteraerlangde op den anvangtijt, soo ist openbaer hoemen die sal vinden op alle ghegeven tijt, want totte plaets des anvangtijts, vervought den eyghen loop van daer af totten ghegheven tijt na de leering deses voorstels, en daer toe noch ghedaen den Eertcloots loop op den selven tijt, men heeft het begheerde. En sghelijcx is oock te verstaen mette duysteringsne.

VER

advance which the mean motion of the apogee of the Moon's orbit is found to make in 90 days, in apparent ecliptical longitude, which according to the procedure amounts to 10°2′. I then take in *BK* the point *L* such that *BL* be the line of eccentricity and I describe about *L* as centre the Moon's orbit *KM,* whose apogee is *K* and whose perigee is *M,* and I also produce *DB* to the circumference, to *N.* PROOF. If the apogee *(F)* had not had any motion of its own, it would — when the Earth had arrived at *B* — be in *DB* produced through *N,* but it has moved backwards from there to *K,* for in that place in the ecliptic it was found according to the procedure; we therefore have to prove that the angle *NBK* makes the aforesaid 78°40′, which is done as follows. Since *BI* is parallel to *DA,* so that from *B* through *I* the same point of the ecliptic is seen that was seen from *D* through *A,* the angle *IBN* must be equal to the angle *ADB* and like the latter must make 81°42′. When from this is subtracted the angle *KBI,* which according to the procedure makes 10°2′, there remains for the angle *NBK* 78°40′. Thus, in the 90 days during which the Earth has moved from *A* to *B* in the order of the degrees the apogee in its own motion against the order of the degrees has moved the angle *NBK,* which makes 78°40′, as we had to prove.

2nd Example, of the Finding of the Motion of the Nodes.

SUPPOSITION. Let the time be one day. WHAT IS REQUIRED. It is required to find in this time the mean motion of the nodes in apparent ecliptical longitude, based on the theory of a moving Earth.

PROCEDURE.

By the 3rd proposition of the 1st book the mean motion of the Earth in one day is	0°;59, 8,17,13,12,31
To this is added the mean amount of the lag which the nodes are found to lag in apparent ecliptical longitude in one day, on the theory of a fixed Earth, which, by the 11th proposition of the 1st book, is	0°; 3,10,41,15,26, 7
The required mean motion of the nodes, against the order of the degrees, in one day becomes	1°; 2,18,58,28,38,38

The proof of which is evident enough from the foregoing proof of the 1st example, as being greatly similar thereto. CONCLUSION. We have thus found the motion of the apogee of the Moon's orbit and of the nodes in a given time, by mathematical operations, based on the theory of a moving Earth; as required.

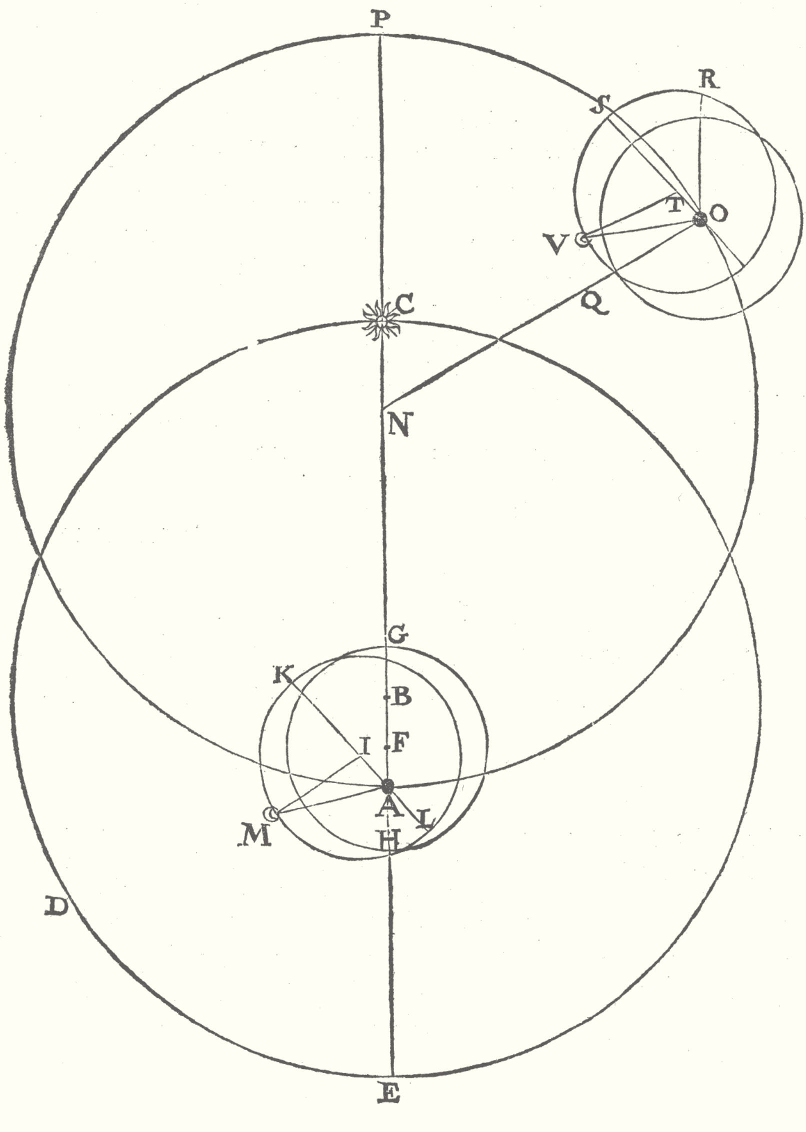
SEQUEL.

Since from the 31st proposition of the 2nd book the apparent ecliptical longitude of the apogee of the Moon's orbit is known at the initial moment, it is clear how it is to be found at any given time, for when to the position of the initial moment we add its own motion from there to the given time, according to the present proposition, and to this we further add the motion of the Earth in the said time, we have found what was required. And the same is also to be understood for the nodes.

VERTOOCH. 12 VOORSTEL.

De Maen ontfangt met ſtelling eens roerenden Eertcloots, de ſelve ſchijnbaer duyſteraerlangde en verheyt vanden Eertcloot dieſe heeft met ſtelling eens vaſten Eertcloots.

T'GHEGHEVEN. Laet voor eerſte ſtelling ghenomen worden t'punt A een vaſten Eertcloot te beteyckenen, van A tot B ſy de Sonwechs uytmiddel.



THEOREM. 12th PROPOSITION.

On the theory of a moving Earth the Moon acquires the same apparent ecliptical longitude and distance from the Earth which it has on the theory of a fixed Earth.

SUPPOSITION. For the latter theory [1]), let the point *A* be assumed to denote a fixed Earth, let the distance from *A* to *B* be the line of eccentricity of the Sun's orbit, and with the semi-diameter *BC* let there be described about *B* as

[1]) For *eerste stelling* in the Dutch text read *tweede stelling*.

punticheytlijn, en mette halfmiddellijn B C sy op B als middelpunt beschreven de Sonwech C D E, waer in C A voortgetrocken tot E, soo is E t'naestepunt, C t'verstepunt, daer na sy tussché den vasten Eertcloot A en B gestelt het punt F als Maenwechs middelpunt, en daer op mette halfmiddellijn F G beschreven den Maenwech G H, sniende A E in H als naestepunt, en A C in G als verstepunt, an t'welck ick voor begin neem de Maen te wesen. Op een tijt lanck daer na sy de Maenwechs middelpunt gecommen van F tot I, waer op ick mette halfmiddellijn I K, even an F G, beschrijf de Maenwech K L, diens verstepunt K, sulcx dat op den boveschreven tijt des Maenwechs verstepunt gecommen is van G tot K, en daerentusschen is de Maen van het verstepunt ghecommen soo verre als van K tot M (met soo veel heele ronden daer toe alst wesen mocht, diemen hier om bekende redenen verlaet) daer na ghetrocken A M, soo sal de Maen uyt A gesien schijnbaerlick soo verre sijn van C, wesende onder des duysteraers 65 tr. 30 ①, als den houck C A M mebrengt.

Dese teyckening des Maenwechs met stelling eens vasten Eertcloots aldus ghedaen sijnde, wy sullen totte teyckening van de ander stelling commen. Tot desen einde stel ick in C A t'punt N, alsoo dat de uytmiddelpunticheytlijn C N even sy an A B, en beschrijf op N als middelpunt mette halfmiddellijn N A die even moet sijn mette halfmiddellijn B C het rondt A O P als Eertclootwech, diens naestepunt P, verstepunt A, waer in ick neem den Eertcloot A mette Maenwech daer rontom ghecommen te sijn tot O, gheloopen hebbende den booch A O, of houck A N O. By aldien nu des Maenwechs verstepunt gheen eyghen roersel ghehadt en hadde te wijle den Eertcloot ghecommen is van A tot O, maer altijt ghebleven hadde tusschen den Eertcloot en t'punt N, t'is kennelick dattet soude wesen inde lini O N an t'punt Q, soo verre van O als van A tot G: Maer het heeft deur het 11 voorstel deses 3 boucx een roersel teghen t'vervolgh der trappen even an den Eertclootloop A N O, soo veel min als sijn schijnbaer voordering inden duysteraer bedraecht, dats den houck G A K, daerom treck ick de lini O R even en evewijdeghe met A G, en soude des Maenwechs verstepunt moeten sijn an R, gheloopen hebbende teghen t'vervolgh der trappen den houck Q O R, waerder niet noch af te trecken een houck even ande schijnbaer voordering G A K, daerom treck ick de lini O S even an A G, en alsoo dat den houck R O S even sy metten houck G F K, teycken daer in t'punt T, soo dat O T even sy met A F, en beschrijf op T als middelpunt, mette halfmiddellijn T S, den Maenwech S V R, diens verstepunt S: Maer want op den boveschreven ghestelden tijt de Maen met stelling eens vasten Eertcloots, ghecommen is van t'verstepunt K tot M, soo stel ick inde Maenwech S V R t'punt V, soo dat de booch S V of houck S T V, even is metten houck K I M, en stel de Maen te wesen an t'pnnt V. Dit soo sijnde, ick segh de Maen V uyt den roerenden Eertcloot O, ghesién te worden onder de selve schijnbaer duysteraerlangde daer de Maen an M onder ghesien wort uyt den vasten Eertcloot A, en dat de verheyt O V even is mette verheyt A M, om dese reden. T'BEWYS. Anghesien O R evewijdeghe is met A G, en den houck R O S even metten houck G A K, soo moet O S evewijdeghe sijn met A K. Voort anghesien den houck S T V, even is metten houck K I M, soo is de lini T V evewijdeghe met I M, boven dien soo heeft den driehouck T O V twee sijden T O, T V, even en evewijdeghe met des driehoucx I A M twee sijden I A, A M, waer deur haer derde sijden O V, A M oock even en evewijdeghe sijn, en daerom wort de Maen an V uyt den roerenden Eertcloot O, ghesien onder de selve schijnbaer duysteraer-

raercentre the Sun's orbit *CDE*, in which, when *CA* is produced to *E, E* is the perigee, *C* the apogee. Thereafter let there be marked between the fixed Earth *A* and *B* the point *F* for the centre of the Moon's orbit, and about this, with the semi-diameter *FG*, let there be described the Moon's orbit *GH*, intersecting *AE* in *H* for the perigee and *AC* in *G* for the apogee, at which I assume the Moon to be, to begin with. At a time long after, let the centre of the Moon's orbit have moved from *F* to *I*, about which with the semi-diameter *IK*, equal to *FG*, I describe the Moon's orbit *KL*, whose apogee is *K*, so that in the above-mentioned time the apogee of the Moon's orbit has moved from *G* to *K*, and meanwhile the Moon has moved from the apogee as far as from *K* to *M* (with as many whole circles as may be, which are here discarded for known reasons). When thereafter *AM* is drawn, the Moon, as seen from *A*, will apparently be as far from *C*, which is at 65°30′ of the ecliptic, as the angle *CAM* implies.

This drawing of the Moon's orbit on the theory of a fixed Earth thus having been made, we shall proceed to draw the situation on the other theory. To this end I mark in *CA* the point *N* such that the line of eccentricity *CN* be equal to *AB*, and I describe about *N* as centre, with the semi-diameter *NA* (which is to be equal to the semi-diameter *BC)*, the circle *AOP* for the Earth's orbit, whose perihelion is *P* and whose aphelion is *A*, in which I assume the Earth *A* with the Moon's orbit around it to have arrived at *O*, having moved the arc *AO* or the angle *ANO*. If the apogee of the Moon's orbit had had no motion of its own while the Earth moved from *A* to *O*, but had always remained between the Earth and the point *N*, it is evident that it would be in the line *ON* at the point *Q*, as far from *O* as from *A* to *G*. But according to the 11th proposition of this 3rd book it has a motion against the order of the degrees, equal to the motion of the Earth *ANO*, as much less as its apparent advance in the ecliptic amounts to, *i.e.* the angle *GAK*. I therefore draw the line *OR* equal and parallel to *AG;* then the apogee of the Moon's orbit would have to be at *R*, having moved, against the order of the degrees, the angle *QOR*, if it were not that an angle equal to the apparent advance *GAK* has to be subtracted from it. I therefore draw the line *OS* equal to *AG* and such that the angle *ROS* be equal to the angle *GAK* [1]), mark therein the point *T* such that *OT* be equal to *AF*, and describe about *T* as centre, with the semi-diameter *TS*, the Moon's orbit *SVR*, whose apogee is *S*. But because in the above-mentioned time the Moon has moved from the apogee *K* to *M* on the theory of a fixed Earth, I mark in the Moon's orbit *SVR* the point *V* such that the arc *SV* or the angle *STV* is equal to the angle *KIM*, and assume the Moon to be at the point *V*. This being so, I say that the Moon *V* is seen from the moving Earth *O* at the same apparent ecliptical longitude at which the Moon at *M* is seen from the fixed Earth *A*, and that the distance *OV* is equal to the distance *AM* for this reason. PROOF. Since *OR* is parallel to *AG* and the angle *ROS* is equal to the angle *GAK*, *OS* must be parallel to *AK*. Further, since the angle *STV* is equal to the angle *KIM*, the line *TV* is parallel to *IM*. Moreover the triangle *TOV* has two sides *(TO, TV)* equal and parallel to the two sides *IA, IM* [2]) of the triangle *IAM*, in consequence of which their third sides *OV, AM* are also equal and parallel, and therefore the Moon at *V* is seen from the moving Earth *O* at the same apparent ecliptical longitude at

[1]) For *GFK* in the Dutch text read *GAK*.
[2]) For *AM* in the Dutch text read *IM*.

raerlangde daer de Maen an M onder gesien wort uyt den vasten Eertcloot A, en de verheyt O V is even mette verheyt A M. T'BESLVYT. De maen dan ontfangt met stelling eens roerenden Eertcloots de selve schijnbaer duysteraerlangde en verheyt vanden Eertcloot diese heeft met stelling eens vasten Eertcloots, t'welck wy bewijsen moesten.

VERVOLGH.

T'is kennelick, datmen om op een ghegheven tijt te vinden de Manens schijnbaer duysteraerlangde gegront op stelling eens roerenden Eertcloots, sal soucken des Maenwechs verstepunts schijnbaer duysteraerlangde na de manier vant vervolgh des 11 voorstels, en daer na de rest ghelijck met stelling eens vasten Eertcloots, maer t'eenemael te rekenen op de stelling eens vasten Eertcloots valt gherievigher, om de redenen die t'haerder plaets breeder verclaert sullen worden.

VIERDE

which the Moon at *M* is seen from the fixed Earth *A*, and the distance *OV* is equal to the distance *AM*. CONCLUSION. On the theory of a moving Earth the Moon thus acquires the same apparent ecliptical longitude and distance from the Earth that it has on the theory of a fixed Earth; which we had to prove.

SEQUEL.

It is evident that in order to find at a given time the Moon's apparent ecliptical longitude, based on the theory of a moving Earth, we have to find the apparent ecliptical longitude of the apogee of the Moon's orbit in the manner of the sequel to the 11th proposition, and thereafter the rest in the same way as on the theory of a fixed Earth, but it is more convenient to make the computation at once on the theory of a fixed Earth, for the reasons to be explained more fully in the proper place.

VIERDE ONDERSCHEYT DES DERDEN BOUCX VAN Saturnus, Iupiters, Mars, Venus eu Mercurius langdeloop met stelling eens roerenden Eertcloots.

CORTBEGRYP DESES VIERDEN ONDERSCHEYTS.

Dit vierde Onderscheyt sal seven voorstellen hebben.

Het eerste wesende in d'oirden het 13, om te vindē de halfmiddellijnen der weghen, de uytmiddelpunticheytlijnen, met meeste en minste verheden van de Dwaelders, in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet, deur wisconstighe werckingghegront op stelling eens roerenden Eertcloots.

Het tweede wesende in d'oirden het 14, van den loop der drie bovenste Dwaelders Saturnus, Iupiter en Mars in haer weghen op een ghegeven tijt, met stelling eens roerenden Eertcloots.

Het derde wesende in d'oirden het 15, dat de drie bovenste Dwaelders Saturnus, Iupiter en Mars, met stelling eens roerenden Eertcloots de selve schijnbaer duysteraerlangde en verheyt vanden Eertcloot ontfanghen, diese hebben met stelling eens vasten Eertcloots.

Het vierde wesende in d'oirden het 16, vanden loop der twee onderste Dwaelders Venus en Mercurius in haer weghen op een ghegheven tijt, met stelling eens roerenden Eertcloots.

Het vijfde wesende in d'oirdē het 17, dat de twee onderste Dwaelders Venus en Mercurius met stelling eens roerenden Eertcloots, de selve schijnbaer duysteraerlangde en verheyt vanden Eertcloot ontfanghen diese hebben met stelling eens vasten Eertcloots.

Het seste wesende in d'oirden het 18, inhoudende verclaring der reden waerom ick inde 8.12.15. en 17 voorstellen, bewesen hebbe de Dwaelders deur stelling eens roerenden Eertcloots bevonden te worden totte selve schijnbaer plaetsen en verheden van malcander, diemense met stelling eens vasten Eertcloots bevint, mette omstandighen van dien.

Het sevende wesende in d'oirden het 19, inhoudende verclaring op

FOURTH CHAPTER

OF THE THIRD BOOK

Of Saturn's, Jupiter's, Mars', Venus', and Mercury's Motion in Longitude, on the Theory of a Moving Earth

SUMMARY OF THIS FOURTH CHAPTER

This fourth Chapter is to contain seven propositions.

The first, which in the sequence is the 13th, to find the semi-diameters of the orbits, the lines of eccentricity, with the greatest and least distances of the Planets, in such parts as the semi-diameter of the Earth's orbit has 10,000, by means of mathematical operations based on the theory of a moving Earth.

The second, which in the sequence is the 14th, of the motion of the three upper Planets Saturn, Jupiter, and Mars in their orbits at a given time, on the theory of a moving Earth.

The third, which in the sequence is the 15th, that the three upper Planets Saturn, Jupiter, and Mars on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth.

The fourth, which in the sequence is the 16th, of the motion of the two lower Planets Venus and Mercury in their orbits in a given time, on the theory of a moving Earth.

The fifth, which in the sequence is the 17th, that the two lower Planets Venus and Mercury on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth.

The sixth, which in the sequence is the 18th, containing an explanation of the reason why in the 8th, 12th, 15th, and 17th propositions I have proved that on the theory of a moving Earth the Planets are found at the same apparent places and mutual distances that they are found on the theory of a fixed Earth, with the circumstances relating thereto.

The seventh, which in the sequence is the 19th, containing an exposition on which theory — to wit, the untrue theory of a fixed Earth or the true one of a moving Earth — it seems most suitable to make the computations of the motion in longitude of the Planets.

vvelcke stelling, te vveten de oneyghen met een vasten Eertcloot, of de eyghen met een roerende, oirboirst schijnt de rekeninghen te maken vande langdeloop der Dvvaelders.

13 VOORSTEL.

Te vinden de halfmiddellijnen der vveghen, de uytmiddelpuntichey tlijnen, met meeste en minste verheden vande Dvvaelders, in sulcke deelen alsser des Eertclootvvechs halfmiddellijn 10000 doet, deur vviſconstighe vvercking gegront op stelling eens roerenden Eertcloots.

Mijn voornemen is hier te ghebruycken de ghetalen by *Ptolemeus* ghevonden, want hoe wel de verstepunten sedert seer verloopen sijn, en dat voorbeelden des teghenwoordighen tijts oirboirder souden moghen wesen om te sien de overeencomminghen deser sloten mette dadelicke ervaringhen, nochtans ghemerckt in *Ptolemeus* beschrijving ghevonden worden veel bequame voorbeelden, soo wel des breedeloops als langdeloops van al de Dwaelders, om daer uyt te bevestighen de voorstellen ghegront op stelling eens roerenden Eertcloots, soo heb ick die vercoren voor ander.

Om dan totte saeck te commen, t'is te anmercken dat de stelders eens vasten Eertcloots, niet wetende dat der drie bovenste Dwaelders schijnbaer inronden daerse in schijnen te loopen, sijn in plaets des Eertclootwechs, en datse daerom mette selve evegroot behooren te wesen, soo en hebbense haer halfmiddellijnen gheen selve ghetal ghegheven, ghelijck sy souden meughen doen die sulcx bekent is. Maer want hier uyt volght dat de ghetalen der halfmiddellijnen van weghen en uytmiddelpunticheytlijnen verscheydener Dwaelders, niet everedelick en sijn mette wesentlicke langden, ghelijck nochtans de sake vereyscht om bequamelick te wercken met stelling eens roerenden Eertcloots, en * Hemelloopstuych der Dwaelders te meughen teyckenen diens deelen everedelick sijn met haer overeencommende deelen des weerelts, soo sal ick op allen een ghemeene maet stellen, te weten des Eertclootwechs halfmiddelijn in 10000 ghedeelt, welcke met reden daer toe vercoren wort, om datse in de rekeningen van elck der ander Dwaelders comt, en dat haer ghetal van 10000 lichticheyt int werck veroirsaeckt.

Instrumenta Astronomica.

SAMING

der halfmiddellijnen en uytmiddelpunticheytlijnen door Ptolemeus *ghevonden en beschreven op stelling eens vasten Eertcloots.*

Sonwechs halfmiddellijn	60 deel.
Sonwechs uytmiddelpunticheytlijn	2 deel 30 (1).
Maenwechs halfmiddellijn	60 deel.
Maenwechs uytmiddelpunticheytlijn (die *Ptolemeus* inronts halfmiddellijn noemde)	5 deel 14 (1).

Andersins heeft *Ptolemeus* int 13 voorstel sijns 5 boucx, in plaets van die twee ghetalen 60 deel en 5 deel 14 (1), ghenomen twee inde selve reden, te weten 59 deel en 5 deel 10 (1), wesen-

13th PROPOSITION.

To find the semi-diameters of the orbits, the lines of eccentricity, with the greatest and least distances of the Planets, in such parts as the semi-diameter of the Earth's orbit has 10,000, by means of mathematical operations based on the theory of a moving Earth.

My intention is here to use the values found in *Ptolemy,* for though the apogees have shifted a good deal since then and examples of the present time might be more suitable to see the correspondences of these results with practical experience, nevertheless, seeing that in *Ptolemy's* description many suitable examples are found both of the motion in latitude and of the motion in longitude of all the Planets, with which to confirm the propositions based on the theory of a moving Earth, I have chosen them in preference to others.

To come to the matter, it is to be noted that those who assume a fixed Earth, not knowing that the apparent epicycles of the three upper Planets, in which they seem to move, have come instead of the Earth's orbit, and that they ought therefore to be of the same magnitude as the latter, have not given their semi-diameters the same value, as those to whom this is known might do. But since it follows from this that the values of the semi-diameters of the orbits and the lines of eccentricity of different Planets are not proportional to the actual lengths, as nevertheless is required for a convenient treatment on the theory of a moving Earth and for making it possible to draw astronomical instruments of the Planets [1]), whose parts are proportional to the corresponding parts of the world, I shall set a common measure for all, to wit, the semi-diameter of the Earth's orbit divided into 10,000, which is chosen for it with good reason because it occurs in the computations of each of the other Planets and because its value of 10,000 facilitates the procedure.

COMPILATION

of the Semi-diameters and Lines of Eccentricity Found and Described by *Ptolemy* on the Theory of a Fixed Earth.

Semi-diameter of the Sun's orbit	60^p [2])
Line of eccentricity of the Sun's orbit	$2^p30'$
Semi-diameter of the Moon's orbit	60^p
Line of eccentricity of the Moon's orbit (which *Ptolemy* called epicycle's semi-diameter)	$5^p14'$

[1]) Stevin does not here allude to instruments for observing the stars, but to structures, mostly of cardboard and parchment, such as were often made in those days to facilitate the understanding of the planetary orbits.

[2]) Sexagesimal division was used in ancient science for linear distances as well. The unit of distance was put 60 parts, and the first- and second-order subdivisions were indicated by the symbols now used for minutes and seconds of arc.

wesende Eertcloots halfmiddellijnen, sulcke alsser de Sonwechs halfmiddellijn 1210 doet.

Saturnus inrontwechs halfmiddellijn	60 deel.
Saturnus inronts halfmiddellijn	6 deel 30 ①.
Saturnus uytmiddelpunticheytlijn	3 deel 25 ①.
Iupiters inrontwechs halfmiddellijn	60 deel.
Iupiters inronts halfmiddellijn	11 deel 30 ①.
Iupiters uytmiddelpunticheytlijn	2 deel 45 ①.
Mars inrontwechs halfmiddellijn	60 deel.
Mars inronts halfmiddellijn	39 deel 30 ①.
Mars uytmiddelpunticheytlijn	6 deel.
Venus inrontwechs halfmiddellijn	60 deel.
Venus inronts halfmiddellijn	43 deel 10 ①.
Venus uytmiddelpunticheytlijn	1 deel 15 ①.
Mercurius inrontwechs halfmiddellijn	60 deel.
Mercurius inronts halfmiddellijn	21 deel 26 ①.
Mercurius uytmiddelpunticheytlijn	5 deel 41 ①.

Om dese ghetalen ghevonden met stelling eens vasten Eertcloots, te verkeeren in getalen met stelling eens roerenden Eertcloots, diens wechs halfmiddellijn 10000 doet als voorseyt is, ick sal metten Eertcloot beginnen, diens wechs halfmiddellijn de selve wesende diemen anders de Sonwech toeschrijft, sy is deur de voorgaende SAMING tot haer uytmiddelpunticheytlijn, in sulcken reden als 60 deel tot 2 deel 30 ①, maer om die te hebben in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet, ick segh 60 deelen gheven 2 deel 30 ①, wat 10000? comt 417, sulcx dat doende des Eertclootwechs halfmiddellijn 10000.

De uytmiddelpunticheytlijn doet 417.

Ende want *Ptolemeus* int 4 Hoofstick sijns 3 boucx de Sonwechs verstepunt stelt onder des duysteraers 65 tr. 30 ①, in wiens plaets nu des roerenden Eertclootwechs naestepunt comt deur het 9 voorstel deses 3 boucx, soo valt des selfden verstepunt onder des duysteraers 245 tr. 30.

Angaende de Maenwechs halfmiddellijn, anghesien die totte halfmiddellijn des Eertclootwechs in sulcken reden is deur de voorgaende SAMING als 1210 tot 59, ick segh 1210, gheeft 59, wat 10000? comt voor de Maenwechs halfmiddellijn 488.

Om te hebben de Maenwechs uytmiddelpunticheytlijn, ick segh Maenwechs halfmiddellijn 59 deel, gheeft uytmiddelpunticheytlijn 5 deel 10 ① deur de voorgaende SAMING, wat Maenwechs halfmiddellijn 488 vierde in d'oirden? comt Maenwechs uytmiddelpunticheytlijn 43.

Die vergaert totte Maenwechs halfmiddellijn 488 vierde in d'oirden, comt voor de meeste verheyt vanden Eertcloot totte Maen 531.

Ende die 43 vijfde in d'oirden, ghetrocken vande Maenwechs halfmiddellijn 488 vierde in d'oirden blijft de minste verheyt 445.

De Maenwechs verstepunts schijnbaer duysteraerlangde op corten tijt groote verandering crijghende en vereyscht hier niet te commen.

On the other hand, in the 13th proposition of his 5th book, *Ptolemy* took, instead of those two values of 60^p and $5^p14'$, two values in the same ratio, to wit, 59^p and $5^p10'$, the units being the semi-diameter of the Earth, of which the semi-diameter of the Sun's orbit has 1,210.

Semi-diameter of Saturn's deferent	60^p
Semi-diameter of Saturn's epicycle	$6^p30'$
Saturn's line of eccentricity	$3^p25'$
Semi-diameter of Jupiter's deferent	60^p
Semi-diameter of Jupiter's epicycle	$11^p30'$
Jupiter's line of eccentricity	$2^p45'$
Semi-diameter of Mars' deferent	60^p
Semi-diameter of Mars' epicycle	$39^p30'$
Mars' line of eccentricity	6^p
Semi-diameter of Venus' deferent	60^p
Semi-diameter of Venus' epicycle	$43^p10'$
Venus' line of eccentricity	$1^p15'$
Semi-diameter of Mercury's deferent	60^p
Semi-diameter of Mercury's epicycle	$21^p26'$
Mercury's line of eccentricity	$5^p41'$

In order to convert these values, found on the theory of a fixed Earth, into values on the theory of a moving Earth, the semi-diameter of whose orbit makes 10,000, as has been said above, I shall begin with the Earth, and since the semi-diameter of its orbit is the same as that otherwise ascribed to the Sun's orbit, it is according to the above Compilation to its line of eccentricity in the ratio of 60^p to $2^p30'$; but to have it in such parts as the semi-diameter of the Earth's orbit has 10,000, I say: 60^p give $2^p30'$; what does 10,000 give? It makes 417, so that if the semi-diameter of the Earth's orbit makes 10,000

the line of eccentricity makes 417

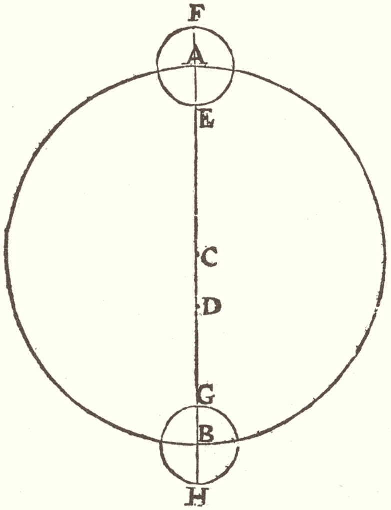
And since *Ptolemy* in the 4th Chapter of his 3rd book puts the apogee of the Sun's orbit at 65°30′ of the ecliptic, which is now replaced by the perihelion of the moving Earth's orbit, by the 9th proposition of this 3rd book, the aphelion of the latter now falls in the ecliptic at 245°30′

As regards the semi-diameter of the Moon's orbit, since this is to the semi-diameter of the Earth's orbit in the ratio of 1,210 to 59, according to the above Compilation, I say: 1,210 gives 59; what does 10,000 give? It gives for the semi-diameter of the Moon's orbit 488

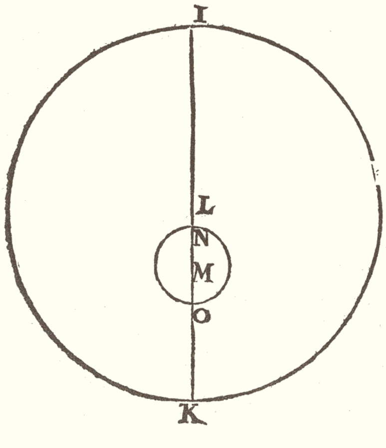
Om te hebben Saturnus wechs halfmiddellijn, ick segh Saturnus inronts halfmiddellijn 6 deel 30 ①, gheeft sijn inrontwechs halfmiddellijn 60 deel deur de voorgaende SAMING, wat 10000? comt voor Saturnuswechs halfmiddellijn 92308.

Om te hebben Saturnus uytmiddelpunticheytlijn, ick segh sijn inronts halfmiddellijn 6 deel 30 ①, geeft sijn uytmiddelpunticheytlijn 3 deel 25 ① deur de voorgaende SAMING, wat 10000? comt Saturnus uytmiddelpunticheytlijn 5256.

Om te hebbē Saturnus meeste en minste verhedē, ick sal op dat alles claerder sy teyckenē twee formē, d'eerste met stelling eens vasten Eertcloots, d'ander eens roerenden. Laet A B Saturnus inrontwech sijn diēs middelpunt C, en de halfmiddellijn C A doet 92308 achtste in d'oirdē, D is den vasten Eertcloot, C D de uytmiddelpunticheytlijn doende 5256 negende in d'oirden, A des inrontwechs



verstepunt, B naestepūt, voort y op A als middelpunt bechreven het inront E F, diens halfmiddellijn A E doet 10000 deur t'ghestelde, eñ des inronts verstepunt is F, naestepunt E, daer na sy op B als middelpunt beschrevē het inront G H even an E F, diens naestepunt G. Dit so sijnde, en om nu te hebben de meeste verheyt D F, soo vergaer ick tot C A 92308 achtste in d'oirden, de uytmiddelpunticheytlijn D C 5256 neghende in d'oirden, mette halfmiddellijn A F 10000, comt t'samen voor de meeste verheyt D F 107564. Ende om te hebbē de minste verheyt D G, ick treck D C 5256 met G B 10000 t'samen 15256, van C B 92308, en blijft voor de minste verheyt D G 77052.



Maer om nu te siē sulcke overeencoming met een roerendē Eertcloot, laet het ront I K sijn Saturnus wech, diens middelpunt L, en halfmiddellijn I L evē an C A doet als die 92308, en t'middelpunt des Eertclootwechs sy M, uytmiddelpunticheytlijn L M doende als C D 5256, Saturnuswechs verstepunt sy I, naestepunt K, voort sy op M als middelpunt beschreven dē Eertclootwech N O, diēs halfmiddellijn M N

even

In order to have the line of eccentricity of the Moon's orbit, I say: semi-diameter of the Moon's orbit 59^p gives line of eccentricity $5^p10'$ according to the above Compilation; what does the semi-diameter of the Moon's orbit 488 give, the fourth in the list? The line of eccentricity of the Moon's orbit becomes 43

When this is added to the semi-diameter of the Moon's orbit 488 (the fourth in the list), the greatest distance of the Earth from the Moon becomes 531

And when 43 (the fifth in the list) is subtracted from the semi-diameter of the Moon's orbit 488 (the fourth in the list), the least distance is 445

Since the apparent ecliptical longitude of the apogee of the Moon's orbit varies widely in a short time, this is not required to be discussed here.

In order to have the semi-diameter of Saturn's orbit, I say: the semi-diameter of Saturn's epicycle $6^p30'$ gives the semi-diameter of its deferent 60^p according to the above Compilation; what does 10,000 give? The semi-diameter of Saturn's orbit becomes 92,308

In order to have Saturn's line of eccentricity, I say: the semi-diameter of its epicycle $6^p30'$ gives its line of eccentricity $3^p25'$ according to the above Compilation; what does 10,000 give? Saturn's line of eccentricity becomes 5,256

In order to have Saturn's greatest and least distances, I will — to make everything clearer — draw two figures, the first on the theory of a fixed Earth, the other on that of a moving Earth. Let *AB* be Saturn's deferent, whose centre be *C,* and the semi-diameter *CA* makes 92,308 (the eighth in the list), *D* is the fixed Earth, *CD* the line of eccentricity making 5,256 (the ninth in the list), *A* the deferent's apogee, *B* its perigee; further let there be described about *A* as centre the epicycle *EF,* whose semi-diameter *AE* makes 10,000 by the supposition, then the epicycle's apogee is *F,* its perigee *E;* thereafter let there be described about *B* as centre the epicycle *GH* equal to *EF,* whose perigee is *G.* This being so, and in order to have the greatest distance *DF,* I add to *CA* 92,308 (the eighth in the list) the line of eccentricity *DC* 5,256 (the ninth in the list), with the semi-diameter *AF* 10,000; this makes together 107,564 for the greatest distance *DF.* And in order to have the least distance *DG,* I subtract *DC* 5,256 with *GB* 10,000, together making 15,256, from *CB* 92,308; then there is left for the least distance *DG* 77,052

But in order to see now such correspondence with a moving Earth, let the circle *IK* be Saturn's orbit, whose centre be *L* and whose semi-diameter *IL* equal to *CA,* like the latter, makes 92,308, and let the centre of the Earth's orbit be *M,* its line of eccentricity *LM* making (like *CD*) 5,256; let the apogee of Saturn's orbit be *I,* its perigee *K;* further let there be described about *M* as centre the Earth's orbit *NO,* even ſijnde met A E doet als die 10000. Dit ſoo weſende de meeſte verheyt O I, moet even ſijn mette boveſchreven D F, en de minſte O K even mette boveſchreven minſte D G, want tot I L 92308, vergaert L M 5256 met M O 10000, comt voor O I (ghelijck boven quam voor D F) als Saturnus meeſte verheyt 107564.

Ende L M 5256 met M O 10000 t'ſamen 15256 ghetrocken van L K 92308, blijft voor O K (ghelijck boven quam voor D G) als Saturnus minſte verheyt 77052.

Ende Saturnuswechs verſtepunt was ten tijde van *Ptolemeus* ſoo hy ſeght int 5 Hoofiſtick ſijns 11 boucx onder des duyſteraers 233 tr.

En ghedaen ſijnde derghelijcke rekeninghen met Iupiter en Mars, men bevint d'uytcomſt als volght

Iupiters wechs halfmiddellijn 52174.

Iupiters uytmiddelpunticheytlijn 2391.

Iupiters meeſte verheyt 64565.

Iupiters minſte verheyt 39783.

Iupiters wechs verſtepunt was ten tijde van *Ptolemeus* ſoo hy ſeght int 1 Hoofiſtick ſijns 11 boucx onder des duyſteraers 161 tr.

Marswechs halfmiddellijn 15190.

Mars uytmiddelpunticheytlijn 1519.

Mars meeſte verheyt 26709.

Mars minſte verheyt 3671.

Marswechs verſtepunt was ten tijde van *Ptolemeus* ſoo hy ſegt int 7 Hoofiſtick ſijns 10 boucx onder des duyſteraers 115 tr. 30.

Om te hebbē Venuswechs halfmiddellijn met ſtelling eens roerenden Eertcloots, ſoo is voor al te weten dat haer wech met ſtelling eens vaſten Eertcloots even ghenomen ſijnde metten Eertcloot-wech 10000, ſoo doet haer inronts halfmiddellijn dā ſulcke 7194, want ſegghende Venus inrontwechs halfmiddellijn 60 deel, geeft haer inronts halfmiddellijn 43 deel 10 (1) deur de voorgaende Saming, wat 10000? comt voor haer inronts halfmiddellijn als vooren 7194. Maer t'gene men met ſtelling eens vaſten Eertcloots noemt Venus inrontwech, is met ſtelling eens roerenden Eertcloots voor des ſelfden roerenden Eertclootswech: Ende het ghene men met ſtelling eens vaſten Eertcloots noemt Venus inront, is met ſtelling eens roerenden Eertcloots voor Venus wech, daerom Venus wechs halfmiddellijn met ſtelling eens roerenden Eertcloots doet 7194.

Om te hebben Venus uytmiddelpunticheytlijn, ick ſegh haer inrontwechs halfmiddellijn 60 deel, gheeft haer uytmiddelpunticheytlijn 1 deel 15 (1) deur de voorgaende Saming, wat 10000? comt Venus uytmiddelpunticheytlijn 208.

Om te hebben Venus meeſte en minſte verheden, ick ſal op dat alles claerder ſy, teyckenen twee formen, d'eerſte met ſtelling eens vaſten Eertcloots, d'ander eens roerenden. Laet A B Venus inrontwech ſijn, diens middelpunt C, en de halfmiddellijn C A doet 10000 deur t'geſtelde, D is den vaſten Eertcloot, C D de uytmiddelpunticheytlijn doende 208 vierentwintichſte in d'oirdē, A des inrontwechs verſtepunt, B naeſtepunt, voort ſy op A als middelpunt beſchreven het inrondt E F, diens halfmiddellijn A E doet whose semi-diameter *MN,* being equal to *AE,* like the latter makes 10,000. This being so, the greatest distance *OI* must be equal to the above-mentioned *DF,* and the least *OK* equal to the above-mentioned least distance *DG,* for if to *IL* 92,308 is added *LM* 5,256 with *MO* 10,000, this makes for *OI* (as was found above for *DF),* as Saturn's greatest distance, 107,564

And if *LM* 5,256 with *MO* 10,000, making together 15,256, is subtracted from *LK* 92,308, there is left for *OK* (as was found above for *DG*), as Saturn's least distance, 77,052

And the apogee of Saturn's orbit was at the time of *Ptolemy,* as he says in the 5th Chapter of his 11th book, in the ecliptic at 233°

And when similar computations are made with Jupiter and Mars, the result is found as follows:

Semi-diameter of Jupiter's orbit	52,174
Jupiter's line of eccentricity	2,391
Jupiter's greatest distance	64,565
Jupiter's least distance	39,783

The apogee of Jupiter's orbit was at the time of *Ptolemy,* as he says in the 1st Chapter of his 11th book, in the ecliptic at 161°

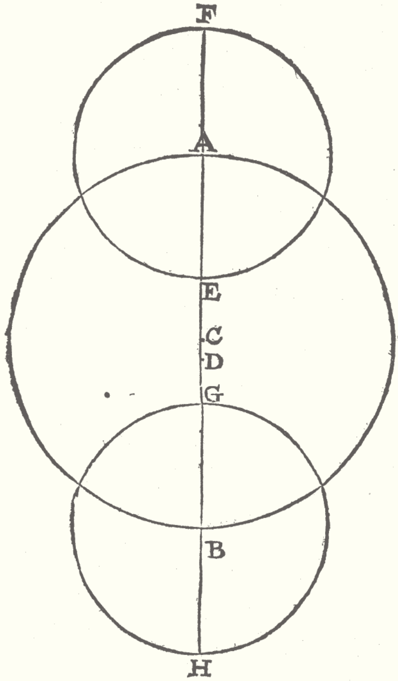
Semi-diameter of Mars' orbit	15,190
Mars' line of eccentricity	1,519
Mars' greatest distance	26,709
Mars' least distance	3,671

The apogee of Mars' orbit was at the time of *Ptolemy,* as he says in the 7th Chapter of his 10th book, in the ecliptic at 115°30′

In order to have the semi-diameter of Venus' orbit on the theory of a moving Earth, it is to be noted first of all that if its orbit on the theory of a fixed Earth is taken equal to the Earth's orbit 10,000, the semi-diameter of its epicycle makes 7,194, for if I say: the semi-diameter of Venus' deferent 60^p gives its epicycle's semi-diameter $43^p10'$ according to the above Compilation, what does 10,000 give? Its epicycle's semi-diameter becomes, as above, 7,194. But what on the theory of a fixed Earth is called Venus' deferent, on the theory of a moving Earth is the orbit of this moving Earth. And what on the theory of a fixed Earth is called Venus' epicycle, on the theory of a moving Earth is Venus' orbit; therefore the semi-diameter of Venus' orbit on the theory of a moving Earth makes 7,194

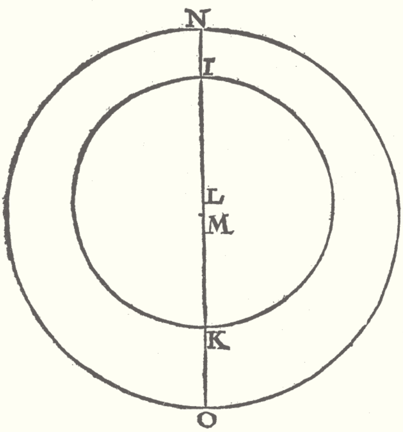
In order to have Venus' line of eccentricity, I say: its deferent's semi-diameter 60^p gives its line of eccentricity $1^p15'$ according to the above Compilation; what does 10,000 give? Venus' line of eccentricity becomes 208

In order to have Venus' greatest and least distances, I will — in order to make everything clearer — draw two figures, the first on the theory of a fixed Earth, the second on that of a moving Earth. Let *AB* be Venus' deferent, whose centre is *C,* and the semi-diameter makes 10,000 by the supposition, *D* is the fixed Earth, *CD* the line of eccentricity, which makes 208 (the twenty-fourth in the list), *A* the



7194 drieentwintichſte in de oirden, en des inronts verſtepunt is F, naeſtepunt E, daer na ſy op B als middelpũt beſchrevẽ het inront G H evẽ an E F, diẽs naeſtepunt G: Dit ſo ſijnde, en om nu te hebbẽ de meeſte verheyt D F, ſo vergaer ick tot C A 10000, de uytmiddelpuntícheytlijn D C 208 vierẽtwintichſte in d'oirden, mette halfmiddellijn A F 7194 drieentwintichſte in d'oirdẽ, comt t'ſamẽ voor de meeſte verheyt D F 17402: Ende om te hebbẽ de minſte verheyt D G, ick treck D C 208 met G B 7194 t'ſamen 7502, van C B 10000, en blijft voor de minſte verheyt D G 2598.

Maer om nu te ſien ſulcke overeencomming met eẽ roerendẽ Eertcloot, laet het ront I K ſijn Saturnus wech, diens middelpunt L, en de halfmiddellijn I L evẽ an A F doet als die 7194, en t'middelpunt des Eertclootwechs ſy M, uytmiddelpunticheytlijn L M, doende als C D 208, Venuswechs verſtepunt van M ſy I, naeſtepunt K, voort ſy op M als middelpunt beſchreven den Eertclootwech N O, diens halfmiddellijn M N evẽ met C A doet als die 10000. Dit ſo weſende, de meeſte verheyt O I moet even ſijn mette boveſchreven D F, en de minſte verheyt N I, evẽ mette boveſchreven minſte D G, want tot I L 7194, vergaert L M 208 met M O 10000, comt voor O I (ghelijck bovẽ quam voor D F) als Venus meeſte verheyt dieſe vandẽ Eertcloot hebben can 17402.



De ſelve getrocken vande heele middellijn N O 20000, blijft voor de minſte verheyt die Venus vanden Eertcloot hebben can 2598.

En Venuswechs verſtepunt was ten tijde van *Ptolemeus*, ſoo hy ſeght in 2 Hoofſtick ſijns 10 boucx, onder des duyſteraers 55 tr.

Ende ghedaen ſijnde dergheliјcke rekeninghen met Mercurius, men bevint d'uytcomſt als volght:

Mercu.

deferent's apogee, *B* its perigee. Further let there be described about *A* as centre the epicycle *EF,* whose semi-diameter *AE* makes 7,194 (the twenty-third in the list), then the epicycle's apogee is *F,* its perigee *E.* Thereafter let there be described about *B* as centre the epicycle *GH* equal to *EF,* whose perigee be *G.* This being so, and in order to have the greatest distance *DF,* I add to *CA* 10,000 the line of eccentricity *DC* 208 (the twenty-fourth in the list), with the semi-diameter *AF* 7,194 (the twentythird in the list); this makes together for the greatest distance *DF* 17,402. And in order to have the least distance *DG,* I subtract *DC* 208 with *GB* 7,194, making together 7,402 [1]), from *CB* 10,000; then there is left for the least distance *DG* 2,598.

But in order to see such correspondence with a moving Earth, let the circle *IK* be Venus' [2]) orbit, whose centre is *L,* then the semi-diameter *IL* equal to *AF,* like the latter, makes 7,194; and let the centre of the Earth's orbit be *M,* the line of eccentricity *LM,* making (like *CD)* 208; let the apogee of Venus' orbit from *M* be *I,* its perigee *K.* Further let there be described about *M* as centre the Earth's orbit *NO,* whose semi-diameter *MN* equal to *CA,* like the latter, makes 10,000. This being so, the greatest distance *OI* must be equal to the above-mentioned *DF,* and the least distance *NI* equal to the above-mentioned least distance *DG,* for if to *IL* 7,194 is added *LM* 208 with *MO* 10,000, this makes for *OI* (as was found above for *DF),* as the greatest distance from the Earth that Venus can have 17,402

When this is subtracted from the whole diameter *NO* 20,000, there is left for the least distance from the Earth that Venus can have 2,598

And the apogee of Venus' orbit was at the time of *Ptolemy,* as he says in the 2nd Chapter of his 10th book, in the ecliptic at 55°

And when similar computations are made with Mercury, the result is found to be as follows:

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1) For 7,502 in the Dutch text read 7,402.
2) For *Saturnus* in the Dutch text read *Venus.*

Mercurius wechs halfmiddellijn	3572.
Mercurius uytmiddelpunticheytlijn	947.
Mercurius meeste verheyt	14519.
Mercurius minste verheyt	5481.
Mercurius wechs verstepunt was tẽ tijde van *Ptolemeus*, soo hy seght int 7 Hoofstick sijns 9 boucx, onder des duysteraers	190 tr.

MERCKT.

De ghetalen der halfmiddellijnen, uytmiddelpunticheytlijnen, verheden, en schijnbaer duysteraerlangden hier boven beschreven gelijckse gevonden wierden, die sal ick nu andermael int corte oirdentlick by een vervoughen, op dat daer uyt int volghende ghebruyck de begheerde ghetalen te gherievelicker gevonden meughen worden.

BYEENVOVGING VANDE Dvvaelders halfmiddellijnen der vveghen, uytmidderpunticheytlijnen, meeste en minste verheden vanden Eertcloot, altemael in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 *doet, metsgaders der verstepuntens schijnbaer duysteraerlangden ten tijde van* Ptolemeus.

	Mercur.	*Venus.*	*Eertcloot.*	*Maen.*	*Mars.*	*Iupiter.*	*Saturnus.*
VVechs halfmiddellijn.	3572	7194	10000	488	15190	52174	92308
Vytmiddelpunticheytlijn.	947	208	417	43	1519	2391	5256
Meeste verheyt.	14519	17402	10417	531	26709	64565	107564
Minste verheyt.	5481	2598	9583	445	3671	39783	77052
Verstepunts schijnbaer duysteraerlangde.	190 tr.	55 tr.	245 tr. 30	- -	115 tr. 30	161 tr.	233 tr.

T'BESLVYT. Wy hebben dan gevondẽ de halfmiddellijnẽ der wegen, de uytmiddelpunticheytlijnen, met meeste en minste verhedẽ vande Dwaelders, in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet, deur wisconstige wercking gegront op stelling eens roerendẽ Eertcloots, na den eysch.

VERVOLGH.

Aldus bekent sijnde de reden deser linien vande Dwaelders weghen en haer inronden, in sulcke deelen alsser des Eertcloots halfmiddellijn 10000 doet, en dat des Eertclootwechs halfmiddellijn in haer begrijpt 1210 halfmiddellijnen des Eertcloots, soo ist openbaer hoemen sal connen beschrijven sulcke voorstellen, als daer vermaen afgedaen is int 2 merck des 49 voorstels vant 2 bouck met stelling eens vasten Eertcloots, te weten sulcke voorstellen als het 23 en 24 vande Son, oock ghelijck het 39 en 40 vande Maen: En boven dien hoemen vinden sal alle linien daer voorvallende, in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet.

14 VOORSTEL.

Te beschrijven den loop der drie bovenste Dvvaelders Saturnus, Iupiter en Mars in haer vveghen, op een ghegeven tijt, met stelling eens roerenden Eertcloots.

Semi-diameter of Mercury's orbit	3,572
Mercury's line of eccentricity	947
Mercury's greatest distance	14,519
Mercury's least distance	5,481

The apogee of Mercury's orbit at the time of *Ptolemy* was, as he says in the 7th Chapter of his 9th Book, in the ecliptic at 190°

NOTE.

I will sum up once more briefly and orderly the values of the semi-diameters, lines of eccentricity, distances, and apparent ecliptical longitudes described above, as they have been found, so that the required values may be found more conveniently in the subsequent application.

LIST

of the semi-diameters of the Planets' orbits, lines of eccentricity, greatest and least distances from the Eearth, all in such parts as the semi-diameter of the Earth's orbit has 10,000, as also the apparent ecliptical longitudes of the aphelia at the time of *Ptolemy*.

	Mercury	Venus	Earth	Moon	Mars	Jupiter	Saturn
Semi-diameter of the orbit	3,572	7,194	10,000	488	15,190	52,174	92,308
Line of eccentricity	947	208	417	43	1,519	2,391	5,256
Greatest distance	14,519	17,402	10,417	531	26,709	64,565	107,564
Least distance	5,481	2,598	9,583	445	3,671	39,783	77,052
Apparent ecliptical longitude of aphelion	190°	55°	245°30′	—	115°30′	161°	233°

CONCLUSION. We have thus found the semi-diameters of the orbits, the lines of eccentricity, with the greatest and least distances of the Planets, in such parts as the semi-diameter of the Earth's orbit has 10,000, by means of mathematical operations based on the theory of a moving Earth; as required.

SEQUEL.

The ratio of these lines of the Planets' orbits and their epicycles thus being known, in such parts as the semi-diameter of the Earth's orbit has 10,000, and the semi-diameter of the Earth's orbit comprising 1,210 semi-diameters of the Earth, it is evident how it is possible to describe propositions such as have been announced in the 2nd note to the 49th proposition of the 2nd book, on the theory of a fixed Earth, to wit, propositions such as the 23rd and 24th on the Sun, as also the 39th and 40th on the Moon [1]); and moreover how any lines that may occur can be found, in such parts as the semi-diameter of the Earth's orbit has 10,000.

14th PROPOSITION.

To describe the motion of the three upper Planets Saturn, Jupiter, and Mars in their orbits in a given time, on the theory of a moving Earth.

[1]) These propositions, on the pages 171, 201 and 221 of the Dutch original, concern the distances of the Earth to the Moon, to the Sun and to Saturn, all expressed in the semi-diameter of the Earth.

T'GHEGEVEN. Om met Saturnus te beginnen soo laet den tijt sijn van een dach. T'BEGEERDE. Men wil daer op vinden den loop van Saturnus in sijn wech, met stelling eens roerenden Eertcloots.

T'WERCK.

Het is te weten dat Saturnus eyghentlick wesende in sijn wech tot sulcken plaets als daermen met stelling eens vasten Eertcloots het inronts middelpunt teyckent, soo en heeft hy eyghentlick gheen ander loop, doende daghelicx als int 18 voorstel des 1 boucx 0 tr. 2. 0. 33. 31. 28. 51. Angaende de voorofachtringhen welcke men hem siet hebben, die en commen niet van weghen een inront datmē hem deur stelling eens vasten Eertcloots toeschrijft, maer deur den Eertcloots loop, sulcx dat hy met soodanich daghelicx roersel eenvoudelick in sijn wech draeyt: Ende desghelijcx is oock te verstaen van Iupiter en Mars.

T'BESLVYT. Wy hebben dan beschreven den loop der drie bovenste Dwaelders Saturnus, Iupiter en Mars in haer weghen, op een ghegheven tijt met stelling eens roerenden Eertcloots, na den eysch.

VERTOOCH. 15 VOORSTEL.

De drie bovenste Dvvaelders Saturnus, Iupiter en Mars, ontfanghen met stelling eens roerenden Eertcloots, de selve schijnbaer duysteraerlangde en verheyt vanden Eertcloot, diese hebbē met stelling eens vastē Eertcloots.

Om dit voorstel oirdentlick te verclaren, ick salt in ses leden verdeelen, waer af int eerste sal sijn de teyckening van een der drie bovenste als Marsloop met stelling eens vasten Eertcloots.

Int tweede de teyckening van Marsloop met stelling eens roerenden Eertcloots.

Int derde t'bewijs dat Mars in d'een en d'ander stelling een selve schijnbaer duysteraerlangde heeft, en de selve verheyt vanden Eertcloot als hy is in sijn inronts verstepunt, en t'inronts middelpunt an sijn wechs verstepunt.

Daer na om te bewijsen sulcx alsoo overal te gheschien, soo sal tot bereyding van dien int vierde lidt verclaert worden, hoe dat de halfmiddellijn van Saturnuswechs middelpunt tot des inronts middelpunt, altijt evewijdeghe is mette halfmiddellijn van des inrontwechs middelpunt, tot des inronts middelpunt: En des Eertclootwechs halfmiddellijn van haer middelpunt totten Eertcloot, altijt evewijdeghe mette halfmiddellijn des inronts vant middelpunt tot Mars.

Int vijfde lidt, dat Mars in d'een en d'ander stelling tot allen plaetsen een selve schijnbaer duysteraerlangde heeft, en de selve verheyt vanden Eertcloot.

Int seste vant verschil datter valt tusschen de werckingen van d'een en d'ander stelling int berekenen der schijnbaer duysteraerlangde des Dwaelders.

1 LIDT *inhoudende de teyckening van Marsloop met stelling eens vasten Eertcloots.*

Laet voor eerste stelling ghenomen worden t'punt A een vasten Eertcloot te beteyckenē, en van A tot B sy des Sonwechs uytmiddelpunticheytlijn doende na

SUPPOSITION. To begin with Saturn, let the time be one day. WHAT IS REQUIRED. It is required to find in that time the motion of Saturn in its orbit, on the theory of a moving Earth.

PROCEDURE.

It is to be noted that since Saturn actually is in its orbit in a place where on the theory of a fixed Earth the epicycle's centre is drawn, in reality it has no other motion, and moves daily, as in the 18th proposition of the 1st book [1]), 0° 2,0,33,31,28,51. As to the advance-or-lag [2]) that it is seen to have, this is not caused by an epicycle ascribed to it on the theory of a fixed Earth, but by the motion of the Earth, so that with this daily motion it simply revolves in its orbit. And the same is also to be understood of Jupiter and Mars.

CONCLUSION. We have thus described the motion of the three upper Planets Saturn, Jupiter, and Mars in their orbits in a given time, on the theory of a moving Earth; as required.

THEOREM. 15th PROPOSITION.

The three upper Planets Saturn, Jupiter, and Mars on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth.

In order to explain this proposition properly, I will divide it into six sections, the first of which is to comprise the drawing of one of the three upper Planets, namely Mars' motion, on the theory of a fixed Earth.

The second, the drawing of Mars' motion on the theory of a moving Earth.

The third, the proof that on either theory Mars has the same apparent ecliptical longitude and the same distance from the Earth when it is at its epicycle's apogee and when the epicycle's centre is at the apogee of its orbit.

Thereafter, in order to prove that this happens thus always, in preparation of this it will be explained in the fourth section how the semi-diameter from the centre of Mars' orbit [3]) to the planet itself [4]) is always parallel to the semi-diameter from the deferent's centre to the epicycle's centre, and the semi-diameter of the Earth's orbit from its centre to the Earth always parallel to the semi-diameter of the epicycle from its centre to Mars.

The fifth section, that in all places Mars has on either theory the same apparent ecliptical longitude and the same distance from the Earth.

The sixth, of the difference between the operations on either theory in the computation of the apparent ecliptical longitude of the Planet.

1st SECTION, comprising the drawing of Mars' motion on the theory of a fixed Earth.

Let it first be assumed that the point *A* denotes a fixed Earth, and let the line from *A* to *B* be the line of eccentricity of the Sun's orbit, according to *Ptolemy's*

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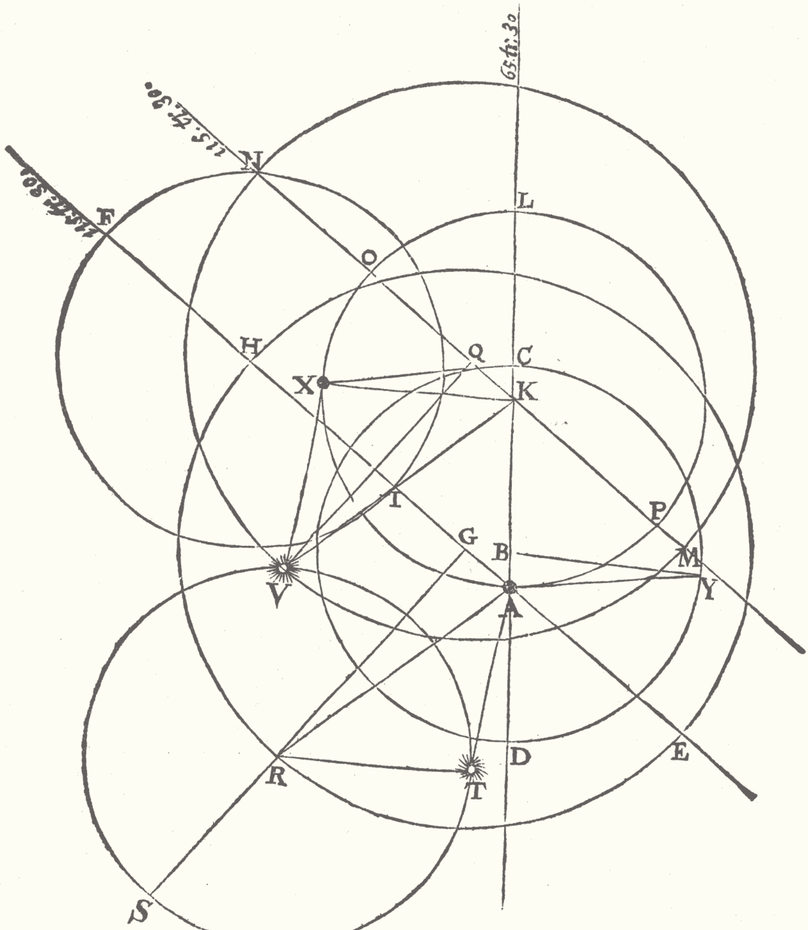
[1]) See Stevin's "2nd Method" in the 19th proposition of the 1st book, p. 97.

[2]) Here advance-or-lag is used, as it was in the 18th Definition of the First Book, for the total deviation, by the epicycle as well as by the eccentricity.

[3]) For *Saturnuswechs* in the Dutch text read *Marswechs*.

[4]) For *des inronts middelpunt* in the Dutch text read an expression equivalent to "the planet itself".

de na *Ptolemeus* rekening t'sijnder tijt sulcke 417, alsser des Sonwechs halfmiddellijn die B C sy 10000 doet deur het 13 voorstel deses 3 boucx, mette selve B C sy op B als middelpunt beschreven de Sonwech C D, waer in C A voortghetrocken tot D, soo is D t'naestepunt, C t'verstepunt wesende tot *Ptolemeus* tijt onder des duysteraers 65 tr. 30 ①. Om hier op nu te doen de teyckening des loops van eenige der drie boveschreven Dwaelders, ick neem tot voorbeelt daer toe als boven gheseyt is Mars, wiens inrontwechs verstepunt ten tijde van



Ptolemeus gheweest hebbende onder des duysteraers 115 tr. 30 ① deur het 13 voorstel deses 3 boucx, ick treck deur den vasten Eertcloot A de lini E F van A tot F na des duysteraers boveschreven 115 tr. 30 ①, te weten soo dat den houck C A F doe 50 tr. dieder sijn vandē 65 tr. 30 ① daer C onder is, tottē 115 tr 30 ① daer F onder is. Ick stel daer na in A F t'punt G, soo dat A G sy Mars inrontwechs uytmiddelpunticheytlijn, doende deur het 13 voorstel sulcke 1519, alsser des Sonwechs halfmiddellijn 10000 doet: Ick teycken daer na inde lini G F t'punt computation making at his time 417 such parts as the semi-diameter of the Sun's orbit, which shall be *BC,* has 10,000, by the 13th proposition of this 3rd book; with the said *BC* let there be described about *B* as centre the Sun's orbit *CD,* in which, when *CA* is produced to *D, D* is the perigee, *C* the apogee, which in *Ptolemy's* time was at 65°30′ of the ecliptic. In order to base on this the drawing of the motion of one of the three aforesaid Planets, I take as example, as has been said above, Mars; the apogee of the latter's deferent having been in *Ptolemy's* time at 115°30′ of the ecliptic, by the 13th proposition of this 3rd book, I draw through the fixed Earth *A* the line *EF* from *A* to *F* towards the point at 115°30′ of the ecliptic described above, to wit, in such a way that the angle *CAF* be the 50° which are from the 65°30′ at which *C* is situated to the 115°30′ at which *F* is situated. I then take on *AF* the point *G* such that *AG* be the line of eccentricity of Mars' deferent, making by the 13th proposition 1,519 such parts as the semi-diameter of the Sun's orbit has 10,000. I then mark on the line *GF* the t'punt H, alsoo dat G H doet 15190 voor Mars inrontwechs halfmiddellijn, welcke deur het 13 voorstel van dier langde is in sulcke deelen alser des Sonwechs halfmiddellijn B C 10000 doet: Ick beschrijf daer na op G als middelpunt, mette halfmiddellijn G H den inrontwech H E, sniende A F in H als haer verstepunt, en E is t'naestepunt: Daer na beschrijf ick tot eenige plaets Mars inront, latet ten eersten sijn op t'verstepunt H als middelpunt, en dat mette halfmiddellijn H F even an B C (want ghelijck B C of H I 10000, tot G H, alsoo *Ptolemeus* gevonden reden van des inronts halfmiddellijn tot haer wechs halfmiddellijn, te weten van 39 deel 30 ①, tot 60 deel deur het 11 voorstel) welck inront sy F I, diens verstepunt F, naestepunt I, waer me Mars voorghenomen teyckening met stelling eens vasten Eertcloots A voldaen is.

2 LIDT inhoudende de teyckening van Marsloop met stelling eens roerenden Eertcloots.

Om nu te commen totte teyckening met een roerenden Eertcloot, soo neem ick den Eertcloot A nu te loopen, en de Son an C vast te staen, en teycken in C A t'punt K, alsoo dat de uytmiddelpunticheytlijn C K even sy an A B, en beschrijf op K als middelpunt, mette halfmiddellijn K A die even is met B C het rondt A L als Eertclootwech, diens naestepunt L, verstepunt A: Om nu hier op te doen de teyckening van Marsloop met stelling eens roerenden Eertcloots, ick treck deur t'punt K de lini M N evewijdeghe met E F, en sal daerom K N oock strecken na des duysteraers 115 tr. 30 ① ghelijck A H; Ick stel daer na in K N t'punt Q, alsoo dat K Q even sy ande uytmiddelpunticheytlijn A G, en teycken t'punt N, soo dat Q N even sy an Mars inrontwechs halfmiddellijn G H, en beschrijf daer me Marswech N M, sniende de lini K N in N als verstepunt, en in M als naestepunt.

3 LIDT inhoudende bewijs dat Mars in d'eē en d'ander stelling een selve schijnbaer duysteraerlangde heeft, en de selve verheyt vanden Eertcloot als hy is in sijn inronts verstepunt, en t'inronts middelpunt an sijn wechs verstepunt.

Ghenomen dat Mars inronts middelpunt H met stelling eens vasten Eertcloots A sy an sijn wechs verstepunt H, en Mars ant inronts verstepunt F, soo salt van A tot F sijn inde grootste verheyt die Mars vanden vasten Eertcloot wesen can, ende dien volghens soo sal Mars met stelling eens roerenden Eertcloots moeten wesen an sijn wechs verstepunt N, en den roerendē Eertcloot an P, want daer me ist vanden roerenden Eertcloot P tot Mars an N, even soo verre als vanden Eertcloot A, tot Mars int inront an F, uyt oirsaeck dat Marswechs halfmiddellijn Q N mette uytmiddelpunticheytlijn Q K, even sijn an des inrontwechs halfmiddellijn G H mette uytmiddelpunticheytlijn G A, en boven dien K P halfmiddellijn des Eertclootwechs, even met H F halfmiddellijn des inronts. Voort want P N en A F evewijdeghe sijn, soo wort Mars in d'een en d'ander stelling schijnbaerlick tot een selve plaets des duysteraers ghesien: Ende om derghelijcke redenen ist openbaer, dat de aldercortste verheyt P M met stelling eens roerenden Eertcloots even moet sijn an d'aldercortste met stelling des vasten Eertcloots, t'welck soude sijn de lini van A tot des inronts naestepunt by aldien op E als middelpunt beschreven waer, maer ongheteyckent ghelaten is om deur veel linien gheen duysterheyt te veroirsaken.

4 LIDT

point *H* such that *GH* makes 15,190 for the semi-diameter of Mars' deferent, which by the 13th proposition is of that length in such parts as the semi-diameter of the Sun's orbit *BC* has 10,000. I then describe about *G* as centre, with the semi-diameter *GH,* the deferent *HE,* intersecting *AF* in *H* as its apogee, and *E* is the perigee. Thereafter I describe in some place Mars' epicycle: let it first be about the apogee *H* as centre, such with the semi-diameter *HF* equal to *BC* (for as *BC* or *HI* 10,000 is to *GH,* so is the ratio of the epicycle's semi-diameter to the semi-diameter of its orbit found by *Ptolemy,* to wit, of $39^{p}30'$ to 60^{p}, by the 11th proposition), which epicycle shall be *FI,* whose apogee is *F* and perigee *I;* with which the proposed drawing of Mars on the theory of a fixed Earth *A* has been completed.

2nd SECTION, comprising the drawing of Mars' motion on the theory of a moving Earth.

In order to come to the drawing on the theory of a moving Earth, I now take the Earth *A* to move and the Sun to be fixed at *C,* and I mark on *CA* the point *K* such that the line of eccentricity *CK* be equal to *AB,* and I describe about *K* as centre, with the semi-diameter *KA,* which is equal to *BC,* the circle *AL* as the Earth's orbit, whose perihelion is *L* and aphelion *A.* In order to base on this the drawing of Mars' motion on the theory of a moving Earth, I draw through the point *K* the line *MN* parallel to *EF,* and therefore *KN* will also tend towards $115°30'$ of the ecliptic, like *AH.* I then take on *KN* the point *Q* such that *KQ* be equal to the line of eccentricity *AG,* and I mark the point *N* such that *QN* be equal to the semi-diameter of Mars' deferent *GH,* and with this I describe Mars' orbit *NM,* intersecting the line *KN* in *N* as apogee and in *M* as perigee.

3rd SECTION, comprising the proof that on either theory Mars has the same apparent ecliptical longitude and the same distance from the Earth when it is at its epicycle's apogee and when the epicycle's centre is at the apogee of its orbit.

Assuming that the centre *H* of Mars' epicycle, on the theory of a fixed Earth *A,* be at the apogee *H* of its orbit, and Mars at the epicycle's apogee *F,* the distance from *A* to *F* will be the greatest distance at which Mars can be from the fixed Earth, and consequently on the theory of a moving Earth Mars will have to be at the apogee *N* of its orbit and the moving Earth at *P,* for thus it is as far from the moving Earth *P* to Mars at *N* as from the Earth *A* to Mars in the epicycle at *F,* because the semi-diameter of Mars' orbit *QN,* with the line of eccentricity *QK,* is equal to the deferent's semi-diameter *GH* with the line of eccentricity *GA,* while moreover *KP,* semi-diameter of the Earth's orbit, is equal to *HF,* semi-diameter of the epicycle. Further, since *PN* and *AF* are parallel lines, on either theory Mars is seen apparently in the same place of the ecliptic. And for similar reasons it is evident that the shortest distance *PM* on the theory of a moving Earth must be equal to the shortest distance on the theory of a fixed Earth, which would be the line from *A* to the epicycle's perigee, if it were described about *E* as centre, but it has not been drawn, so as not to cause obscurity on account of a multitude of lines.

4 *LIDT dat de halfmiddellijn van Marsvvechs middelpunt tot des inronts middelpunt, altijt evevvijdeghe is mette halfmiddellijn van des inrontsvvechs middelpunt tot des inronts middelpunt: En des Eertclootvvechs halfmiddellijn van haer middelpunt totten Eertcloot, altijt evevvijdeghe mette halfmiddellijn des inronts vant middelpunt tot Mars.*

Laet Mars inronts middelpunt ghecommen sijn van H tot R, deur welcke R getrocken de rechte G R S, soo sy S middelverstepunt, van t'welck daerentusschen Mars met stelling eens vasten Eertcloots ghecommen sy tot T: En hier om sal Mars met stelling eens roerenden Eertcloots op dien tijt ghecommen sijn van N tot V, soo dat de booch N V even is ande booch H R, en ghetrocken Q V sy is even en evewijdeghe met G R.

Oock sal den roerenden Eertcloot die doen was an P, van daer gedaen hebben den loop P L X, even ande boveschreven twee als P L even neem ick an S T, en L X ghelijck met N V, of den houck L K X even mettẽ houck N Q V, het welck soo wesen moet deur het 33 voorstel des 1 boucx, alwaer bewesen is Mars verstepuntsloop met sijn inrontsloop t'samen even te wesen mette Sonloop, t'welck hier oock is metten Eertclootloop.

Nu dan den roerenden Eertcloot gesien uyt K, sal sulcken loop gedaen hebben in schijnbaer duysteraerlangde als de booch P L X mebrengt: En dergelijcke loop in schijnbaer duysteraerlangde sal oock gedaen hebben Mars met stelling eens vasten Eertcloots. Dit soo wesende, den roerenden Eertcloot gesien uyt sijn wechs middelpunt K, sal sulcken loop ghedaen hebben in schijnbaer duysteraerlangde, als Mars met stelling eens vasten Eertcloots ghesien uyt sijn inronts middelpunt, en daerom moeten hun twee halfmiddellijnen K X, R T, evewijdeghe sijn ghelijckse waren int begin des loops, te weten H F met K P, en ghelijck de strecking vant middelpunt K totten Eertcloot P, doen was na de tegenoversijde der strecking vant middelpunt H tot Saturnus F, also is nu oock de strecking vant middelpunt K totten Eertcloot X, na de teghenoversijde der strecking vant middelpunt R tot Saturnus T.

5 *LIDT dat Mars in d'een en d'ander stelling tot allen plaetsen een selve schijnbaer duysteraerlangde heeft, en de selve verheyt vanden Eertcloot.*

Om tottet bewijs te commen ick treck de vier linien A R, AT, KV, V X, en segh daer me aldus: Angesien des driehoucx K V Q sijde K Q, evẽ en evewijdeghe is met des driehoucx A R G sijde A G, sgelijcx Q V even en evewijdeghe met G R deur het 4 lidt, soo moet de derde sijde K V even en evewijdeghe sijn mette derde A R: Voort segh ick dat anghesien des driehoucx K V X sijde K V, even en evewijdeghe is met A R, en K X even en evewijdeghe met R T, deur het 4 lidt, soo moet de derde sijde X V even en evewijdeghe sijn mette derde A T, en daerom sietmen Mars an V uyt den roerenden Eertcloot X, schijnbaerlick totte selve plaets des duysteraers daermẽ hem an T siet uyt den vasten Eertcloot A, en is soo verre van V tot X, als van T tot A.

6 LIDT

4th SECTION, that the semi-diameter from the centre of Mars' orbit to Mars itself (QV) [1] is always parallel to the semi-diameter from the deferent's centre to the epicycle's centre. And the semi-diameter of the Earth's orbit from its centre to the Earth always parallel to the semi-diameter of the epicycle from its centre to Mars.

Let the centre of Mars' epicycle have travelled from H to R, and if through this R is drawn the straight line GRS, let S be the mean apogee, from which meanwhile let Mars on the theory of a fixed Earth have travelled as far as T. And for this reason, on the theory of a moving Earth Mars will in that time have travelled from N to V [2], so that the arc NV is equal to the arc HR, and when QV is drawn, it is equal and parallel to GR.

The moving Earth, which then was at P, will also have performed thence the motion PLX, equal to the above-mentioned two, namely PL, which I take to be equal to ST, and LX equal to NV, or the angle LKX equal to the angle NQV, which must be so by the 33rd proposition of the 1st book, where it has been proved that the motion of Mars' apogee together with the motion in its epicycle is equal to the Sun's motion, which is here also equal to the Earth's motion.

Now the moving Earth, seen from K, will have performed the same motion in apparent ecliptical longitude as the arc PLX amounts to. And a similar motion in apparent ecliptical longitude will also have been performed by Mars on the theory of a fixed Earth. This being so, the moving Earth, seen from the centre of its orbit K, will have performed the same motion in apparent ecliptical longitude as Mars on the theory of a fixed Earth, seen from the centre of its epicycle, and therefore their two semi-diameters KX, RT must be parallel, as they were at the beginning of the motion, to wit, HF to KP, and just as the direction from the centre K to the Earth P then was opposite to the direction from the centre H to Mars [3] F, so the direction from the centre K to the Earth X is now opposite to the direction from the centre R to Mars [4] T.

5th SECTION, that in all places Mars has on either theory the same apparent ecliptical longitude and the same distance from the Earth.

In order to come to the proof, I draw the four lines AR, AT, KV, VX, and then say as follows: Since the side KQ of the triangle KVQ is equal and parallel to the side AG of the triangle ARG, and likewise QV equal and parallel to GR, by the 4th section, the third side KV must be equal and parallel to the third side AR. Further I say that since the side KV of the triangle KVX is equal and parallel to AR, and KX equal and parallel to RT, by the 4th section, the third side XV must be equal and parellel to the third side AT, and therefore Mars is seen at V from the moving Earth X, apparently in the same place of the ecliptic where it is seen at T from the fixed Earth A, and it is as far from V to X as from T to A.

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[1]) For *des inronts middelpunt* in the Dutch text read the equivalent of "Mars itself". Here the parallelism of QV to GR is demonstrated: QV belongs to the theory of the moving Earth, in which no epicycles occur, V is the position of Mars.

[2]) Note that V is not really situated on the epicycle ST, but only on the orbit MN.

[3]) For *Saturnus* in the Dutch text read *Mars*.

[4]) For *Saturnus* in the Dutch text read *Mars*.

6 LIDT van t'verschil datter valt tusschen de werckinghen van d'een en d'ander stelling, int berekenen der schijnbaer duysteraerlangde der Dwaelders.

Met stelling een vasten eertcloots ontmoett ons int rekenen der soucking van Mars schijnbaer duysteraerlangde deses voorbeelts den gemeenen vierden houck A G R T, met vijf bekende palen, te weten drie sijden A G, G R, R T, altijt van een selve bekende langde: Voort den houck A G R als halfrontschil des bekenden houcx H G R, middelloop van des inronts middelpunt, en den houck G R T als halfrontschil des bekenden houcx S R T middelloop van Mars int inront, waer me deur het 6 voorstel inde Byvough der platte veelhoucken ghevonden sijnde den onbekenden houck G A T, en die vervought totte bekende duysteraerlangde daer A G henē streckt, dats na den 115 tr. 30 ①, men heeft t'begheerde.

Maer met stelling eens roerenden Eertcloots ontmoet ons hier den cruysvierhouck K Q V X, met vijf bekende palen, te weten drie sijden K Q, Q V, K X, altijt van een selve bekende langde, voort den houck K Q V, als halfrontschil des bekenden houcx N Q V middelloop van Saturnus in sijn wech, en dē houck Q K X, wesende des Eertcloots middellangde L K X, min den houck L K O van 50 tr. waer me deur het 6 voorstel inde Byvough der platte veelhoucken ghevonden sijnde den onbekenden houck K X V, en die ghetrocken vande bekende duysteraerlangde daer X K henen streckt, t'welck is de schijnbaer duysteraerlangde der Middelson K, men heeft t'begheerde, en moet nootsakelick t'selve besluyt voortbrenghen datmen deur d'eerste wercking heeft.

Sulcx als hier is gheweest het bewijs van Mars, soo salt oock sijn van Saturnus en Iupiter.

T B E S L V Y T. De drie bovenste Dwaelders dan Saturnus, Iupiter en Mars ontfanghen met stelling eens roerenden Eertcloots de selve schijnbaer duysteraerlangde, en verheyt vanden Eertcloot, diese hebben met stelling eens vasten Eertcloots, t'welck wy bewijsen moesten.

1 MERCK.

Hier machmen nu sien d'oirsaeck hoet inde oneyghen stelling eens vasten Eertcloots by comt, dat des inronts middelpunts loop in sijn wech, en des Dwaelders loop in sijn inront, t'samen even vallen ande Sonloop, en niet int wesen te bestaen dat die Dwaelders totte Son een opsicht nemen als tot haer Coninck, ghelijck int 6 voorstel deses 3 boucx gheseyt is dat sommige hemlien daer in verwonderen, want inde wesentlicke sake en isser niet dan den Dwaelder, en den Eertcloot, elck met sijn eygen roersel in sijn wech, waer me de oneyghen versierde stelling des vasten Eertcloots gheraeckt sulcken overeencomminghen te crijghen.

2 MERCK.

Om in dit voorbeelt noch te sien de overeencomming vande Son metten Eertcloot en Saturnus, in d'een en d'ander stelling, ick treck de lini X C vandē roerenden Eertcloot X totte vaste Son C: Maer want in die stelling den roerenden Eertcloot is ghecommen van A over P en L tot X, soo sal op den selven tijt met stelling eens vasten Eertcloots, de roerende Son moeten ghecommen sijn van C over D tot Y, sulcx dat de booch C D Y, even sy mette booch

A P L X:

6th SECTION, of the difference between the operations on either theory in the computation of the apparent ecliptical longitude of the Planets.

On the theory of a fixed Earth we meet, in the computation of the finding of Mars' apparent ecliptical longitude in this example, with the ordinary quadrilateral *AGRT*, with five known terms, to wit: three sides *AG, GR, RT*, always of the same known length; further the angle *AGR* as supplement of the known angle *HGR*, mean motion of the epicycle's centre, and the angle *GRT* as supplement of the known angle *SRT*, mean motion of Mars in the epicycle; and thus, the unknown angle *GAT* being found, by the 6th proposition in the Supplement of Plane Polygons [1]), and added to the known ecliptical longitude towards which *AG* tends, *i.e.* 115°30′, the value required is obtained.

But on the theory of a moving Earth we meet here with the crossed quadrilateral *KQVX*, with five known terms, to wit: three sides *KQ, QV, KX*, always of the same known length, further the angle *KQV* as supplement of the known angle *NQV*, mean motion of Mars [2]) in its orbit, and the angle *QKX*, being the Earth's mean longitude *LKX minus* the angle *LKO* of 50°; and thus, the unknown angle *KXV* being found, by the 6th proposition in the Supplement of plane polygons [1]), and subtracted from the known ecliptical longitude towards which *XK* tends — which is the apparent ecliptical longitude of the Mean Sun *K* — the value required is obtained, and this operation must necessarily lead to the same conclusion as that obtained by the first operation.

As the proof for Mars has been here, such it will also be for Saturn and Jupiter.

CONCLUSION. The three upper Planets Saturn, Jupiter, and Mars therefore on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth; which we had to prove.

1st NOTE.

Here we can see the cause why it is that according to the untrue theory of a fixed Earth the motion of the epicycle's centre in its orbit and the Planet's motion on its epicycle together are equal to the Sun's motion, and that in reality those Planets do not take their guidance from the Sun as from their King, as has been said in the 6th proposition of this 3rd book that some people wonder about it, for in reality there is nothing but the Planet and the Earth, each with its own movement in its orbit, as a result of which the untrue, fictitious theory of the fixed Earth obtains such correspondences.

2nd NOTE.

In order to see in this example the correspondence of the Sun with the Earth and Mars [3]), on either theory, I draw the line *XC* from the moving Earth *X* to the fixed Sun *C*. But since on this theory the moving Earth has travelled from *A via P* and *L* to *X*, the moving Sun in the same time, on the theory of a fixed

[1]) Stevin's *Trigonometry*, Work XI; i, 12, Appendix on Plane Polygons. See Vol. II B, p. 755. In the 6th proposition he considers a quadrangle and shows how all 8 angles and sides may be found, if 5 independent ones are given.

[2]) For *Saturnus* in the Dutch text read *Mars*.

[3]) For *Saturnus* in the Dutch text read *Mars*.

APLX: T'welck soo wesende A Y moet even en evewijdeghe sijn met X C, en vervolghens de vaste Son C, wort uyt den roerenden Eertcloot X ghesien onder de selve duysteraerlangde als de roerende Son an Y, uyt den vasten Eertcloot A. Voort want X C evewijdeghe is met A Y, en X V met A T, soo is den houck V X C, even metten houck T A Y, waer deur oock de schijnbaer verheyt van Saturnus V totte vaste Son C, ghesien uyt den roerenden Eertcloot X, even is ande schijnbaer verheyt van Saturnus T totte roerende Son Y ghesien uyt den vasten Eertcloot A.

3 MERCK

Anghesien de twee stellinghen des eersten en tweeden lidts, d'een met een vasten Eertcloot, d'ander met een roerenden, t'samen in een form staen, deur dien t'bewijs sulcx daer vereyschte, en dattet tot meerder claerheyt can dienen, de selve elck besonderlick te sien, soo heb ickse hier van malcander ghescheyden als blijckt.

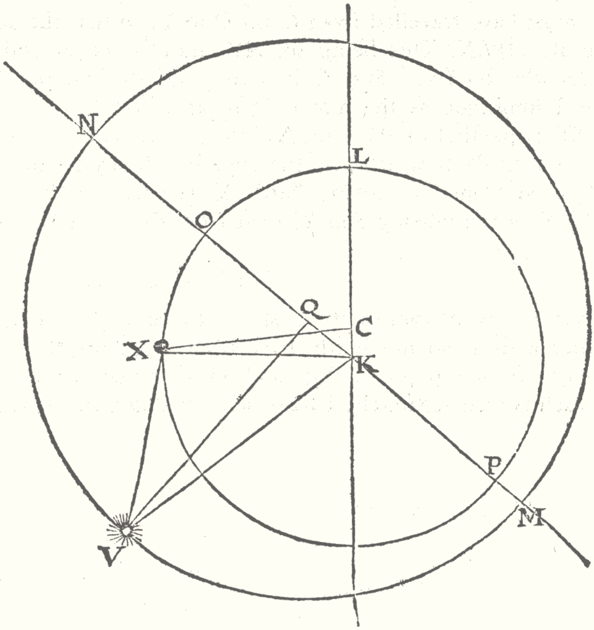
 16 VOOR-

Earth, must have travelled from *C via D* to *Y*, so that the arc *CDY* shall be equal to the arc *APLX*. This being so, *AY* must be equal and parallel to *XC*, and consequently the fixed Sun *C* is seen from the moving Earth *X* at the same ecliptical longitude as the moving Sun at *Y* from the fixed Earth *A*. Further, since *XC* is parallel to *AY*, and *XV* to *AT*, the angle *VXC* is equal to the angle *TAY*, as a result of which also the apparent distance from Saturn *V* to the fixed Sun *C*, seen from the moving Earth *X*, is equal to the apparent distance from Mars [1]) *T* to the moving Sun *Y*, seen from the fixed Earth *A*.

3rd NOTE.

Since the two theories of the first and the second section, one with a fixed and the other with a moving Earth, have been given together in one figure, because the demonstration required this, and since it may serve to make things clearer when each is seen separately, I have here separated them, as appears hereafter.

[1]) For *Saturnus* in the Dutch text read *Mars*.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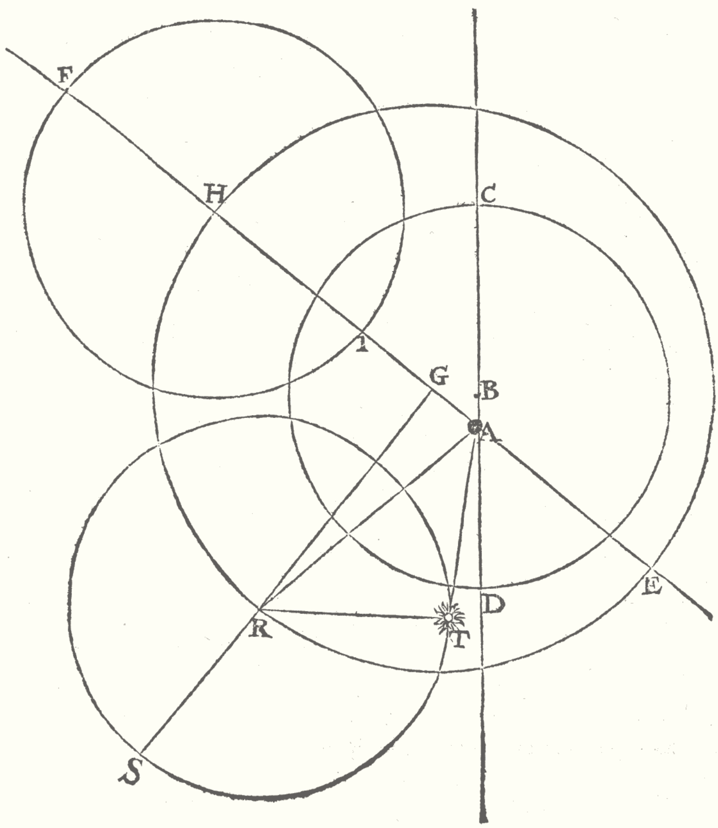
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16 VOORSTEL.

Te beschrijven den loop der tvvee onderste Dvvaelders Venus en Mercurius in haer vveghen, op een ghegeven tijt, met stelling eens roerenden Eertcloots.

T'GHEGHEVEN. Om met Venus te beginnen, soo laet den tijt sijn van een dach. T'BEGHEERDE. Men wil daer op vinden dē loop in haer wech met stelling eens roerenden Eertcloots.

T'WERCK.

Totten loop van haer inronts middelpunt met stelling eens vasten Eertcloots doende op een dach deur het 36 voorstel des 1 boucx	0 tr. 59. 8. 17. 13. 12. 31.
Vergaert haer inronts loop eens dachs, doende deur het 41 voorstel des 1 boucx	0 tr. 36. 59. 25. 53. 11. 28.
Comt t'samen voor den begheerden loop eens dachs van Venus met stelling eens roerenden Eertcloots	1 tr. 36. 7. 43. 6. 23. 59.

Ghemerckt het volghende * vertooch wesende het 17 voorstel tot bewijs deses * werckstucx dient, soo sullen wy dat daer voor laten verstrecken. Ende sghelijcx sal oock sijn den voortganck met Mercurius. T'BESLVYT. Wy hebben dan beschrevē den loop der twee onderste Dwaelders Venus en Mercurius in haer weghen, op een ghegheven tijt, met stelling eens roerenden Eertcloots, na den eysch. *Theorema.* *Probelematis.*

VERTOOCH. 17 VOORSTEL.

De tvvee onderste Dvvaelders Venus en Mercurius, ontfangen met stelling eens roerenden Eertcloots de selve schijnbaer duysteraerlangde, en verheyt vanden Eertcloot, diese hebben met stelling eens vasten Eertcloots.

Om dit voorstel oirdentlick te verclaren, ick salt in sulcke ses leden verdeelen, als int 15 voorstel mette drie bovenste ghedaen is, ghebruyckende Venus tot voorbeelt.

1 LIDT inhoudende de teyckening van Venus loop met stelling eens vasten Eertcloots.

Laet voor eerste stelling A ghenomen worden een vasten Eertcloot te beteyckenen, en van A tot B sy des Sonwechs uytmiddelpunticheytlijn, doende na *Ptolemeus* rekening t'sijnder tijt sulcke 417 alser des Sonwechs halfmiddellijn die B C sy 10000 doet deur het 13 voorstel deses 3 boucx, mette selve B C sy op B als middelpunt beschreven de Sonwech C D, waer in C A voortghetrocken tot D, soo is D t'naestepunt, C t'verstepunt, wesende tot *Ptolemeus* tijt onder des duysteraers 65 tr. 30 ♊. Om hier op nu te doē de teyckening des loops van eenige der twee boveschreven Dwaelders, ick neem tot voorbeelt daer toe als boven gheseyt is Venus, wiens inrontwechs verstepunt ten tijde van *Ptole-*

16th PROPOSITION.

To describe the motion of the two lower Planets Venus and Mercury in their orbits, in a given time, on the theory of a moving Earth.

SUPPOSITION. To begin with Venus, let the time be one day. WHAT IS REQUIRED. It is desired to find its motion in that time in its orbit on the theory of a moving Earth.

PROCEDURE.

When to the motion of its epicycle's centre on the theory of a fixed Earth,

being in one day, by the 36th proposition of the 1st book	0° 59, 8,17,13,12,31
there is added its epicyclic motion in one day, being by the 41st proposition of the 1st book	0° 36,59,25,53,11,28
this makes together, for the required motion of Venus in one day on the theory of a moving Earth	1° 36, 7,43, 6,23,59

Considering that the subsequent theorem, being the 17th proposition, serves as proof of this problem, we shall use it as such. And the procedure with Mercury will be similar.

CONCLUSION. We have thus described the motion of the two lower Planets Venus and Mercury in their orbits, in a given time, on the theory of a moving Earth; as required.

THEOREM. 17th PROPOSITION.

The two lower Planets Venus and Mercury on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth.

In order to explain this proposition properly, I shall divide it into six sections, as has been done in the 15th proposition for the three upper Planets, taking Venus as example [1]).

1st SECTION, comprising the drawing of Venus' motion on the theory of a fixed Earth.

First let *A* be assumed to denote a fixed Earth, and let the line from *A* to *B* be the line of eccentricity of the Sun's orbit, according to *Ptolemy's* computation making at his time 417 such parts as the semi-diameter of the Sun's orbit, which shall be *BC*, has 10,000, by the 13th proposition of this 3rd book. With the said *BC*, about *B* as centre, let there be described the Sun's orbit *CD*, in which, when *CA* is produced to *D*, *D* is the perigee, *C* the apogee, which in *Ptolemy's* time was at 65°30′ of the ecliptic. In order to base on this the drawing of the motion of either of the two aforesaid Planets, I take as example, as has been said above,

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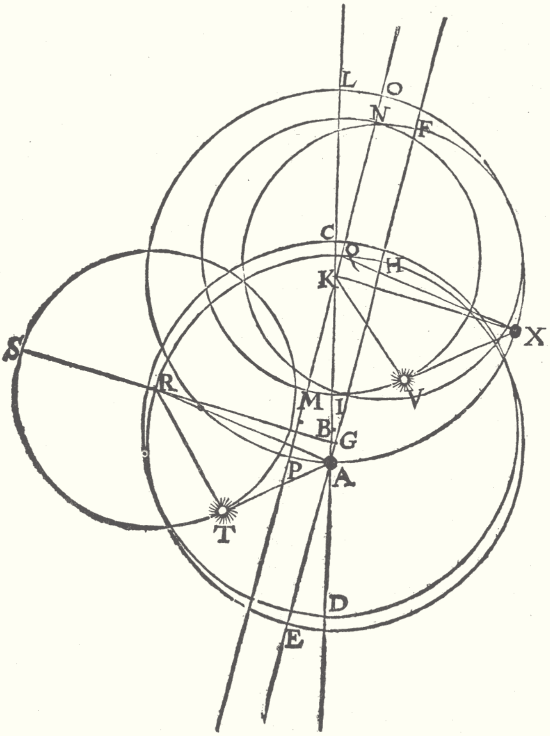
[1]) For this theory, see the explanatory diagrams and text in the Introduction, p. 16—17.



meus gheweest hebbende onder des duysteraers 55 tr. deur het 13 voorstel deses 3 boucx, ick treck deur den vasten Eertcloot A, de lini E F van A tot F, na des duysteraers boveschreven 55 tr. te wetē soo dat dē houck C A F doe 10 tr. 30 ①, dieder sijn vanden 55 tr. daer F onder is, tottē 65 tr. 30 ①, daer C onder is: Ick stel daer nain A F t'punt G, so dat A G sy Venus in rontwechs uytmiddelpunticheytlijn, doende deur het 13 voorstel sulcke 208, alsser des Sonwechs halfmiddellijn B C 10000 doet: Maer want Venus inronts wech even is metten Eertclootwech, ghelijck ick gheseyt heb int 13 voorstel, en den selven Eertclootwech even mette Sonwech, soo beschrijf ick op t'punt G als middelpunt, mette halfmiddellijn G A evē an des Sonwechs halfmiddellijn B C, dē inrontwech H E, sniende H F in H als haer verstepunt, en E is t'naestepunt: Ick beschrijf daer na tot eenige plaets Venus inrontwech, latet ten eersten sijn opt verstepunt H als middelpunt, en dat mette halfmiddellijn H F, doende deur t'ghene verclaert is int 13 voorstel 7194, welck inront sy F I, waer me Venus teyckening met stelling eens vasten Eertcloots A voldaen is.

2 *LIDT inhoudende de teyckening van Venusloop met stelling eens roerenden Eertcloots.*

Om nu te commen totte teyckening met een roerendē Eertcloot, soo neem ick den Eertcloot A nu te loopen, en de Son an C vast te staen, en teycken in C A t'punt K, alsoo dat de uytmiddelpunticheytlijn C K even sy an A B, en beschrijf op K als middelpunt, mette halfmiddellijn K A die even is met B C, het inront A L als Eertclootwech, diens naestepunt L, verstepunt A: Om hier op te doen de teyckening van Venusloop met stelling eens roerenden Eertcloots, ick treck deur t'punt K de lini M N evewijdege met E F, sniende den Eertclootwech in O en P, en sal daerom K N oock streckē na des duysteraers 55 tr. gelijck A H: Ick stel daer na in K N t'punt Q, alsoo dat K Q even sy ande uytmiddelpunticheytlijn A G, en teyckē t'punt N, soo dat Q N even sy an Venus inronts halfmiddelijn H F, en beschrijf daer me Venuswech N M, sniende de lini K N in N als verstepunt, en in M als naestepunt.

3 *LIDT*

Venus; the apogee of the latter's deferent having been in *Ptolemy's* time at 55° of the ecliptic, by the 13th proposition of this 3rd book, I draw through the fixed Earth *A* the line *EF* from *A* to *F* towards the aforesaid point at 55° of the ecliptic, to wit, in such a way that the angle *CAF* be the 10°30′ which are from the 55° at which *F* is situated to the 65°30′ at which *C* is situated. I then take on *AF* the point *G* such that *AG* be the line of eccentricity of Venus' deferent, making by the 13th proposition 208 such parts as the semi-diameter of the Sun's orbit *BC* has 10,000. But because Venus' deferent is equal to the Earth's orbit, as I have said in the 13th proposition, and the said Earth's orbit is equal to the Sun's orbit, I describe about the point *G* as centre, with the semi-diameter *GH* [1]) equal to the semi-diameter of the Sun's orbit *BC*, the deferent *HE* intersecting *HF* at *H* as its apogee, and *E* is the perigee. Thereafter I describe in some place Venus' epicycle [2]), let it be firstly about the apogee *H* as centre, such with the semi-diameter *HF*, making — by what has been explained in the 13th proposition — 7,194, which epicycle shall be *FI*, with which the drawing of Venus on the theory of a fixed Earth *A* is completed.

2nd SECTION, comprising the drawing of Venus' motion on the theory of a moving Earth.

In order to come to the drawing on the theory of a moving Earth, I now take the Earth *A* to move and the Sun to be fixed at *C*, and I mark on *CA* the point *K* such that the line of eccentricity *CK* be equal to *AB*, and I describe about *K* as centre, with the semi-diameter *KA* which is equal to *BC*, the circle [3]) *AL* as the Earth's orbit, whose perihelion is *L* and aphelion *A*. In order to base on this the drawing of Venus' motion on the theory of a moving Earth, I draw through the point *K* the line *MN* parallel to *EF*, intersecting the Earth's orbit in *O* and *P*, and therefore *KN* will also tend towards 55° of the ecliptic, like *AH*. Thereafter I take on *KN* the point *Q* such that *KQ* be equal to the line of eccentricity *AG*, and I mark the point *N* such that *QN* be equal to the semi-diameter of Venus' epicycle *HF*, and with this I describe Venus' orbit *NM*, intersecting the line *KN* in *N* as aphelion and in *M* as perihelion.

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[1]) For *GA* in the Dutch text read *GH*.
[2]) For *inrontswech* in the Dutch text read *inront*.
[3]) For *inront* in the Dutch text read *ront*.

3 *LIDT inhoudende bevvijs dat Venus in d'een en d'ander stelling een selve schijnbaer duysteraerlangde heeft, en de selve verheyt vanden Eertcloot als sy is in haer inronts verstepunt, en t'inronts middelpunt an sijn vvechs verstepunt.*

Ghenomen dat Venus inronts middelpunt H met stelling eens vasten Eertcloots A, sy in sijn wechs verstepunt H, en Venus ant inronts verstepunt F, soo salt van A tot F sijn inde grootste verheyt die Venus vanden vasten Eertcloot wesen can, en dien volghens soo sal Venus met stelling eens roerenden Eertcloots moeten wesen an haer wechs verstepunt N, en den roerenden Eertcloot an P, want daer me ist vanden roerendē Eertcloot P tot Venus an N, even soo verre als vandē vasten Eertcloot A, tot Venus int inront an F, uyt oirsaeck dat des Eertclootwechs halfmiddellijn K P, mette uytmiddelpunticheytlijn Q K, even sijn an des inrontwechs halfmiddellijn G H mette uytmiddelpunticheytlijn G A, en boven dien Q N halfmiddellijn van Venuswech, even met H F halfmiddellijn des inronts: Voort want P N en A F evewijdeghe sijn, soo wort Venus in d'een en d'ander stelling schijnbaerlick tot een selve punt des duysteraers ghesien: En om derghelijcke redenen ist openbaer dat de aldercortste verheyt N O met stelling eens roerendē Eertcloots, even moet sijn an d'andercortste met stelling des vasten Eertcloots, t'welck soude sijn de lini van A tot des inronts naestepunt, by aldient op E als middelpunt beschreven waer, maer ongheteyckent ghelaten is om deur veel linien gheen duysterheyt te veroirsaken.

4 *LIDT dat de halfmiddellijn van des Eertclootvvechs middelpunt totten Eertcloot, altijt evevvijdeghe is mette halfmiddellijn van des inrontvvechs middelpunt, tot des inronts middelpunt: En Venusvvechs halfmiddellijn vant middelpunt tot Venus, altijt evevvijdeghe mette halfmiddellijn des inronts vant middelpunt tot Venus.*

Laet Venus inronts middelpunt ghecommen sijn van H tot R, deur welcke R ghetrocken de rechte lini G R S, soo sy S middelverstepunt, van t'welck daerentusschen Venus met stelling eens vasten Eertcloots ghecommen sy tot T: Ende want den loop van Venus in haer wech even is ande twee loopen, d'een vant inronts middelpunt dats hier den houck H G R, d'ander, van Venus int inront, dats hier den houck S R T, soo moet Venus inde wech van N af daerentusschen gedaen hebben een loop even an die twee voorschreven houcken, welcke sy den verkeerden houck N Q V, ende ghetrocken Q V, sy moet even en evewijdeghe sijn met R T, om t'bewijs dat op derghelijcke ghedaen is van Mars int 4 lidt des 13 voorstels deses 3 boucx: Oock sal den roerenden Eertcloot die doen was an P, van daer ghedaen hebben den loop P X, even an des inronts middelpunts loop H R, waer deur oock de twee halfmiddellijnen K X, G R, dier twee even ronden even en evewijdeghe moeten sijn.

5 *LIDT dat Venus in d'een en d'ander stelling tot allen plaetsen eenselve schijnbaer duysteraerlangde heeft, en de selve verheyt vanden Eertcloot.*

3rd SECTION, comprising the proof that Venus on either theory has the same apparent ecliptical longitude and the same distance from the Earth when it is at its epicycle's apogee and when the epicycle's centre is at the orbit's apogee.

When it is assumed that on the theory of a fixed Earth *A* the centre of Venus' epicycle *H* is in its orbit's apogee *H*, and Venus at the epicycle's apogee *F*, from *A* to *F* will be the greatest distance at which Venus can be from the fixed Earth, and consequently, on the theory of a moving Earth, Venus will have to be at its orbit's apogee *N*, and the moving Earth at *P*, for thus it is just as far from the moving Earth *P* to Venus at *N* as from the fixed Earth *A* to Venus on the epicycle at *F*, because the semi-diameter of the Earth's orbit *KP*, and the line of eccentricity *QK*, are equal to the semi-diameter of the deferent *GH* and the line of eccentricity *GA*, while moreover *QN*, the semi-diameter of Venus' orbit, is equal to *HF*, the semi-diameter of the epicycle. Further, since *PN* and *AF* are parallel lines, Venus on either theory is apparently seen at the same point of the ecliptic. And for similar reasons it is evident that the very shortest distance *NO* on the theory of a moving Earth must be equal to the very shortest [1] distance on the theory of a fixed Earth, which would be the line from *A* to the epicycle's perigee, if it had been described about *E* as centre; but it has not been drawn, in order not to obscure the drawing by a multitude of lines.

4th SECTION, that the semi-diameter from the centre of the Earth's orbit to the Earth is always parallel to the semi-diameter from the deferent's centre to the epicycle's centre; and the semi-diameter of Venus' orbit from the centre to Venus always parallel to the semi-diameter of the epicycle from the centre to Venus.

Let Venus' epicycle's centre have travelled from *H* to *R*, and when through this *R* is drawn the straight line *GRS*, *S* shall be the mean apogee, from which meanwhile Venus, on the theory of a fixed Earth, shall have arrived at *T*. And because the motion of Venus in its orbit is equal to the two motions, one of the epicycle's centre, that is here the angle *HGR*, the other of Venus on the epicycle, that is here the angle *SRT*, Venus must meanwhile have performed in the orbit from *N* a motion equal to the two aforesaid angles, which shall be the opposite angle *NQV*; and when *QV* is drawn, it must be equal and parallel to *RT*, because of the proof furnished in a similar case for Mars in the 4th section of the 13th proposition of this 3rd book. The moving Earth, which then was at *P*, will also have performed from there the motion *PX*, equal to the motion of the epicycle's centre *HR*, in consequence of which the two semi-diameters *KX* and *GR* of those two equal circles must also be equal and parallel.

5th SECTION, that in all places Venus has on either theory the same apparent ecliptical longitude and the same distance from the Earth.

[1]) For *andercortste* in the Dutch text read *aldercortste*.

Om tottet bewijs te commen ick treck de vier linien A R, A T, Q V, V X, en segh daer me aldus: Anghesien des driehoucx K X Q sijde K Q, even en evewijdeghe is met des driehoucx A R G sijde A G, sghelijcx Q X even en evewijdeghe met G R, soo moet de derde sijde K X, even en evewijdeghe sijn mette derde A R: Voort segh ick dat anghesien des driehoucx Q V X sijde Q X, even en evewijdeghe is met A R, en Q V even en evewijdeghe met R T deur het 4 lidt, so moet de derde sijde X V, even en evewijdege sijn mette derde A T: En daerom sietmen Venus an V uyt den roerendẽ Eertcloot X, schijnbaerlick totte selve plaets des duysteraers datmẽ Venus siet uyt den vasten Eertcloot A, en is soo verre van V tot X, als van T tot A.

6 LIDT van t'verschil datter valt tusschen de vverckinghen van d'een en d'ander stelling, int berekenen der schijnbaer duysteraerlangde der Dvvaelders.

Met stelling eens vasten Eertcloots ontmoet ons int rekenen der soucking van Venus schijnbaer duysteraerlangde deses voorbeelts den ghemeenen vierhouck A G R T, met vijf bekende palen, te weten de drie sijden A G, G R, R T. altijt van een selve bekende langde: Voort den houck A G R, als halfrontschil des bekenden houcx H G R, middelloop van des inronts middelpunt, en den houck G R T, als halfrontschil des bekendẽ houcx S R T middelloop van Venus int inront, waer me deur het 6 voorstel inde byvough der platte veelhoucken ghevonden sijnde den onbekenden houck G A T, en die vervought totte bekende duysteraerlangde daer A G henen streckt, dats na den 55 tr. men heeft het begheerde.

Maer met stelling eens roerendẽ Eertcloots ontmoet ons hier dẽ cruysvierhouck K Q V X met vijf bekende palen, te weten drie sijden K Q, Q V, K X, altijt van een selve bekende langde, voort den houck K Q V, als halfrontschil des bekendẽ houcx N Q V rontschil des middelloops van Venus in haer wech en dẽ houck Q K X, wesende rontschil van des Eertcloots middellangde, waer me deur het 6 voorstel inde byvough der platte veelhoucken, ghevonden sijnde den onbekendẽ houck K X V, en die vergaert totte bekende duysteraerlangde daer X K henen streckt, t'welck is de schijnbaer duysteraerlangde der middelson K men heeft t'begeerde, en moet nootsakelick het eerste besluyt voortbrenghen datmen deur d'eerste wercking heeft.

Sulcx als hier is gheweest het bewijs van Venus, soo salt oock sijn van Mercurius.

T'BESLVYT. De twee onderste Dwaelders dan Venus en Mercurius, ontfanghen met stelling eens roerenden Eertcloots de selve schijnbaer duysteraerlangde en verheyt vanden Eertcloot, diese hebben met stelling eens vasten Eertcloots, t'welck wy bewijsen moesten.

MERCKT.

T'is kennelick datmen de twee stellinghen des eersten en tweeden lidts van Venus t'samen in een form ghemengt sijnde, soude meughen scheyden, ghelijck van Mars ghedaen wiert int 3 merck des 15 voorstels.

18 VOORSTEL.

Te verclaren de reden vvaerom ick inde 8. 12. 15. en 17 voor-

In order to come to the proof, I draw the four lines *AR, AT, QV, VX,* and then say as follows: Since the side *KQ* of the triangle *KXQ* is equal and parallel to the side *AG* of the triangle *ARG,* and likewise *QX* equal and parallel to *GR,* the third side *KX* must be equal and parallel to the third side *AR.* Further I say that since the side *QX* of the triangle *QVX* is equal and parallel to *AR,* and *QV* equal and parallel to *RT* by the 4th section, the third side *XV* must be equal and parallel to the third side *AT.* And for this reason Venus is seen at *V* from the moving Earth *X,* apparently in the same place of the ecliptic where Venus is seen from the fixed Earth *A;* and it is as far from *V* to *X* as from *T* to *A.*

6th SECTION, of the difference between the operations on either theory, in the computation of the apparent ecliptical longitudes of the Planets.

On the theory of a fixed Earth we meet, in the computation of the finding of Venus' apparent ecliptical longitude in this example, with the ordinary quadrilateral *AGRT,* with five known terms, to wit: the three sides *AG, GR, RT,* always of the same known length; further the angle *AGR,* as supplement of the known angle *HGR,* mean motion of the epicycle's centre, and the angle *GRT,* as supplement of the known angle *SRT,* mean motion of Venus on the epicycle. And thus, the unknown angle *GAT* being found, by the 6th proposition in the Supplement of Plane Polygons [1]), and added to the known ecliptical longitude towards which *AG* tends, *i.e.* the point at 55°, the value required is obtained.

But on the theory of a moving Earth we here meet with the crossed quadrilateral *KQVX,* with five known terms, to wit: three sides *KQ, QV, KX,* always of the same known length; further the angle *KQV,* as supplement of the known angle *NQV,* supplementing to 360° the mean motion of Venus in its orbit, and the angle *QKX,* supplementing to 360° the mean longitude of the Earth. And thus, the unknown angle *KXV* being found, by the 6th proposition in the Supplement of Plane Polygons, and added to the known ecliptical longitude towards which *XK* tends, which is the apparent ecliptical longitude of the mean sun *K,* the value required is obtained, and this operation must necessarily lead to the first result, obtained by the first operation.

As the proof for Venus has been here, such is also that for Mercury.

CONCLUSION. The two lower Planets Venus and Mercury therefore on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth; which we had to prove.

NOTE.

It is obvious that the two theories of the first and the second section of Venus, which are combined in one figure, might be separated, as was done for Mars in the 3rd Note of the 15th proposition.

18th PROPOSITION.

To explain the reason why in the 8th, 12th, 15th, and 17th propositions I have proved that the Planets on the theory of a moving Earth are found in the same

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[1]) See note [1]) on p. 189.

voorstellen bevvesen hebbe de Dvvaelders deur stelling eens roerenden Eertcloots, bevonden te vvorden totte selve schijnbaer plaetsen, en verheden van malcander, diemense met stelling eens vasten Eertcloots bevint, mette omstandighen van dien.

Copernicus neemt wel, soot schijnt, dat de tweede stellinghen, d'eene met een vasten d'ander met een roerenden Eertcloot (welverstaende alsmen uyt d'een treckt *Ptolemeus* byvoughsels der onbekende oneventheden, en uyt d'ander de sijne) een selve besluyt voortbrenghen van der Dwaelders plaetsen en verhe-deden van malcander, maer hy en doetter geen bewijs af, achtende soot schijnt, de sake soo claer datse t'bewijs niet en behouft, t'welck my nochtans tot gron-delicker kennis der oirsaken dochte meerder versekerheyt te vereysschen, om verscheyden dinghen die my daer af int gedachte te vooren quamen: Ten eer-sten soo was ick een tijt lanck van meyning (ghelijcker meer sijn onder de ge-ne die hun inde stelling eens roerendē Eertcloots oeffenen) datmen mette roe-rende Son deur t'ghedacht vast te stellen ter plaets des vasten Eertcloots, en daer teghen den Eertcloot roerende int rondt der Son, datmen daer me soude heb-ben t'begin der spiegheling des roerenden Eertcloots: Maer want de voorstel-len op sulcken gront ghebout ghebreckich vielen, soo bevant ick de boveschre-ven versetting des Eertcloots int ront t'welck te vooren de Sonwech gheweest hadde, niet te moeten gheschieden, maer dat om de vaste Son een ander rondt als Eertclootwech behoort beschreven te worden. Ende want my dese din-ghen soo kennelick niet en dochten datter gheen verclaring af en soude behou-ven, soo beschreef ick van dies het 8 voorstel deses 3 boucx: Ick bevant oock dat Saturnus met d'ander Dwaelders ghesien uyt de vaste Son, niet de selve duy-steraerlangde en hadden die des inronts middelpunt heeft ghesien uyt den va-sten Eertcloot: Als by ghelijckenis int 15 voorstel deses 3 boucx, de lini A R niet even en evewijdeghe te wesen met C V by aldiense ghetrocken waer, maer wel met K V. Voort sach ick dat met stelling eens roerenden Eertcloots haer wechs verstepunt valt op de teghenoversijde des verstepunts vande Sonwech met stelling eens vasten Eertcloots, t'welck my oock met een opsicht soo ken-nelick niet en docht dattet gheen bewijs en soude behouven, sulcx dat ick t'sel-ve verclaerde int 9 voorstel des 3 boucx.

Boven dien hoewel de Sonnens schijnbaer duysteraerlangde gesien uyt den Eertcloot het teghenoverpunt is van des Eertcloots schijnbaer duysteraerlang-de gesien uyt de Son, waer deurmen lichtelick soude meynen, dat wanneer de Son met d'een stelling waer in des wechs eerste halfront hebbende achtring, dat op den selven tijt den Eertcloot met d'ander stelling soude sijn in des wechs tweede halfront hebbende voordering niet evegroot wesende mette Sonnens achtring: Nochtans sijn de Son en den Eertcloot in haer roerende stelling op een selve tijt elck inde wechs eerste halfront met achtring, of elck inde wechs tweede halfront met voordering, en beyde evegroot, welcke dinghen my oock dochten haer bewijs te vereysschen, en daerom heb ickt gedaen int 10 voorstel.

Ten tweeden so is my indē handel des Maenloops dit bejegent: T'is kenne-lick dat den Eertcloot in een uytmiddelpuntige wech draeyende, de selve one-venheyt of voorofachtring crijcht diemen met stelling eens vasten Eertcloots de Son toeschrijft, maer den Hemel der Maen den Eertcloot omvanghende en t'samen een cloot wesende, soo ist daervoor te houden, dat dien grooter cloot,

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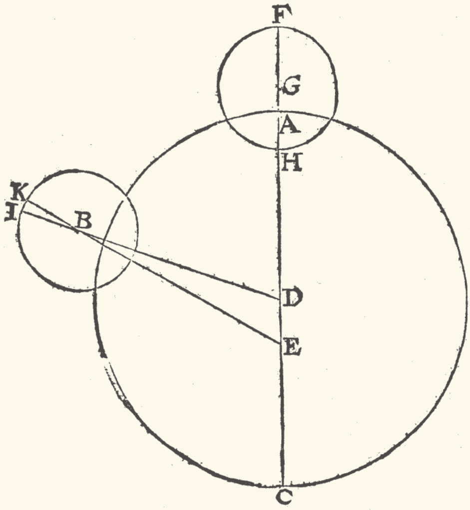
apparent places and distances from each other that they are found on the theory of a fixed Earth, with the circumstances relating thereto.

Copernicus indeed assumes, as it seems, that the two theories, one with a fixed and the other with a moving Earth (that is to say, if from the one are removed *Ptolemy's* additions of the unknown inequalities, and from the other, his own), lead to the same result with regard to the Planets' places and mutual distances, but he does not prove it, apparently considering the matter so clear that it does not call for any proof; but it appeared to me, with a view to the more thorough knowledge of the causes, to require greater certainty, on account of various things that occurred to me in this connection. In the first place, for some time I held the view (as there are more among those who study the theory of a moving Earth) that by imagining the moving Sun to be in the place of the fixed Earth, and on the other hand the Earth as moving in the circle of the Sun, one would have the beginning of the theory of the moving Earth. But since the propositions based on such a foundation turned out defective, I found that the aforesaid displacement of the Earth into the circle which had previously been the Sun's orbit should not be effected, but that about the fixed Sun another circle ought to be described as the Earth's orbit. And since these things did not appear to me so obvious as not to need an explanation, I described of this the 8th proposition of this 3rd book. I also found that Saturn and the other Planets, when seen from the fixed Sun, did not have the same ecliptical longitude that the epicycle's centre has, when seen from the fixed Earth; for example, in the 15th proposition of this 3rd book the line *AR* would not be equal and parallel to *CV*, if this line were drawn, but is equal and parallel to *KV*. Further I saw that on the theory of a moving Earth the aphelion of its orbit falls on the opposite side to the apogee of the Sun's orbit on the theory of a fixed Earth, which also did not appear to me so obvious at simple view as not to require any proof, so that I have explained it in the 9th proposition of the 3rd book.

Moreover, though the Sun's apparent ecliptical longitude, when seen from the Earth, is the opposite to the Earth's apparent ecliptical longitude, when seen from the Sun — in consequence of which one might easily think that if the Sun on the one theory were in the orbit's first semi-circle, being in lag, in the same time the Earth on the other theory would be in the orbit's second semi-circle, being in advance for an amount not equal to the Sun's lag — yet the Sun and the Earth, when assumed to be moving, are in the same time each in the orbit's first semi-circle with lag, or each in the orbit's second semi-circle with advance, and both equal in amount, which things also appeared to me to require a proof, and therefore I have given it in the 10th proposition.

Secondly, I met with the following in dealing with the Moon's motion. It is obvious that the Earth, when it revolves in an eccentric orbit, shows the same inequality or advance-or-lag that is ascribed to the Sun on the theory of a fixed Earth; but since the Heaven of the Moon contains the Earth and they are together one sphere, it is to be assumed that this greater sphere shows the de selve voorschreven onevenheyt crijcht, die de cleender den Eertcloot heeft waer uyt men niet onbillich en schijnt te besluyten dat haer verstepunt daer af oock de selve onevenheyt crijcht, t'welck my eerst dede twijffelen of dit geen oirsaeck vande tweede onbekende onevenheyt en mocht wesen die *Ptolemeus* meent bemerct te hebbē. Maer om hier in vaster te gaen, ick heb de saeck voorder overdocht, en daer toe een form gheteyckent op dese wijse: Laet A B C den Eertcloot wech beteyckenen, diens middelpunt D, de Son E, deur welcke twee punten D, E, ghetrocken de middellijn A D E C, soo beteyckent A t'verstepunt, C t'naestepunt, en den Eertcloot sy an A. Daer na verlangt D A tot F, en tusschen A en F ghestelt sijnde t'punt G, ick beschrijf daer op als middelpunt de Maenwech F H, diēs verstepunt F: Maer want het roersel des selfdē verstepunts tottet gene wy verclaren willen gheen verandering by en brengt, om dattet sonder onevenheyt is, soo neem ick de Maenwech geen eygen roersel te hebben, maer het middelverstepunt als F altijt te blijven inde lini ghetrocken uyt het middelpunt D deur den Eertcloot. Dit soo wesende, laet den Eertcloot met haer Maenwech gecommen sijn van A tot B, en sal haer middelverstepunt volghens t'voorgaende ghestelde, sijn in de voortghetrocken lini



D B, het welck sy in I, ick treck daer na van E deur B tot inde Maenwech de lini E B K, soo dat K t'verstepunt bediet, en den houck E B D of I B K de achtring des Eertcloots B, en daerom oock des heelen Maenwechs, en vervolghens des middelverstepunts I, t'welck tot een ander plaets gesien wort uyt E dan uyt D: Nu is de vraech of t'selve verstepunt I, mette comst des Eertcloots van A tot B, gheen onevenheyt ghecreghen en heeft van weghen des Eertcloots uytmiddelpuntighe wech? Maer alles wel overdocht sijnde, soo valter onderscheyt te maken tusschen de plaetsen des sienders, want soomen het oogh stelt ande Son E als duysteraers middelpunt, t'verstepunt I sal van daer ghesien een onevenheyt ghecreghen hebben: Maer want de dadelicke siening daer t'gheschil af is, niet en ghebeurt uyt E, maer uyt den Eertcloot B, soo moetmense daer nemen, t'welck doende t'punt K wijst an des Eertcloots schijnbaer duysteraerlangde, als commende de lini E B K uyt des duysteraers middelpunt E, sulcx dat by aldiē t'midddelverstepunt daer gesien wierde, het soude mette comst des Eertcloots van A tot B de onevenheyt ghecregen hebben des houcx I B K: Maer t'wort an I ghesien, soo veel voorwaert als den houck K B I bedraecht, en daerom salt mette comst des Eertcloots van A tot B, schijnbaerlick inden duysteraer soo veel gheloopen hebben als den booch A B of den houck A D B bedraecht, dat is effen soo veel als den Eertcloot eyghentlick geloopen heeft, sonder eenige onevenheyt: Inder vougen dat ick doen sach, dat hoewel den Eertcloot en oock den heelen Maenwech de selve onevenheyt heeft diemen met

stelling

same aforesaid inequality that the smaller — the Earth — has, from which it seems to be concluded not unjustly that their apogee also receives from this the same inequality; which first caused me to doubt whether this could not be the cause of the second unknown inequality which *Ptolemy* thinks he has perceived. But to be more certain in this, I considered the matter further, and for this purpose drew a figure in the following manner: Let *ABC* denote the Earth's orbit, its centre *D,* the Sun *E,* and when through these two points *D, E* is drawn the diameter *ADEC, A* denotes the aphelion, *C* the perihelion, and the Earth shall be at *A.* When thereafter *DA* is produced to *F,* while between *A* and *F* is marked the point *G,* I describe about this as centre the Moon's orbit *FH,* whose apogee is *F.* But since the motion of this apogee does not make any difference to what we wish to explain, because it is without inequality, I assume that the Moon's orbit has no proper motion, but that the mean apogee, namely *F,* always remains on the line drawn from the centre *D* through the Earth. This being so, let the Earth with the Moon's orbit have travelled from *A* to *B,* then its mean apogee according to the foregoing supposition will be on the line *DB* produced, which shall be in *I;* thereafter I draw from *E* through *B* into the Moon's orbit the line *EBK,* so that *K* denotes the apogee, and the angle *EBD* or *IBK* the lag of the Earth *B,* and therefore also of the whole Moon's orbit, and consequently of the mean apogee *I,* which is seen from *E* in another place than it is from *D.* Now the question is whether this apogee *I* has not, upon the passage of the Earth from *A* to *B,* acquired an inequality, on account of the Earth's eccentric orbit? But when everything has been well considered, a distinction has to be made between the places of the observer, for if the eye is put at the Sun's place *E* as centre of the ecliptic, the apogee *I,* when seen from there, will have acquired an inequality. But since the practical observation, with which the question is concerned, does not take place from *E,* but from the Earth *B,* it must be taken there; and when this is done, the point *K* indicates the Earth's apparent ecliptical longitude, since the line *EBK* comes from the ecliptic's centre *E,* so that, if the mean apogee were seen there, upon the passage of the Earth from *A* to *B* it would have acquired the inequality of the angle *IBK.* But it is seen at *I,* so much forwards as the angle *KBI* amounts to, and therefore upon the passage of the Earth from *A* to *B* it will apparently have moved in the ecliptic as much as the arc *AB* or the angle *ADB* amounts to, that is exactly as much as the Earth has really moved, without any inequality. In such a manner that I then saw that, though the Earth and also the whole Moon's orbit has the same inequality that is ascribed ſtelling eens vaſten Eertcloots de Son toeſchrijft, dat nochtans des Maenwechs verſtepunt uyt den Eertcloot gheſien die onevenheyt niet en crijcht, twelck my uyt de boveſchreven twijffeling brocht: Ick bevant oock datter gheen verandering en viel deur het ſtellen van des verſtepunts eyghen loop, ſdaechs op 52 ① 27 ② tegen t'vervolgh der trappē als int 11 voorſtel deſes 3 boucx, dat met ſtelling eens vaſten Eertcloots is van 6 ① 41 ② na t'vervolgh der trappen deur het 11 voorſtel des 1 boucx: Sghelijcx oock datter gheen verandering en viel deur het ſtellen des duyſteringſnees eyghen loop ſdaechs van 1 tr. 2. 19 tegen t'vervolgh der trappen als int ſelve 11 voorſtel deſes 3 boucx, dat met ſtelling eens vaſten Eertcloots is van 3 ① 11 ② tegen t'vervolgh der trappen deur het 11 voorſtel des 1 boucx.

Maer hoewel my deſe voorſchreven dinghen alſoo bekent wierden, dat en vertoonde ſich niet ſoo openbaerlick, dat icker uyt ſoude willen beſluyten de overeencommingen des Maenloops deur ſtelling eens roerenden Eertcloots, mette Maenloop deur ſtelling eens vaſten Eertcloots gheen verclaring te behouven, ſulcx dat ick daer af beſchreef het 12 voorſtel.

Ten derden ſoo is my int overlegghen des loops van d'ander Dwaelders, te weten Saturnus, Iupiter, Mars, Venus en Mercurius, dit te vooren ghecommen: T'is kennelick datmen tottet berekenen der Dwaelderplaetſen met ſtelling eens vaſten Eertcloots, verſcheyden linien treckt uyt des Sonwechs uytmiddelpuntichpunt, dats den ſelven vaſten Eertcloot, tot d'ander noodighe punten, als inde form des 15 voorſtels de linien A R, A T, ghetrocken uyt den vaſten Eertcloot A. Ende dien volghens men ſoude lichtelick meynen dat tottet berekenen der Dwaelderplaetſen met ſtelling eens roerenden Eertcloots, ſulcke linien als K X, K V, behooren ghetrocken te worden niet uyt K, maer uyt des Eertclootwechs uytmiddelpuntichpunt C, dats de vaſte Son, gemerckt die plaets ſulcx wat is inde ſtelling eens roerenden Eertcloots, gelijck de plaets des vaſten Eertcloots inde ſtelling der roerende Son: Inder voughen dat my dit oock docht bewijs te vereyſchen.

Ten vierden machmen hier noch by voughen de groote verandering dieder valt, deur datmen in des roerenden Eertcloots ſtelling het inrondt wech doet, den Dwaelder ſeght an des inronts middelpunt te weſen, en weerom daer teghen datmen den Eertcloot die vaſt gheſtelt wiert, ſulcken loop gheeſt als gheſien is.

Al dit en dochten my gheen dinghen om an te nemen voor eyntlick een ſelve beſluyt uyt te brenghen, ſonder daer af de reden te weten en bewijs te doen: T'is oock daer voor te houden dat ſy die tot kennis dier gelijckheyt eerſt gherochten, ſeker middelen tot inleyding hadden waer deur ſy ſulcx voor gewis hielden. Maer angheſien *Copernicus* die niet beſchreven en heeft, nochtans al t'ghene hy van dies ſtelt vaſt gaet (als de Maen met ſtelling eens roerenden Eertcloots ſoo te berekenen ghelijck met ſtelling eens vaſten Eertcloots, ſonder van weghen de ſtelling des Eertclootloops verandering te crijghen, voort in d'ander Dwaelders de linien als de boveſchreven niet te trecken uyt de Son als weerelts middelpunt, maer uyt des Eertclootwechs middelpunt) t'gheeft billichlick vermoeden een van tween, te weten of dat hem d'oirſaeck deſer ghelijckheyt anders bejeghent is dan my, en t'ſelve met ſulcken lichticheyt dat hem onnoodich docht verclaring daer af te doen, of dies niet, dat hy by ghevalle ghecreghen heeft eenighe oude boucken vande beſchrijving des loopenden Eertcloots, ſonder ſulcke redenen daer in vervatet te weſen, en dat hy daerom ſulcx alſoo mach uytgheghεven hebben.

Maer,

to the Sun on the theory of a fixed Earth, nevertheless the apogee of the Moon's orbit, when seen from the Earth, does not acquire that inequality, which removed my aforesaid doubts. I also found that no change was caused by putting the true motion of the apogee at 52′27″ a day, contrary to the order of the degrees, as in the 11th proposition of this 3rd book, which on the theory of a fixed Earth is 6′41″ in the order of the degrees, by the 11th proposition of the 1st book. Likewise that no change was caused by putting the proper motion of the line of nodes at 1°2′19″ a day contrary to the order of the degrees, as in the said 11th proposition of this 3rd book, which on the theory of a fixed Earth is 3′11″ contrary to the order of the degrees, by the 11th proposition of the 1st book.

But though these aforesaid things thus became known to me, this was not manifested so clearly that I should be inclined to conclude from it that the correspondences of the Moon's motion on the theory of a moving Earth with the Moon's motion on the theory of a fixed Earth do not require any explanation; so that I described thereof the 12th proposition.

Thirdly, the following occurred to me in considering the motion of the other Planets, to wit, Saturn, Jupiter, Mars, Venus, and Mercury. It is evident that for the computation of the places of the Planets on the theory of a fixed Earth various lines are drawn from the point of eccentricity of the Sun's orbit, *i.e.* the said fixed Earth, to the other necessary points, namely in the figure of the 15th proposition the lines *AR, AT,* drawn from the fixed Earth *A.* And consequently one might easily think that for the computation of the places of the Planets on the theory of a moving Earth such lines, as *KX, KV,* ought to be drawn, not from *K,* but from the point of eccentricity of the Earth's orbit *C, i.e.* the fixed Sun, observing that this place is the same in the theory of a moving Earth as the place of the fixed Earth in the theory of the moving Sun; so that I thought that this, too, required to be proved.

Fourthly, there may be added the great change which is caused by the fact that in the theory of the moving Earth the epicycle is taken away, the Planet is said to be at the epicycle's centre, and again, on the other hand, that the Earth, which was assumed to be fixed, is given such a motion as has been seen.

All these appeared to me not to be things that could be taken to lead ultimately to the same result, without knowing the reason thereof and giving a proof. It is also to be assumed that those who first attained to knowledge of this equality had secure aids to guide them, in consequence of which they considered this certain. But since *Copernicus* has not described these, while yet all that he states about this is firmly established (such as computing the Moon on the theory of a moving Earth in the same manner as on the theory of a fixed Earth, without its undergoing any change because of the assumption of the Earth's motion; further, with the other Planets, not to draw the lines described above from the Sun as the world's centre, but from the centre of the Earth's orbit), it gives a just surmise of either one thing or the other, to wit, either that the cause of this equality appeared different to him from what it does to me, and this with such easiness that he thought it unnecessary to give an explanation of it, or otherwise that he had perhaps access to some old books describing the moving Earth, which did not contain such reasons, and that he may therefore have edited them like this.

Maer, mocht nu ymant segghen, laet een van beyden wesen, wat nut is uyt soodanich verhael te trecken? Dit: By aldien d'oirsaeck der gelijckheyt en overcomming deser twee stellinghen eens roerenden en vasten Eertcloots, voor *Copernicus* soo claer gheweest heeft, datse hem docht gheen uytlegging te behouven, t'sal anderen meughen beweghen daer op te letten, en naerder wech te beschrijven, om niet deur een langher te doen dat deur een corter can ghedaen worden. Maer soo hy sulcx ghecreghen heeft uyt eenighe oude boucken hem ter handt ghecommen, t'soude hope gheven datter vande wetenschappen des wijsentijts herwaerts of derwaerts in d'een of d'ander * bouckcasse noch meer overblijfsels meughen sijn dan die, en oirsaeck gheven van vlietelicker daer na te vernemen dan men ghedaen heeft. T'BESLVYT. Hier is dan verclaert de reden waerom ick inde 8. 12. 15. en 17 voorstellen bewesen hebbe de Dwaelders deur stelling eens roerenden Eertcloots bevonden te worden totte selve schijnbaer plaetsen en verheden van malcander, diemense met stelling eens vasten Eertcloots bevint, mette omstandighen van dien, na den eysch.

Bibliotheca.

19 VOORSTEL.

Te verclaren op vvelcke stelling, te vveten de oneygen met een vasten Eertcloot, of de eyghen met een roerende, oirboirst schijnt de rekeninghen te maken vande langdeloop der Dvvaelders.

Int voorgaende 1 bouck beschreven sijnde den loop der Dwaelders deur ervarings dachtafels, en int 2 deur wisconstighe wercking beyde ghegront op de oneyghen stelling eens vasten Eertcloots, en dat ick volghens mijn voornemen daer na in dit 3 bouck beschreven heb dien loop ghegront op de eyghen stelling des waren roerenden Eertcloots, soo mocht ymant dencken, dat nadien wy voor t'beginnen deses handels wisten die stelling oneyghen te wesen, en dese warachtich, oft niet beter en waer gheweest dien oneygen onbeschreven te laten, en den tijt mettet ondersoucken der selfde niet deur te brenghen, maer in die plaets ten eersten ant warachtighe te vallen: Hier op antwoorde ick int ghemeen, dat ick de kennis van d'een en d'ander seer noodich acht, en om daer af int besonder breeder te spreken, segh aldus: Dese twee stellingen sulcke ghelijckheyt hebbende als int voorgaende blijckt, soo acht ick onnoodich te beschrijven nieuwe werckstucken vande vinding der Dwaelderlangden, effening der daghen, saminghen, teghestanden, duysteringhen, en meer anderen, ghegront op stelling eens roerenden Eertcloots: Ia mijn meyning is dat de voorvallende rekeninghen, deurgaens bequamelicker souden gedaen worden deur vervouging des ghedachts op de oneyghen stelling eens vasten Eertcloots, en op gheteyckende formen na den eysch van dien, dan op de eyghen des roerenden Eertcloots, al waren sy oock volcommelick beschreven.

Om van t'welck by voorbeelt breeder redenen te verclaren, ghenomen dat ymant in een varende schip bevale een pack thien ghemeten voeten achterwaert te legghen: Die sulck bevel na wil commen, stelt uyt sijn ghedacht het roersel des schips, die thien voeten daer in metende even al oft stil laghe, want anders verstaen sijnde, tusschen den tijt dattet bevel gheschiede, en t'werck gedaen wiert, mach t'schip 1000 voet voort ghevaren sijn, inder voughen datmen het pack in plaets van 10 voeten, soude moeten 1010 voeten achterwaert legghen, But, someone might now say, let it be one of the two; what profit is to be gained from such a story? This: If the cause of the equality and correspondence of these two theories of a moving and a fixed Earth was so clear to *Copernicus* that he thought it did not require any explanation, it may induce others to pay heed to this and describe a shorter course, in order not to do by a longer procedure what can be done by a shorter. But if he has learned this from some old books that came into his hands, this would raise the hope there may be more remains than these of the knowledge of the Age of the Sages, here or there in some library or other, and would cause us to inquire thereafter more diligently than has been done.

CONCLUSION. Here the reason has thus been explained why I have proved in the 8th, 12th, 15th, and 17th propositions that the Planets on the theory of a moving Earth are found in the same apparent places and distances from each other that they are found on the theory of a fixed Earth, with the circumstances relating thereto; as required.

19th PROPOSITION.

To explain on which theory, to wit, the untrue theory with a fixed Earth or the true theory with a moving Earth, it seems most suitable to base the computations of the motion in longitude of the Planets.

Since in the foregoing 1st book the motion of the Planets has been described by means of empirical ephemerides, and in the 2nd by mathematical operations, both based on the untrue theory of a fixed Earth, and since according to my intention I have thereafter described in this 3rd book the said motion according to the true theory of the moving Earth, it might be thought (since before we began this discussion, we knew the former theory to be untrue and the latter true) whether it would not have been better to leave the untrue one undescribed and not to waste time in studying it, but instead to begin at once with the true theory. To this I reply in general that I consider the knowledge of one theory and the other as highly necessary, and to speak of it specially in more detail, I say as follows: Since these two theories have such equality as appears in the foregoing, I deem it unnecessary to describe new problems about the finding of the Planets' longitudes, equality of time, conjunctions, oppositions, eclipses, and other things, based on the theory of a moving Earth. Nay, I am of opinion that the computations needed would generally be made more easily by directing our thought to the untrue theory of a fixed Earth and to figures drawn in accordance with its requirements than to the true theory of the moving Earth, even if they were described perfectly.

In order to explain this more fully by an example, assume that a man in a sailing vessel should order a parcel to be put ten paced feet backwards. He who wants to obey this order, puts out of his mind the movement of the ship, since those ten feet have exactly the same length as they would if it were lying still; for if it were understood differently: between the time when the order was given and that when the work was performed the ship may have sailed on 1,000 feet, so that, instead of 10 feet, the parcel would have to be put 1,010 feet ghen, dat waer buyten t'schip misschien int water: T'welck de meyning niet wesende, soo ist in sulcken ghevalle beter, het spreken en t'doen te voughen na het schijnbaerlick, dats na de schijnbaer stilstandt der inwendighe stoffen des schips, dan na het eyghen. Maer sooder een ander gheschil waer, niet van een pack te verleggen, maer van een pael, neem ick, int water te moeten slaen, thien voeten achterwaert van een seker plaets ant schip, daer verstaetmen de saeck eyghentlick, te weten thien voeten achterwaert, van die plaets (t'schip mach daerentusschen ghevaren hebben hoet wil) daer den beveelder af sprack. Sulcx dat in smenschen handelingen tweederley wijse van spreken en doen valt, d'eene ghegront opt schijnbaer, d'ander opt eyghen, waer af men altijt die behoort te verkiesen, deur welcke men t'voornemen best can verstaen en uytrechten. Dit soo toeghelaten, t'schijnt oirboir in dit Hemelloopschrift t'ghedacht een gront te gheven oock op tweederley stelling, d'eene schijnbaer, d'ander eyghen elck na den eysch van t'voornemen daer de saeck lichtelicxt deur can verstaen worden. Te weten opt schijnbaer, int leeren der beginselen, en in t'maken der boveschreven rekeninghen, om datmen de woorden dier stof op desen loopenden Eertcloot ghebruyckt al offe stil laghe: Als wanneermen spreeckt van der weereltlichten opganck boven den sichteinder, onderganck onder den sichteinder, comste tottet middachront, en veel dierghelijcke, t'welck eyghentlick heel verkeert sichteinders onderganck, en opganck is, en comste des middachronts totte lichten, welcke woorden duyster souden vallen: Ia en sijn by *Copernicus* self niet ghebesicht, hoewel sijn voornaemste wit was vanden roerenden Eertcloot te schrijven. Voort ghelijckt int varende schip nutter was sijn roersel uyt het ghedacht te stellen, en te houden al oft stil stonde, en t'schijnbaer te nemen al oft eygen waer, alsoo ist in desen ghevalle bequamer totte leering, het roersel des Eertcloots uyt het ghedacht te stellen, en te houden al offe stil stonde, het schijnbaer nemende al oft eyghen waer. Maer wesende gheschil vande breedeloop der Dwaelders Saturnus, Iupiter, Mars, Venus, en Mercurius, van welcke int volghende gheseyt sal worden, daer ist reden (ghelijck vant slaen der pael int water gheseyt is) sijn rekeningen eerst te gronden op formen gheteyckent na den eysch der eygen stelling des roerenden Eertcloots, uyt oirsaeck dat wy daer deur beter connen gheraken tot oirsakelicke kennis des breedeloops, en dat de wercking ghegront op de versierde stelling eens vasten Eertcloots daer uyt ghetrocken wort op de wijse als ick hier na beschrijven sal.

Dits nu soo mijn ghevoelen, en op sulex heb ick dese beschrijving geformt; doch soo ymant ander wichtigher redenen hadde my onbekent, deur welcke hy oirboirder bevonde anders te doen, hy soude die meughen volghen.

Tot hier toe vande langdeloop der Dwaelders gheseyt wesende, ick sal nu commen tottet beschrijven vande breedeloop.

VANDE

backwards, which would be outside the ship, perhaps in the water. This not being what was intended, in such a case it is better to speak and act according to appearance, *i.e.* the apparent rest of the internal materials of the ship, than according to the truth. But if there were another dispute, not of moving a parcel, but, for example, of having to strike a pole in the water, ten feet behind a given place in the ship, there the matter is taken literally, to wit, ten feet behind that place (the ship may meanwhile have sailed in any desired direction) of which the commander spoke. So that in men's actions there are two ways of speaking and doing, one based on appearance and the other on the truth, of which we should always choose that one by means of which the intention can best be understood and carried into effect. If this is admitted, it seems suitable in this book on the Heavenly Motions to base our thoughts also on two theories, one apparent, the other true, each according to the requirement of what is intended, by which the matter can be understood most easily. To wit, on the apparent theory in the learning of the elements and in the making of the aforesaid computations, because the words of this subject matter are applied to this moving Earth as if it were lying still. For example, when we speak of the luminaries rising above the horizon, setting beneath the horizon, their arrival in the meridian, and many similar things, which in reality, quite the reverse, is the setting and rise of the horizon, and arrival of the meridian at the luminaries, which words would be obscure. Nay, they have not been employed by *Copernicus* himself, though his chief object was to write about the moving Earth. Further, just as in the sailing ship it was more useful to put its movement out of one's mind and to assume that it lay still, and to take appearance as if it were the truth, thus in this case it is better for didactic purposes to put the motion of the Earth out of one's mind, and to assume that it stands still, taking appearance as if it were the truth. But when there is a dispute about the motion in latitude of the Planets Saturn, Jupiter, Mars, Venus, and Mercury, which will be discussed in what follows, there is good reason (as it has been said of the striking of a pole in the water) for first basing one's computations on figures drawn in accordance with the requirements of the true theory of the moving Earth, because thus we can better attain to knowledge of the causes of the motion in latitude, and because the operation based on the fictitious theory of a fixed Earth is inferred from it in the manner I shall describe hereinafter.

This is my opinion, and on this I have based this description, but if anyone had other, weightier reasons, unknown to me, for which he should find it more suitable to proceed otherwise, he may follow them up.

The motion in longitude of the Planets having been discussed up to this point, I shall now come to the description of the motion in latitude.

VANDE BREEDE LOOP.

VYFDE ONDERSCHEYT DES DERDEN BOUCX VANde breedeloop der vijf Dvvaelders Saturnus, Iupiter, Mars, Venus, en Mercurius, met stelling eens roerenden Eertcloots.

CORTBEGRYP DESES VYFDEN ONDERSCHEYTS.

ANghesien den roerenden Eertcloot en de stilstaende Son, deur t'ghestelde altijt inden duysteraer sijn sonder afvvijcking of breede, en dat de Manens breedeloop beschreven int 2 bouck beginnende ant 34 voorstel met stelling eens vasten Eertcloots, geen verschil en heeft dat verclaring behouft mette breedeloop der stelling eens roerenden Eertcloots, soo en valt van hemlien breede hier niet te seggghen, maer alleenelick van die der vijf ander als volght:

Ghelijck de Ouden inde beschrijving vande Dvvaelders langdeloop billichlick beginnen met dadelicke ervaringen van yders loop op bekenden tijt, om daer uyt ghemeene regelen te trecken dienende tottet vinden haers loops, in toecommende tijden, alsoo vereyscht oock sulcx de natuerlicke oirden inde beschrijving des breedeloops, inder vougen dat ick daer me in elck Dvvaelders beschrijving beginnen sal: Ende vvant de reghel van eenen Dvvaelder voor allen dient, soo sal ick alleenelick vanden eersten of oppersten Saturnus gheformde voorstellen beschrijven, en van d'ander sulck vermaen doen als breeder verclaring vereyscht. Hier af sal ick ses voorstellen beschrijven.

Het eerste vvesende in d'oirdē het 20, is beschrijving van Ptolemeus *dadelicke ervaringen van Saturnus schijnbaer duysteraerbreede, dienende om daer uyt ghemeene regel te trecken van sijn breedeloops eyghenschappen.*

Het 2 vvesende in d'oirden het 21, om te vinden de vvechlangden der

tvvee

OF THE MOTION IN LATITUDE

FIFTH CHAPTER

OF THE THIRD BOOK

Of the Motion in Latitude of the Five Planets Saturn, Jupiter, Mars, Venus, and Mercury, on the Theory of a Moving Earth

SUMMARY OF THIS FIFTH CHAPTER.

Since the moving Earth and the fixed Sun by supposition are always in the ecliptic without any deviation or latitude, and since the motion in latitude of the Moon (described in the 2nd book, starting with the 34th proposition) on the theory of a fixed Earth has no such difference from its motion in latitude on the theory of a moving Earth as would require an explanation, it is not necessary here to speak of their latitude, but only of that of the five others, as follows.

Just as the Ancients in describing the Planets' motion in longitude rightly start with practical experiences of the motion of each in a known time, in order to derive therefrom common rules serving to find their motion in future times, the same is also required by the natural order in the description of the motion in latitude, so that I will start therewith in the description of each Planet. And because the rule of one Planet applies to all, I will describe propositions with figures only of the first or upper Planet Saturn, and for the others I will give such description as is necessary for a fuller explanation. I will describe hereof six propositions.

The first, which is the 20th in the sequence, is a description of *Ptolemy's* practical experiences of Saturn's apparent ecliptical latitude, serving to derive therefrom a common rule for the properties of its motion in latitude.

tvvee punten van Saturnusvvechs schijnbaer grootste afvvijckinghen van den duysteraer, metsgaders de cortste verheden vanden Eertclootvvech totte selve tvvee punten: Oock me de langde der heele lini vant eenpunt dier grootste afvvijcking tottet ander, in sulcke deelen alßer des Eertclootvvechs halfmiddellijn 10000 doet, deur vvisconstighe vvercking ghegront op stelling eens roerenden Eertcloots.

Het 3 vvesende in d'oirden het 22, om te vinden Saturnusvvechs afvvijcking vandē duysteraer: Metsgaders hoe verre de duysteraersne van des Eertclootvvechs middelpunt valt, in sulcke deelen alsser des Eertclootvvechs halfmiddellijn 10000 doet, deur vvisconstige vvercking gegront op stelling eens roerenden Eertcloots.

Het 4 vvesende in d'oirden het 23, om te vinden de vvechlangde der tvvee uyterste punten vande duysteraersne, en vande tvvee uyterste punten der afvvijcking in Saturnusvvech: Oock der lini die van Saturnusvvechs middelpunt op de duysteraersne rechthouckich valt, in sulcke deelen alßer des Eertclootvvechs halfmiddellijn 10000 doet, deur vvisconstighe vvercking ghegront op stelling eens roerenden Eertcloots.

Het 5 vvesende in d'oirden het 24, om te vinden de langde der lini die van een gegevē punt in Saturnusvvech rechthouckich valt opt plat des duysteraers, in sulcke deelē alsser des Eertclootvvechs halfmiddellijn 10000 doet, deur vvisconstige vvercking gegront op stelling eens roerenden Eertcloots.

Het 6 vvesende in d'oirden het 25, om te vinden Saturnus schijnbaer duysteraerbreede op een ghegheven tijt, deur vvisconstighe vvercking ghegront op stelling eens roerenden Eertcloots.

Daer na sal volgen het bovescbreven vermaen van d'ander Dvvaelders, sonder daer af ghesormde voorstellen te maken.

EERST VAN SATVRNVS BREEDELOOP.

20 VOORSTEL.

Te beschrijven *Ptolemeus* dadelicke ervaringhen van Saturnus schijnbaer duysteraerbreeden, dienende om daer uyt gemeene reghel te trecken van sijn breedeloops eyghenschappen.

1 LIDT.

Want mijn gevoelen is datter by de menschen eertijts een grondelicker ervarentheyt gheweest heeft, vande Dwaelders langdeloop met stelling eens roerenden Eertcloots, soo vermoede ick daer uyt by hemlien oock kennis geweest

The 2nd, which is the 21st in the sequence, to find the orbital longitudes of the two points of the apparently greatest deviations of Saturn's orbit from the ecliptic, as well as the shortest distances from the Earth's orbit to these two points; also the length of the whole line from the one point of that greatest deviation to the other, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth.

The 3rd, which is the 22nd in the sequence, to find the deviation of Saturn's orbit from the ecliptic, as well as how far the line of nodes is from the centre of the Earth's orbit, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth.

The 4th, which is the 23rd in the sequence, to find the orbital longitudes of the two extremities of the line of nodes and of the two extremities of the deviation in Saturn's orbit; also of the line which from the centre of Saturn's orbit is dropped perpendicular to the line of nodes, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth.

The 5th, which is the 24th in the sequence, to find the length of the line which from a given point in Saturn's orbit is dropped perpendicular to the plane of the ecliptic, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth.

The 6th, which is the 25th in the sequence, to find Saturn's apparent ecliptical latitude at a given time, by mathematical operation based on the theory of a moving Earth.

This is to be followed by the above-mentioned description of the other Planets, without making thereof illustrated propositions.

FIRST OF SATURN'S MOTION IN LATITUDE.

20th PROPOSITION.

To describe *Ptolemy's* practical experiences of Saturn's apparent ecliptical latitudes, serving to derive therefrom a common rule for the properties of its motion in latitude.

1st SECTION.

Because it is my opinion that in earlier times people had more thorough experience of the Planets' motion in longitude on the theory of a moving Earth, te sijne van der selve breedeloop, om datse, alsmender wat op let ghenouchsaem heur selven daer in openbaert, sulcx dat ick meen *Ptolemeus* gagheslaghen ervaringhen van dies niet d'eerste te wesen, maer datmender voor hem al dapperlick op ghelet, en spiegheling daer af gheformt heeft, doch de selve ter handt van hem noch niemant anders gecommen sijnde, datmen weet, soo sullen wy danckbaerlick annemen den vlietighen arbeyt by hem hier in ghedaen, sonder welcke het nu swaerlick soude by commen tot bescheyt deses handels te geraken, ghemerckt men te weynich gaslaghers vindt. Om dan totte sake te commen ick segh aldus: Ghelijckmen tottet soucken der ghedaente vande Sonnens schijnbaer evenaerbreede, en vande Manens schijnbaer duysteraerbreede, ten eersten tracht dadelick te vinden haer meeste afwijckinghen na het Zuyden en Noorden, om daer deur te commen tot kennis der plaets vande duysteraersne, en de rest dies angaende, alsoo heeft *Ptolemeus* int dadelick ondersoucken der ghedaente van Saturnus schijnbaer duysteraerbreede, ten eersten ghetracht na sijn meeste afwijckingen, en die bevonden op de Noortsijde van 3 tr. 2 ① (soo staetse in sijn tafel) ghebeurende altijts als sijn inronts middelpunt schijnbaerlick was 50 tr. voor sijn wechs verstepunt, te wetē onder des duysteraers 183 tr. en Saturnus an des inronts naestepunt: Maer buyten het naestepunt wesende, soo was sijn Noordersche afwijcking voor datmael cleender, ende ten minsten doen hy alsoo ant verstepunt was, want hoewel hy dat dadelick niet sien en conde, deur dien Saturnus doen by de Son was, soo merckte hijt nochtans by gissing deur de daghelicksche minderinghen die hy in sijn uyterste verschijninghen gade slouch.

2 LIDT.

Op de Zuytsijde bevant hy de meeste afwijcking van 3 tr. 5 ①, ghebeurende altijt als sijn inronts middelpunt was onder het teghenoverpunt des boveschreven 183 tr. dats onder des duysteraers 3 tr. en Saturnus an sijn inronts naestepunt: Maer buyten het naestepunt wesende, soo was sijn Zuydersche afwijcking voor datmael cleender, en ten minsten doe men hem alsoo an het verstepunt vandt.

3 LIDT.

Tot hier toe is gheseyt van Saturnus eyghenschappen wesende het inronts middelpunt an sijn wechs grootste afwijckingen, maer wesende tusschen beyden in een der duysteraersneen twee uytersten, soo bevant hy Saturnus altijt int plat des duysteraers sonder breede, tot wat plaets des inronts hy oock mocht wesen.

4 LIDT.

Het schijnbaerlickste dat *Ptolemeus* bedencken conde van d'oirsaeck deses seltsaem roersels, om daer op een spiegheling te gronden, deur welcke men Saturnus breede op alle toecommende tijden berekenen en van te vooren weten mocht, heeft hy beschreven int 3 hooftstick sijns 13 boucx, waer af den sin dusdanich is.

Laet A B t'plat des duysteraers beteyckenen overcant ghesien, diens middelpunt dats den vasten Eertcloot C, waer deur ghetrocken is de wech D E, en streckende C D op de Noortsijde na des duysteraers 183 tr. als int 1 lidt, sulcx dat

I suppose from this that they also were acquainted with their motion in latitude, because, if some heed is paid thereto, it is sufficiently revealed therein, so that I think that *Ptolemy's* observations were not the first in this respect, but that before his day it had already been attentively observed and theories had been framed on it; but since these have not been handed down to him or anyone else as far as is known, we will gratefully accept the diligent work he has done in this respect, without which it would now be difficult to attain to knowledge of this matter, seeing that too few observers are to be found. To come to the matter, I say as follows. Just as, in order to find the nature of the Sun's apparent equatorial latitude and of the Moon's apparent ecliptical latitude, it is first attemped to find directly their greatest deviations towards the South and the North, in order thus to know the place of the line of nodes and the rest relating thereto; thus *Ptolemy* in practically investigating the character of Saturn's apparent ecliptical latitude first inquired into its greatest deviations and found them on the North side to be 3°2′ (thus it appears in his table [1]), which always occurred when its epicycle's centre was apparently at 50° ahead of its orbit's apogee, to wit at 183° of the ecliptic, and Saturn at the epicycle's perigee. But when it was outside the perigee, its Northerly deviation was for that case smaller, and smallest when it was thus at the apogee; for though he could not see this actually, because Saturn then was near the Sun, yet he found this by guessing, from the daily decrements he observed in its extreme appearances.

2nd SECTION.

On the South side he found the greatest deviation to be 3°5′, which always occurred when the epicycle's centre was at the point opposite the above-mentioned 183°, *i.e.* at 3° of the ecliptic, and Saturn at its epicycle's perigee. But when it was outside the perigee, its Southerly deviation was for that case smaller, and smallest when it was thus found at the apogee.

3rd SECTION.

Up to this point Saturn's properties have been described when the epicycle's centre was at its orbit's greatest deviations, but when it was between the two at one of the two extremities of the line of nodes, he always found Saturn in the plane of the ecliptic without any latitude, no matter in what place of the epicycle it might be.

4th SECTION.

The most plausible explanation that *Ptolemy* could think of as the cause of this curious movement, to base thereon a theory by means of which one might calculate and know in advance Saturn's latitude at all future times, he has described in the 3rd chapter of his 13th book, the meaning of which is as follows.

Let *AB* denote the plane of the ecliptic, seen transversely, whose centre is the fixed Earth *C,* through which has been drawn the orbit *DE, CD* tending on the North side towards 183° of the ecliptic, as in the 1st section, so that the two

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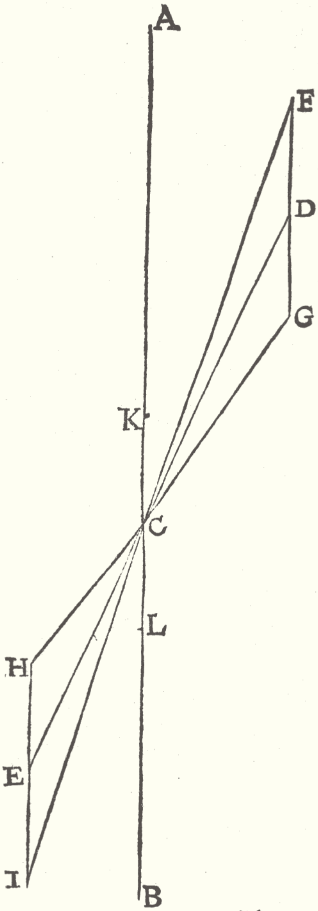
[1]) *Syntaxis* XIII, 6 (Manitius II, p. 376), where also the tables for the other planets are found.



dat de twee houckē A C D,B C E des wechs afwijcking bediē,en op ſijn uyterſte punt D na het Noordē als middelpunt is beſchrevē tinront F G bycans evewijdich mettē duyſteraer A B,diēs naeſtepunt G,verſtepunt F,vā welcke getrockē ſijn de twee liniē G C,E C. Ick heb hier geſeyt bycans evewijdich,deur dien t'verſchil ſeer cleē is, alleenelick vā 2 tr. 4 Ⓘ,want dē houck A C D berekēde hy op 2 tr. 26 Ⓘ,en C D G op 4 tr. 30 Ⓘ, welcke twee houckē om volcommē evewijdicheyt te hebben evegroot ſoudē moetē ſijn. Deſe boveſchrevē rondē wordē altemael gelijck vanden duyſteraer geſeyt is verſtaē overcant geſiē te ſijn. Voort weſende Saturnus ant inronts naeſtepunt G,ſo wiert ſijn ſchijnbaer afwijcking vandē duyſteraer, dats dē houck A C G,ten grootſtē bevonden van 3.tr.2 Ⓘ daer afgheſeyt is int 1 lidt. Maer buyten het naeſtepūt ſijnde,als neem ick voor D, of an F, ſo was ſijn Noorderſche afwijcking voor datmael cleender,en ten minſten ant verſtepunt F weſende, want cleender is dē houck A C F dā A C D. Maer om dergelijcke verclaring oock te doen op de Zuytſijde, ſo laet op des wechs uyterſte punt E als middelpūt. beſchreven ſijn het inront H I,oock bycans evewijdich mettē duyſteraer A B,diens naeſtepunt H,verſtepunt I,van welcke getrockē ſijn de twee liniē H C,I C. Voort weſende Saturnus ant inronts naeſtepunt H, ſo wiert ſijn afwijcking vanden duyſteraer, dats den houck B C H, tē grootſtē bevonden vā 3 tr. 5 Ⓘ,daer af geſeyt is int 2 lidt, maer buyten het naeſtepunt ſijnde, als neem ick voor E of in I,ſoo was ſijn Zuyderſche afwijcking voor datmael cleender,en tē minſten ant verſtepunt weſende, want cleender is den houck B C I dan B C E.

Deſe twee eyghenſchappen der afwijcking als het inront is ande uyterſten D E aldus beſchreven ſijnde, wy ſullen nu commen totte verclaring van d'oirſaeck der eyghenſchappen verhaelt int 3 lidt, te weten alſt tuſſchen beyden is. *Ptolemeus* ghevoelt dat commende het inront van D na de duyſteraerſne, het blijft daeren tuſſchen altijt bycans evewijdich metten duyſteraer, te weten ſoo gematicht, dat weſende het inront F G voor C,ſijn plat is dan altijt teenemael int plat des duyſteraers,als ter plaets van K L,en alſdan ſoo moetmē gelijck int 3 lidt gheſeyt is, Saturnus altijt bevinden inden duyſteraer ſonder breede, tot wat plaets des inronts hy oock weſen mocht: Ende op ſulcke ſpiegheling heeft *Ptolemeus* ſijn voorſtellen,tafelen en rekeninghen ghemaeckt, welcke ick na mijn ſtijl beſchreven hadde, op dattet anderen tot hulp mochte ſtrecken die na beter ſpiegheling trachten wilden, maer daer na ghevonden hebbende t'gene ick meen uyt kennis der oirſaken te commen, ghelijck ick dat int volgende beſchrijven ſal,heb ſulcx onghedruckt ghelaten, ghenouch achtende de eyghen-

 ſchapangles *ACD, BCE* denote the deviations of the orbit. And about its most northerly point *D* as centre has been described the epicycle *FG,* almost parallel to the ecliptic *AB,* whose perigee is *G,* apogee *F,* from which have been drawn the two lines *GC, FC* [1]). I have here said "almost parallel", because the difference is very small, only 2°4′, for he calculated the angle *ACD* to be 2°26′, and *CDG* to be 4°30′, which two angles would have to be equal for perfect parallelism. The above-mentioned circles are all understood to be seen transversely, as has been said of the ecliptic. Further, when Saturn was at the epicycle's perigee *G,* its apparent deviation from the ecliptic, *i.e.* the angle *ACG,* was found at most to be 3°2′, as has been said in the 1st section. But when it was outside the perigee, I assume in front of *D* or at *F,* its Northerly deviation was for that case smaller, and smallest when it was at the apogee *F,* for the angle *ACF* is smaller than *ACD.* But to give a similar explanation also for the South side, let there be described about the orbit's extremity *E* as centre the epicycle *HI,* also almost parallel to the ecliptic *AB,* whose perigee is *H,* apogee *I,* from which have been drawn the two lines *HC, IC.* Further, when Saturn was at the epicycle's perigee *H,* its deviation from the ecliptic, *i.e.* the angle *BCH,* was found to be at most 3°5′, as has been said in the 2nd section, but when it was outside the perigee, I assume in front of *E* or at *I,* its Southerly deviation was for that case smaller, and smallest when it was at the apogee, for the angle *BCI* is smaller than *BCE.*

These two properties of the deviation when the epicycle is at the extremities *D, E* thus having been described, we now come to the explanation of the cause of the properties described in the 3rd section, to wit, when it is between the two. *Ptolemy* is of opinion that when the epicycle moves from *D* to the nodes, meanwhile it always remains almost parallel to the ecliptic, to wit, so moderately that when the epicycle *FG* is in front of *C,* its plane is always altogether in the plane of the ecliptic, as at *KL,* and in that case, as has been said in the 3rd section, Saturn must always be found in the ecliptic without any latitude, no matter in what place of the epicycle it may be. And on this theory *Ptolemy* has based his propositions, tables, and calculations, which I had described in my own manner, in order that it might help others, who wanted to find a better theory. But since after that I found what in my opinion results from knowledge of the causes, as I shall describe in the following pages, I have left it unprinted, considering it sufficient that the properties of the first experiences, observed by *Ptolemy*

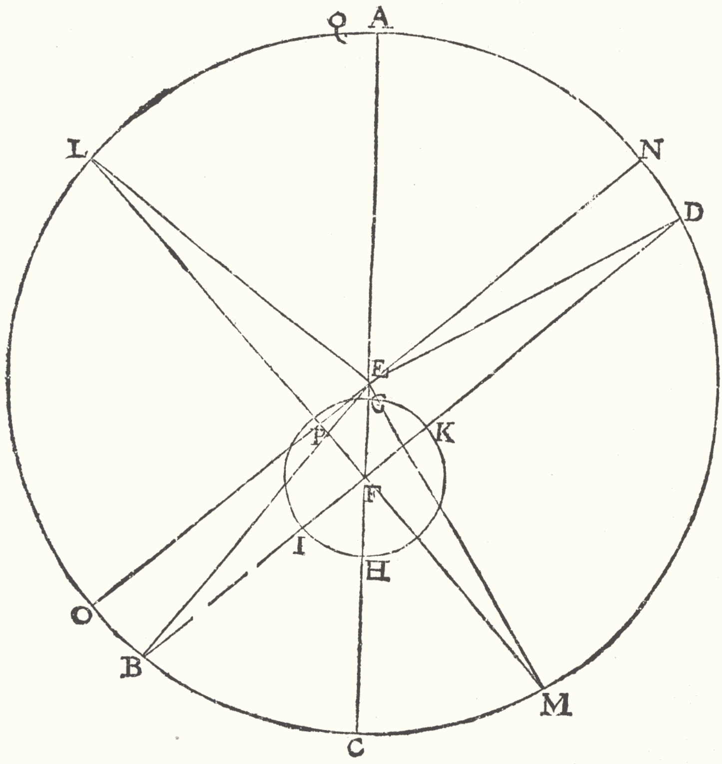
[1]) For *EC* in the Dutch text read *FC.*

schappen der eerste ervaringhen deur *Ptolemeus* met seltsaem yver en neersticheyt gagheslagen, int gemeen aldus verclaert te wesen, en daer me te commen totte voorghenomen spiegheling ghegront op stelling eens roerenden Eertcloots. T'BESLVYT. Wy hebben dan beschreven *Ptolemeus* dadelicke ervaringhen van Saturnus schijnbaer duysteraerbreeden, dienende om daer uyt gemeene reghel te trecken van sijn breedeloops eyghenschappen, na den eysch.

21 VOORSTEL.

Te vinden de vvechlangden der tvvee punten van Saturnusvvechs schijnbaer grootste afvvijckingen vanden duysteraer, metsgaders de cortste verheden vanden Eertclootvvech totte selve tvvee punten. Oock me de langde der heele lini vant een punt dier grootste afvvijcking tottet ander, in sulcke deelē allser des Eertclootvvechs halfmiddellijn 10000 doet, deur vvisconstighe vvercking ghegront op stelling eens roerenden Eertcloots.

T'GHEGHEVEN. Laet A B C D Saturnusweeh beteyckenen, diens middelpunt E, des eertclootwechs middelpunt F, waer op beschreven is den Eert-



cloot.

with exceptional zeal and diligence, had thus been explained generally and that thus I had come to the intended theory based on the theory of a moving Earth. CONCLUSION. We have thus described *Ptolemy's* practical experiences of Saturn's apparent ecliptical latitudes, serving to derive therefrom a common rule for the properties of its motion in latitude; as required.

21st PROPOSITION.

To find the orbital longitudes of the two points of the apparently greatest deviations of Saturn's orbit from the ecliptic, as well as the shortest distances from the Earth's orbit to these two points; also the length of the whole line from the one point of that greatest deviation to the other, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Let *ABCD* denote Saturn's orbit, whose centre is *E,* the centre clootwech G H, en ghetrocken deur de twee punten F E Saturnuswechs middellijn A C, soo is A t'verstepunt, wesende onder des duysteraers 233 tr. deur het 13 voorstel deses 3 boucx, ende want deur het 1 lidt des 20 voorstels, Saturnus grootste Noordersche afwijcking gebeurde als hy schijnbaerlick was 50 tr. voor des duysteraers 233 tr. soo treck ick deur t'punt F de lini L F D, sniende den Eertclootwech in I en K, soo dat den houck A F D doe de selve 50 tr. en sullen daer me D en B sijn de twee punten van Saturnuswech daer hy schijnbaerlick in sijn grootste afwijckinghen vanden duysteraer can gesien worden, en dat uyt de punten K en I.

T'BEGHEERDE. Wy moeten vinden de wechlangde der twee punten B en D, dat sijn de twee boghen A B en A B D, voort de cortste twee verheden K D, I B, vanden Eertclootwech totte selve twee punten, oock me de langde der heele lini vant een punt dier grootste afwijcking tottet ander, in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet.

T'BEREYTSEL. Laet ghetrocken sijn E B, E D.

T'WERCK.

De driehouck E F D heeft drie bekende palē, te wetē de uytmiddelpunticheytlijn E F 5256, des wechs halfmiddellijn E D 92308 deur de Byeenvouging van het 13 voorstel deses 4 boucx, en den houc E F D 50 tr. deur t'gegevē: Hier me gesocht dē houck E D F, wort bevonden deur het 5 voorstel der platte driehoucken van	2 tr. 30.
Welcke vergaert totten houck A F D 50 tr. deur t'ghegheven, comt voor den houck A E D of booch A D	52 tr. 30.
Maer t'verstepunt A is t'begin des wechs, daerom A D 52 tr. 30 (1) ghetrocken vant heel ront 360 tr. blijft voor de begeerde wechlangde van t'punt der schijnbaer grootste uyterste Noordersche afwijcking D, dats den booch A B D	307 tr. 30.
Ende de sijde F D wort bevonden van 95610, waer af getrocken des Eertclootwechs halfmiddellijn F K 10000, blijft voor de begeerde cortste verheyt K D vanden Eertclootwech tottet punt der schijnbaer grootste Noordersche afwijcking D	85610.
Ende sghelijcx gedaen metten driehouck E F B, soo wort den houck E B F bevonden van	2 tr. 30.
Welcke ghetrocken vanden houck B F C 50 tr. (als even sijnde met haer teghenoverhouck A F D) blijft voor den houck C E B	47 tr. 30.
De selve ghetrocken vant halfront A B C, blijft voor de begheerde wechlangde vā t'punt der schijnbaer grootste Zuydersche afwijcking B, dats de booch A B	132 tr. 30.
Ende de sijde F B wort bevonden van 88849, waer af ghetrocken des Eertclootwechs halfmiddellijn F I 10000, blijft voor de begeerde cortste verheyt I B, vanden Eertclootwech tottet punt der schijnbaer grootste Zuydersche afwijcking B	78849.
En om te hebben de langde der heele lini B D, t'is kennelick dat die deur de drie deelen B I 78849, I K 20000, en K D 85610, t'samen bevonden wort voor t'begheerde van	184459.

Waer af t'bewijs deur t'werck openbaer is. T'BESLVYT. Wy hebben dan ghevonden de wechlangden der twee punten van Saturnuswechs schijnbaer grootste afwijckingen vandē duysteraer, metsgaders de cortste verhedē vanden Eertclootwech totte selve twee punten, oock me de langde der heele lini vant of the Earth's orbit *F,* about which has been described the Earth's orbit *GH;* and when through the two points *F, E* is drawn the diameter of Saturn's orbit *AC, A* is the apogee, which is at 233° of the ecliptic by the 13th proposition of this 3rd book. And because by the 1st section of the 20th proposition Saturn's greatest Northerly deviation occurred when it was apparently at 50° before 233° of the ecliptic, I draw through the point *F* the line *BFD,* intersecting the Earth's orbit in *I* and *K,* so that the angle *AFD* shall make the said 50°; then *D* and *B* will be the two points of Saturn's orbit where it can apparently be seen in its greatest deviations from the ecliptic, such from the points *K* and *I.*

WHAT IS REQUIRED. We have to find the orbital longitudes of the two points *B* and *D, i.e.* the two arcs *AB* and *ABD,* further the two shortest distances *KD, IB* from the Earth's orbit to these two points; also the length of the whole line from the one point of that greatest deviation to the other, in such parts as the semi-diameter of the Earth's orbit has 10,000.

PRELIMINARY. Let there be drawn *EB, ED.*

PROCEDURE.

The triangle *EFD* has three known terms, to wit, the line of eccentricity *EF* = 5,256, the orbit's semi-diameter *ED* = 92,308 by the Compilation of the 13th proposition of this 3rd book [1]), and the angle *EFD* = 50° by the supposition. When the angle *EDF* is sought therewith, this is found, by the 5th proposition of plane triangles [2]), to be 2°30′

When this is added to the angle *AFD* = 50° by the supposition, the angle *AED* or arc *AD* becomes 52°30′

But the apogee *A* is the beginning of the orbit, therefore when *AD* = 52°30′ is subtracted from the whole circle = 360°, there is left for the required orbital longitude of the point of the apparently greatest extreme Northerly deviation *D, i.e.* the arc *ABD,* 307°30′

And the side *FD* is found to be 95,610, and when from this is subtracted the semi-diameter of the Earth's orbit *FK* = 10,000, there is left for the required shortest distance *KD* from the Earth's orbit to the point of the apparently greatest Northerly deviation *D* 85,610

And when the same is done with the triangle *EFB,* the angle *EBF* is found to be 2°30′

When this is subtracted from the angle *BFC* = 50° (as being equal to its opposite angle *AFD),* there is left for the angle *CEB* 47°30′

When this is subtracted from the semi-circle *ABC,* there is left for the required orbital longitude of the point of the apparently greatest Southerly deviation *B, i.e.* the arc *AB,* 132°30′

And the side *FB* is found to be 88,849, and when from this is subtracted the semi-diameter of the Earth's orbit *FI* = 10,000, there is left for the required shortest distance *IB,* from the Earth's orbit to the point of the apparently greatest Southerly deviation *B,* 78,849

And in order to have the length of the whole line *BD,* it is obvious that by the addition of the three parts *BI* = 78,849, *IK* = 20,000, and *KD* = 85,610, this required value is found to be 184,459

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1) For *deses 4 boucx* in the Dutch text read *deses 3 boucx.*

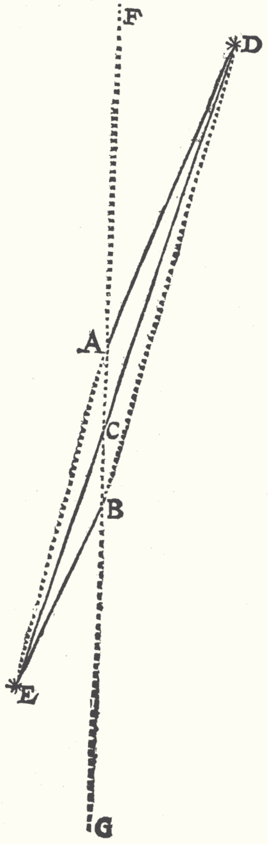
2) Stevin's *Trigonometry* (Work XI; i, 12), p. 152. — See Vol. II B, p. 755.

een punt dier grootste afwijcking tottet ander, in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet, deur wisconstighe wercking ghegront op stelling eens roerenden Eertcloots, na den eysch.

22 VOORSTEL.

Te vinden Saturnusvvechs afvvijcking vandē duysteraer. Metsgaders hoe verre de duysteraersne van des Eertclootvvechs middelpunt valt, in sulcke deelen alsser des Eertclootvvechs halfmiddellijn 10000 doet, deur vvisconstighe vvercking ghegront op stelling eens roerenden Eertcloots.

T'GHEGHEVEN. Laet de lini A B den Eertclootwech bedien overcant ghesien, die ick even teycken an des Eertclootwechs middellijn I K inde form



des 21 voorstels, C sy t'middelpunt, daer na treck ick dese D E, even met die D B inde selve form des 21 voorstels, doēde 1844959, en dese C D evē met die F D, dese C E even met die F B, en sniende dese twee linien malcander in of ontrent C, sulcx dat dē houck der afwijcking dier twee platten is A C D, en A D sy de lini vanden Eertcloot an A, tot Saturnuswechs uyterste noortsche punt der awijcking doēde deur het 21 voorstel 85610, wesende daer de lini K D: Sgelijcx sy B E de lini vanden Eertcloot an B, tot Saturnuswechs uyterste zuytsche punt der afwijcking E, doende deur het selve 21 voorstel 78849, welcke daer beteyckent is met F B, daer na A B an weder sijden verlangt wesende tot F en G, soo sijn A F en B G int plat des duysteraers, en Saturnus meeste noordersche breede sy dē houck F A D, doende deur het 5 lidt des 20 voorstels 3 tr. 2 ①, maer sijn meeste zuydersche breede dē houck G B E, doende deur het 2 lidt des 20 voorstels 3 tr. 5 ①.

T'BEGEERDE. Wy moeten vinden Saturnuswechs afwijcking vanden duysteraer, dats dē houck A C D, metsgaders hoe verre de duysteraer sne van des Eertclootwechs A B middelpunt C valt.

T' WERCK.

De cruysvierhouck A B D E heeft vijf bekende palen, te weten de sijde A B middellijn des Eertclootwechs 20000 deur t'ghestelde: Boven dien soo doet deur t'ghegheven A D 85610, B E 78849, den houck D A B 176 tr. 58 ①, want soo veel blijfter alsmen treckt F A D 3 tr. 2 ①, vant halfront, en den houck E B A 176 tr. 55 ①, want soo veel blijfter

The proof of which is evident from the procedure. CONCLUSION. We have thus found the orbital longitudes of the two points of the apparently greatest deviations of Saturn's orbit from the ecliptic, as well as the shortest distances from the Earth's orbit to these two points; also the length of the whole line from the one point of this greatest deviation to the other, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth; as required.

22nd PROPOSITION.

To find the deviation of Saturn's orbit from the ecliptic, as well as how far the line of nodes is from the centre of the Earth's orbit, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Let the line *AB* denote the Earth's orbit, seen transversely, which I draw equal to the diameter of the Earth's orbit *IK* in the figure of the 21st proposition; let *C* be the centre. Thereafter I draw this *DE* equal to that *DB* in the said figure of the 21st proposition, making 1,844,959, and this *CD* equal to that *FD,* this *CE* equal to that *FB,* and these two lines intersecting each other in or about *C,* so that the angle of the deviation between those two planes is *ACD,* then *AD* shall be the line from the Earth at *A* to the extreme northerly point of the deviation of Saturn's orbit, making by the 21st proposition 85,610, which there is the line *KD.* In the same way let *BE* be the line from the Earth at *B* to the extreme southerly point of the deviation of Saturn's orbit *E,* making by the said 21st proposition 78,849, which is there denoted by *FB.* Thereafter, *AB* being produced on either side to *F* and *G, AF* and *BG* are in the plane of the ecliptic, and Saturn's greatest northerly latitude shall be the angle *FAD,* making by the 1st [1]) section of the 20th proposition 3°2′, but its greatest southerly latitude the angle *GBE,* making by the 2nd section of the 20th proposition 3°5′.
WHAT IS REQUIRED. We have to find the deviation of Saturn's orbit from the ecliptic, *i.e.* the angle *ACD,* as well as how far the line of nodes is from the centre *C* of the Earth's orbit *AB.*

PROCEDURE.

The crossed quadrilateral *ABDE* has five known terms, to wit, the side *AB* (the diameter of the Earth's orbit) = 20,000 by the supposition; moreover, by the supposition *AD* makes 85,610, *BE* 78,849, the angle *DAB* 176°58′, for that is what is left when *FAD* = 3°2′ is subtracted from the semi-circle, and the angle *EBA* = 176°55′, for that is what is left when *GBE* = 3°5′ is subtracted

[1]) For 5 in the original read 1.

blijfter alsmen treckt G B E 3 tr. 5 ① vant halfront: Merct noch dat benevens de boveschreven vijf bekende palen, tot meerder gerief bekent sijn drie ander, te weten D E lini tusschen de twee punten der uyterste breeden van Saturnus, doende deur het 21 voorstel deses 3 boucx 1844959, de lini A E 98849, als evē ghenouch sijnde met A B 20000, en B E 78849 t'samen: Voort de lini B D 105610, als even genouch sijnde met A B 20000, en A D 85610 t'samen: Hier me ghesocht den houck A C D, wort bevonden deur het 6 voorstel inde Byvough der platte veelhoucken voor Saturnuswechs begeerde afwijcking vanden duysteraer 2 tr. 43 ①, waer voor *Ptolemeus* int 3 Hoofstick sijns 13 boucx al tastende vandt 2 tr. 26 ①.

Angaende de verheyt der duysteraersne van des Eertclootwechs middelpunt C, sy wort ghenouchsaem bevonden daer in te vallen, want onder verscheyden neminghen der palen daer t'werck deur gedaen can worden, soo viel my deur de ghene die ick nam den houck A D C van 19 ①, waer me des driehoucx D A C drie bekende palen, te weten de twee houcken A D C, D C A, en de sijde A D my uytbrochten de lini A C van 10294, t'welck te veel is 294, want om volcommen te wesen soudet sijn 10000, maer t'schilt soo weynich dat nemende voor den houck A D C 18 ①, in plaets van 19 ① als vooren, soo comt dan voor A C 9732, t'welck 268 weynigher is dan de volcommenheyt vereyscht, sulcx datment daer voor houden mach de duysteraersne te vallen deur des Eertclootwechs middelpunt C, overeencommende mette rekening die *Ptolemeus* op sijn spiegheling ghemaeckt heeft, aldaer vallende deur den vasten Eertcloot, want de selve in die stelling sulcx wat is als des Eertclootwechs middelpunt inde hare, ghelijck ick int 18 voorstel deses 3 boucx daer af wat verclaring ghedaen heb.

Merckt noch dat ander manier van wercking dan de voorgaende mach gedaen worden, met eerst te vinden de lini C D (dat is van des Eertclootwechs middelpunt tot Saturnus ter ghegheven plaets sijns wechs) deur het 6 lidt des 15 voorstels deses 3 boucx, want die bekent sijnde, soo heeft den driehouck A D C drie bekende palen, te weten benevens die C D noch de lini A D, en dē houck A C D, waer me d'ander onbekende palen ghevonden worden.

T'BESLVYT. Wy hebben dan gevonden Saturnuswechs afwijcking vanden duysteraer: Metsgaders hoe verre de duysteraersne van des Eertclootwechs middelpunt valt, in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet, deur wisconstighe wercking ghegront op stelling eens roerenden Eertcloots, na den eysch.

MERCK INHOVDENDE VERclaring dat Saturnus breedeloop seker ghetuychnis gheeft vant roersel des Eertcloots.

Alsmen met stelling eens vasten Eertcloots seght Saturnus inronts middelpunt te wesen ande duysteraersne, soo bevintmen hem metter daet altijt int plat des duysteraers sonder breede, tot wat plaets hy oock int inront sijn mocht deur het 3 lidt des 20 voorstels, t'welck voor een wonder gehouden sijnde, soo heeft *Ptolemeus* daer toe verdocht sulcke spiegheling als beschreven is int selve 20 voorstel, maer met dese stelling eens roerendē Eertcloots sietmē alles nootsakelick te moeten volghen uyt de eenvoudighe draeying van Saturnus hemel op haer as, sonder yet nieus of vreemts daer by te moeten versieren, en datmen sich met reden verwonderen mocht, soo ment anders bevonde, want commen-from the semi-circle. Note also that besides the above-mentioned five known terms for greater convenience three more are known, to wit, *DE*, the line between the two points of the extreme latitudes of Saturn, making by the 21st proposition of this 3rd book 1,844,959, the line *AE* = 98,849, as being sufficiently equal to *AB* = 20,000 and *BE* = 78,849 together. Further the line *BD* = 105,610 as being sufficiently equal to *AB* = 20,000 and *AD* = 85,610 together. When the angle *ACD* is sought therewith, by the 6th proposition in the Supplement of plane polygons [1]) there is found for [it, *i.e.* for] the required deviation of Saturn's orbit from the ecliptic 2°43′, for which *Ptolemy* in the 3rd Chapter of his 13th book tentatively found 2°26′.

As to the distance of the line of nodes from the centre of the Earth's orbit *C*, it is found substantially to fall therein, because among several assumptions of the terms by means of which the operation can be performed I found among those I took the angle *ADC* equal to 19′, by means of which the three known terms of the triangle *DAC*, to wit, the two angles *ADC, DCA* and the side *AD*, yielded me the line *AC* = 10,294, which is 294 too much, for in order to be perfect it would have to be 10,000; but the difference is so small that, taking the angle *ADC* equal to 18′ instead of 19′, as above, I get *AC* = 9,732, which is 268 less than perfection requires; so that it may be assumed that the line of nodes falls through the centre of the Earth's orbit *C*, which corresponds to the calculation performed by *Ptolemy* on the basis of his theory, where it falls through the fixed Earth, for the latter in that theory is the same as the centre of the Earth's orbit is in this theory (of the moving Earth), as I have explained in the 18th proposition of this 3rd book.

Note also that another method of operation than the preceding one may be used by first finding the line *CD* (*i.e.* from the centre of the Earth's orbit to Saturn at the given point of its orbit) by the 6th section of the 15th proposition of this 3rd book, for when that is known, the triangle *ADC* has three known terms, to wit, besides the said *CD* also the line *AD* and the angle *ACD*, by means of which the other unknown terms are found.

CONCLUSION. We have thus found the deviation of Saturn's orbit from the ecliptic, as well as how far the line of nodes falls from the centre of the Earth's orbit, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth; as required.

NOTE

consisting in a statement that Saturn's motion in latitude furnishes certain evidence of the motion of the Earth.

When it is said on the theory of a fixed Earth that the centre of Saturn's epicycle is in the line of nodes, it is indeed always found in the plane of the ecliptic without any latitude, no matter in what place it may be on the epicycle, by the 3rd section of the 20th proposition; and because this was considered a wonder, *Ptolemy* devised for it the theory that has been described in the said 20th proposition; but with the present theory of a moving Earth it is seen that everything must necessarily follow from the simple rotation of Saturn's heaven about its axis, without anything new or strange having to be invented; and that one would

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1) See p. 189, note 1.

de by voorbeelt ghesteyt, Saturnus van E tot dat hy is ande duysteraersne voor C, soo is hy dan int plat des duysteraers, en wantter den Eertcloot nummermeer buyten en loopt, soo volght daer uyt dat tot wat plaets haers wechs den selven Eertcloot dan is, soo en can Saturnus van daer niet dan inden duysteraer ghesien worden: En is onder anderen hier me het roersel des Eertcloots soo openbaer, datment by de ghene diet ontkennen, voor ghebreck van ervarentheyt houden mach.

23 VOORSTEL.

Te vinden de vvechlangde der tvvee uyterste punten vande duysteraersne, en vande tvvee uyterste punten der afvvijcking in Saturnusvvech, oock der lini die vã Saturnusvvechs middelpunt op de duysteraersne rechthouckich valt in sulcke deelen alsser des Eertclootvvechs halfmiddellijn 10000 doet, deur vviseonstige vvercking ghegront op stelling eens roerenden Eertcloots.

T'GHEGHEVEN. Anghesien ick om mijn voornemen wel te verclaren hier soude moeten verteyckenen de form des 21 voorstels, om daer noch by te vervougẽ t'gene in dit voorstel vereyscht is, soo sal ick de voorschrevẽ form des 21 voorstels self daer toe gebruyckẽ, tot welcken einde ick aldus segh: Nadien de duysteraersne deur des Eertclootwechs middelpunt streckt, volgens t'inhout des 22 voorstels, soo treck ick inde form vant 21 voorstel deur des Eertclootwechs middelpunt F de lini L M, als duysteraersne rechthouckich op D B, van wiens twee uyterste punten L, M, ick treck de twee linien L E en M E: Treck daer na deur t'punt E Saturnuswechs middellijn N O evewijdeghe met D B, en sniende L M in P. T'BEGHEERDE. Wy moeten vinden de wechlangden van L, M, wesende de twee uyterste punten vande duysteraersne L M; Sghelijcx de wechlangde van N O, wesende de uyterste punten der afwijcking in Saturnuswech, ende E P langde der lini die van Saturnuswechs middelpunt E op de duysteraersne L M rechthouckich comt.

T'WERCK.

Vanden rechthouck L F D doende 90 tr. ghetrocken den houck A F D, doende deur het 1 lidt des 20 voorstels 50 tr. blijft voor den houck L F E — 40 tr.

De driehouck L F E heeft drie bekende palen, te weten L E als halfmiddellijn des wechs 92308, de uytmiddelpunticheytlijn E F 5256 deur het Byeenvoughsel vant 13 voorstel deses 3 boucx, en den houck L F E 40 tr. eerste in d'oirden: Hier me ghesocht den houck F L E, wort bevonden deur het 5 voorstel der platte driehoucken van — 2 tr. 6.

Welcke vergaert totte 40 tr. des houcx L F E, comt voor den houck A E L, of booch A L, als begheerde wechlangde vant een punt der duysteraersne L — 42 tr. 6.

Maer om te vindẽ de wechlangde vant ander punt M, t'is kennelick den houck F M E, even te wesen metten houck F L E, en te doen

als die

wonder with reason if one found it otherwise, for when *e.g.* Saturn moves from *E* until it is in the line of nodes in front of *C*, it is in the plane of the ecliptic, and because the Earth never moves outside it, it follows that no matter in what place of its orbit the said Earth then is, Saturn cannot thence be seen anywhere but in the ecliptic. And from this, among other reasons, the motion of the Earth is so evident that those who deny it are to be considered lacking in experience.

23rd PROPOSITION.

To find the orbital longitudes of the two extremities of the line of nodes, and of the two extremities of the deviation in Saturn's orbit, also of the line which from the centre of Saturn's orbit is dropped perpendicular to the line of nodes, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Since in order to explain my intention properly I should have to draw anew the figure of the 21st proposition and add thereto what is required in the present proposition, I will use the aforesaid figure of the 21st proposition itself, to which end I say as follows: Since the line of nodes passes through the centre of the Earth's orbit, according to the contents of the 22nd proposition, I draw in the figure of the 21st proposition, through the centre of the Earth's orbit *F*, the line *LM* as the line of nodes at right angles to *DB*, from whose two extremities *L*, *M* I draw the two lines *LE* and *ME*. I then draw through the point *E* the diameter of Saturn's orbit *NO* parallel to *DB* and intersecting *LM* in *P*. WHAT IS REQUIRED. We have to find the orbital longitudes of *L*, *M*, which are the two extremities of the line of nodes *LM*; also the orbital longitudes of *N*, *O*, which are the extremities of the deviation in Saturn's orbit, and *EP*, the length of the line which from the centre of Saturn's orbit *E* is dropped perpendicular to the line of nodes *LM*.

PROCEDURE.

When from the right angle *LFD*, making 90°, is subtracted the angle *AFD*, making by the 1st section of the 20th proposition 50°, there is left for the angle *LFE* 40°

The triangle *LFE* has three known terms, to wit, *LE* (as the semi-diameter of the orbit) = 92,308, the line of eccentricity *EF* = 5,256 by the Compilation of the 13th proposition of this 3rd book, and the angle *LFE* = 40° (the first in the present list); when the angle *FLE* is sought therewith, it is found by the 5th proposition of plane triangles [1]) to be 2°6′

When this is added to the 40° of the angle *LFE*, I get for the angle *AEL* or arc *AL*, as the required orbital longitude of the one point of the line of nodes *L*, 42°6′

But in order to find the orbital longitude of the other point *M*, it is obvious that the angle *FME* is equal to the angle *FLE* and, like the latter, makes 2°6′; when this is subtracted from the angle *CFM* which, being the opposite angle to *LFE* = 40° (the first in the present list), has the same value, there is left for the angle *FEM*, that is also *CEM*, or for the arc *CM*, 37°54′. When this is added to the semi-circle *ABC* = 180°, the required orbital longitude of the point *M* is 217°54′

[1]) See p. 223, note 2.

als die 2 tr. 6 ①, de selve ghetrocken vanden houck C F M, die als teghenoverhouck van L F E 40 tr. eerste in d'oirden oock soo veel doet, blijft voor den houck F E M, dats oock C E M, of voor de booch C M 37 tr. 54 ①: De selve vergaert tottet halfrondt A B C 180 tr, comt voor begheerde wechlangde des punts M 217 tr. 54.

Maer om nu te vinden de wechlangde vande twee uyterste punten der afwijcking in Saturnuswech C, (t'welck om bekende redenen niet en sijn de twee puntē B, D, hoewel Saturnus daer uyt dē Eertcloot inde aldergrootste schijnbaer duysteraerbreede can ghesien wordē, maer N, O) ick segh aldus: Angesien dē houck E D F doet deur het 21 voorstel 2 tr. 30 ①, en dat dē houck N E D evē is an E D F, om dat E D is tusschen de twee evewijdegen E N, F D, soo doet dē houck E N D, of de booch D N oock 2 tr. 30 ①, die vergaert totte wechlangde des punts D, doende deur het 21 voorstel 307 tr. 30 ①, comt voor begheerde wechlangde des punts N 310 tr.

Ende O teghenoverpunt van dien moet sijn in des wechs 130 tr.

Om nu te vinden de lini E P, soo heeft daer toe den driehouck E L P drie bekende palen, te weten de halfmiddellijn E L 92308 deur de Byeenvougingh van het 13 voorstel deses 3 boucx, den houck E L P 2 tr. 6 ① tweede in d'oirden, en den houck E P L recht: Hier me ghesocht de sijde E P, wort bevonden deur het 4 voorstel der platte driehoucken voor begheerde lini van 3378.

Waer af t'bewijs deur t'werck openbaer is. T'BESLVYT. Wy hebben dan ghevonden de wechlangde der twee uyterste punten vande duysteraersne, en vande twee uyterste punten der afwijcking in Saturnuswech, oock der lini die vā Saturnuswechs middelpunt op de duysteraersne rechthouckich valt, in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet, deur wisconstighe wercking ghegront op stelling eens roerenden Eertcloots, na den eysch.

VERVOLGH.

T'is kennelick dat Saturnus in sijn wech vanden 42 tr. 6 ① derde in d'oirden, dats inde form des 21 voorstels van L over B tot M, altijt Zuyderlick moet wesen, maer vanden 217 tr. 54 ① vierde in d'oirden, totten 42 tr. 6 ①, dats van M over A tot L altijt Noorderlick: Om t'welck noch opentlicker te verclaren, merckt dat de voorschreven booch van L over B tot M doende 217 tr. 54 ①, inde form des 22 voorstels anghewesen is deur de lini C F, waer me t'selve rontsdeel L B M beteyckent wort overcant ghesien te wesen: T'welck soo sijnde, Saturnus en can in C E der form des 22 voorstels tot sulcken plaets niet wesen, dat hy ghesien uyt den Eertcloot tot wat plaets die oock in haer wech A B mocht wesen, anders verschijne dan op de Zuytsijde des duysteraers F G: En sghelijcx en can hy int deel C D der form des 22 voorstels (beteyckenende het boveschreven rontsdeel M A L der form des 21 voorstels) tot sulckē plaets niet wesen, dat hy ghesien uyt den Eertcloot, tot wat plaets die oock in haer wech A B mocht wesen, anders verschijne dan op de Noortsijde des duysteraers F G.

24 VOORSTEL.

Te vinden de langde der lini die van een gegeven punt in Saturnusvvech rechthouckich valt opt plat des duy-

steraers

But in order now to find the orbital longitudes of the two extremities of the deviation in Saturn's orbit *C* (which for known reasons are not the two points *B, D,* though Saturn can there be seen from the Earth at the greatest apparent ecliptical latitude, but *N, O),* I say as follows: Since the angle *EDF* by the 21st proposition makes 2°30′, and the angle *NED* is equal to *EDF,* because *ED* is situated between the two parallel lines *EN, FD,* the angle *NED* [1] or the arc *DN* also makes 2°30′.
When this is added to the orbital longitude of the point *D,* which by the 21st proposition makes 307°30′, the required orbital longitude of the point *N* becomes 310°

And *O,* the opposite point thereto, must be in the orbit at 130°

In order now to find the line *EP,* the triangle *ELP* has three known terms, to wit, the semi-diameter *EL* = 92,308 by the Compilation of the 13th proposition of this 3rd book, the angle *ELP* = 2°6′ (the second in the present list), and the angle *EPL* right. When the side *EP* is sought therewith, the required line is found, by the 4th proposition of plane triangles [2], to be 3,378

The proof of which is evident from the procedure. CONCLUSION. We have thus found the orbital longitudes of the two extremities of the line of nodes, and of the two extremities of the deviation in Saturn's orbit, also of the line which from the centre of Saturn's orbit is dropped perpendicular to the line of nodes, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth; as required.

SEQUEL.

It is obvious that Saturn in its orbit from longitude 42°6′ (the third in the present list), that is in the figure of the 21st proposition from *L via B* to *M* (*i.e.* to 217°54′), must always be on the South side, but from longitude 217°54′ (the fourth in the present list) to longitude 42°6′, that is from *M via A* to *L,* always on the North side. In order to explain this even more plainly, note that the aforesaid arc from *L via B* to *M,* making 175°48′ [3], in the figure of the 22nd proposition is designated by the line *CE* [4], by which this part of the circle *LBM* is denoted to be seen transversely. This being so, Saturn cannot in *CE* of the figure of the 22nd proposition be in such a place that, when seen from the Earth — no matter in what place the latter may be in its orbit *AB* — it appears anywhere but on the South side of the ecliptic *FG.* And in the same way, in the part *CD* of the figure of the 22nd proposition (denoting the above-mentioned part of the circle *MAL* of the figure of the 21st proposition) it cannot be in such a place that, when seen from the Earth — no matter in what place the latter may be in its orbit *AB* — it appears anywhere but on the North side of the ecliptic *FG.*

24th PROPOSITION.

To find the length of the line which from a given point in Saturn's orbit is dropped perpendicular to the plane of the ecliptic, in such parts as the semi-

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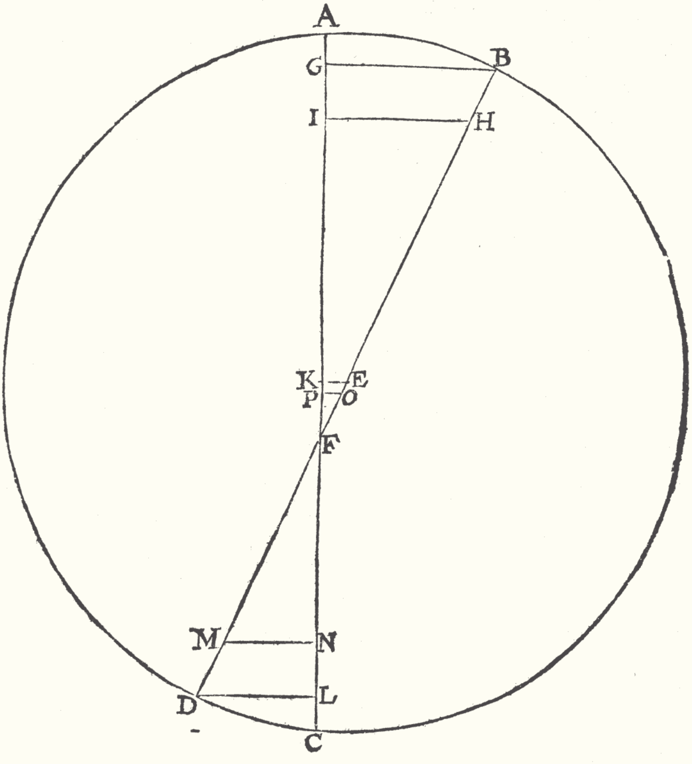
1) For *END* in the Dutch text read *NED.*
2) Stevin's *Trigonometry* (Work XI; i, 12), p. 147.
3) For 217°54′ in the Dutch text read 175°48′.
4) For *CF* in the Dutch text read *CE.*

ſteraers, in ſulcke deelen alſſer des Eertclootvvechs halfmiddellijn 10000 doet, deur vviſconſtighe vverckinggegront op ſtelling eens roerenden Eertcloots.

T'GHEGHEVEN. Laet A B C D Saturnus hemel ſijn, diens middelpunt E, waer deur ghetrocken is de rechte B E D (van gedaente als N E O inde form des 21 voorſtels) beteyckenende Saturnuswech overcant geſien, daer in t'punt



F ſy de duyſteraerſne oock overcant gheſien, ſoo dat E F (even an E P int 21 voorſtel) is de lini die van Saturnuswechs middelpunt op de duyſteraerſne rechthouckich comt, doende deur het 23 voorſtel 3378, daer na ghetrocken deur t'punt F de rechte A F C, bediende t'plat des duyſteraers overcant gheſien, ende den houck B F A, weſende de afwijcking des wechs van t'plat des duyſteraers doet deur het 22 voorſtel deſes 3 boucx 2 tr. 43 ①. T'WERCK.

1 VOORBEELT.

Ghenomen ten eerſten dat hier inghevonden moet ſijn de langde der lini B G, die van des wechs meeſte afwijcking B, rechthouckich valt opt plat des duyſteraers A C. Om daer toe te commen ick ſegh des wechs halfmiddellijn E B te doen 92308 deur het Byeenvoughſel vant 13 voorſtel deſes 3 boucx, en F E 3378 deur t'ghegheven, welcke vergaert maken t'ſamen 95686 voor de lini F B, waer me de driehouck B F G drie bekende palen heeft, te weten de ſijde

FB

diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Let *ABCD* be Saturn's Heaven, whose centre is E, through which has been drawn the straight line *BED* (of the same character as *NEO* in the figure of the 21st proposition), designating Saturn's orbit, seen transversely, wherein the point *F* shall be the line of nodes, also seen transversely, so that *EF* (equal to EP in the 21st proposition) is the line which from the centre of Saturn's orbit is dropped perpendicular to the line of nodes, by the 23rd proposition making 3,378. Thereafter through the point *F* is drawn the straight line *AFC,* denoting the plane of the ecliptic seen transversely, and the angle *BFA,* which is the deviation of the orbit from the plane of the ecliptic, by the 22nd proposition of this 3rd book makes 2°43′.

PROCEDURE.

1st EXAMPLE.

Let it firstly be assumed that in this has to be found the length of the line *BG,* which from the greatest deviation of the orbit *B* is dropped perpendicular to the plane of the ecliptic *AC.* In order to come to this, I say that the semidiameter of the orbit *EB* makes 92,308 by the Compilation of the 13th proposition of this 3rd book, and *FE* 3,378 by the supposition, which when added together make 95,686 for the line *FB,* so that the triangle *BFG* has three known terms, to wit, the side *FB* = 95,686, the angle *BFG* = 2°43′, and

FB 95686, den houck B F G 2 tr. 43 ①, en den houck B G F recht deur t'ghegheven: Hier me gheſocht de ſijde B G, wort bevonden deur het 4 voorſtel der platte driehoucken voor t'begheerde van 4535.

2 VOORBEELT.

Ghenomen ten tweeden dat gevonden moet ſijn de lini H I, commende uyt des wechs 30 trap, van t'punt der uyterſte afwijcking B ghetelt, maer want die lini even is ande lini dieder valt uyt des wechs halfmiddellijn E B, van t'punt H als uyterſte des houckmaet pijls B H, van 30 tr. ſoo laet ons E B anſien voor wechs halfmiddellijn, waer in E H ſal ſijn houckmaet van 60 tr, dats van t'vierendeelrontſchil der ghegeven 30 tr. doende die houckmaet 8660. Dit ſoo ſijnde ick ſegh aldus: Doende de halfmiddellijn E B 10000, ſoo doet E H 8660, wat ſal E H doen weſende E B gheſtelt op 92308? comt alſdan voor E H 79939, daer toe vergaert E F doende deur t'ghegeven 3378, comt voor F H 83317, waer me de driehouck H F I drie bekende palen heeft, te weten de ſijde F H 83317, den houck H F I 2 tr. 43 ①, en den houck H I F recht: Hier me gheſocht de ſijde H I, wort bevonden deur het 4 voorſtel der platte driehoucken voor het begheerde van 3949. Eñ is kennelick H I te commen uyt Saturnuswechs 340 tr. want deſe B wort inde form des 21 voorſtels beteyckent met N, weſende in des wechs 310 tr. deur het 23 voorſtel, waer toe de 30 tr. comt als vooren 340 tr.

Merckt nu dat ſulcx als hier gheweeſt is de manier van t'vinden der lini H I tuſſchen E en B, ofte int halfront E B, alſoo ſalſe oock ſijn tot allen plaetſen int ſelve halfront.

3 VOORBEELT.

Ghenomen ten derden dat gevonden moet ſijn de lini E K, commende uyt des wechs 90 tr. van t'punt der uyterſte afwijcking D ghetelt. Om daer toe te commen, ick ſegh dat angheſien E F doet 3378 deur t'ghegeven, ſoo heeft den driehouck E F K drie bekende palen, te weten F E 3378, den houck E F K 2 tr. 43 ①, en den houck E K F recht: Hier me gheſocht de ſijde E K wort bevonden deur het 4 voorſtel der platte driehoucken voor t'begheerde van 160. Duſdanich dan gheweeſt hebbende den voortganck int halfront E B, wy ſullen nu derghelijcke doen int ander halfront E D.

4 VOORBEELT.

Ghenomen ten 4 dat ghevonden moet ſijn de langde der lini D L, die van des wechs meeſte Zuyderſche afwijcking D, rechthouckich valt opt plat des duyſteraers A C: Om daer toe te commen, ick ſegh des wechs halfmiddellijn E D te doen 92308 deur het Byeenvougſel vant 13 voorſtel deſes 3 boucx, en F E 3378 deur t'ghegheven, welcke ghetrocken vande 92308, blijft 88930, voor de lini F D, waer me de driehouck D F L drie bekende palen heeft, te weten de ſijde F D 88930, den houck D F L 2 tr. 43 ①, en den houck D L F recht deur t'ghegheven: Hier me gheſocht de ſijde D L, wort bevonden deur het 4 voorſtel der platte driehoucken voor t'begheerde van 4215.

5 VOORBEELT.

Ghenomen ten vijfden dat ghevonden moet ſijn de lini M N, commende uyt des wechs 30 trap van t'punt der uyterſte afwijcking D ghetelt, maer want die lini the angle *BGF* right by the supposition. When the side *BG* is sought therewith, the required value is found by the 4th proposition of plane triangles to be 4,535.

2nd EXAMPLE.

Let it secondly be assumed that the line *HI* has to be found, which proceeds from 30° of the orbit, taken from the point of the greatest deviation *B;* but because this line is equal to the line which extends from the semi-diameter of the orbit *EB,* from the point *H* as extremity of the versed sine *BH* of 30°, let us regard *EB* as the semi-diameter of the orbit, in which *EH* shall be the sine of 60°, *i.e.* the complement of the given 30°, said sine making 8,660. This being so, I say as follows: When the semi-diameter *EB* makes 10,000, *EH* makes 8,660. What will *EH* be when *EB* is taken 92,308? Then *EH* becomes 79,939. When to this is added *EF,* making by the supposition 3,378, *FH* becomes 83,317, so that the triangle *HFI* has three known terms, to wit, the side *FH* = 83,317, the angle *HFI* = 2°43′, and the angle *HIF* right. When the side *HI* is sought therewith, the required value is found by the 4th proposition of plane triangles [1]) to be 3,949. Then it is obvious that *HI* proceeds from 340° of Saturn's orbit, for this *B* in the figure of the 21st proposition is denoted by *N,* which is situated at 310° of the orbit by the 23rd proposition; when to this is added the 30°, the required value is 340°, as above.

Now note that such as here has been the method of finding the line *HI* between *E* and *B,* or in the semi-circle *EB,* the same it will also be in any place in this semi-circle.

3rd EXAMPLE.

Let it thirdly be assumed that the line *EK* has to be found, which proceeds from 90° of the orbit, taken from the point of the greatest deviation *D.* In order to come to this, I say that since *EF* makes 3,378 by the supposition, the triangle *EFK* has three known terms, to wit, *FE* = 3,378, the angle *EFK* = 2°43′, and the angle *EKF* right. When the side *EK* is sought therewith, the required value is found by the 4th proposition of plane triangles to be 160. The procedure in the semi-circle *EB* having been thus, we shall now do the same in the other semi-circle *ED.*

4th EXAMPLE.

Let it fourthly be assumed that the length of the line *DL* has to be found, which from the greatest Southerly deviation of the orbit *D* is dropped perpendicular to the plane of the ecliptic *AC.* In order to come to this, I say that the semi-diameter of the orbit *ED* makes 92,308 by the Compilation of the 13th proposition of this 3rd book, and *FE* 3,378 by the supposition. When the latter is subtracted from the 92,308, there is left 88,930 for the line *FD,* so that the triangle *DFL* has three known terms, to wit, the side *FD* = 88,930, the angle *DFL* = 2°43′, and the angle *DLF* right by the supposition. When the side *DL* is sought therewith, the required value is found by the 4th proposition of plane triangles to be 4,215.

5th EXAMPLE.

Let it fifthly be assumed that the line *MN* has to be found, which proceeds from 30° of the orbit, taken from the point of the greatest deviation *D,*

[1]) Stevin's *Trigonometry* (Work XI; i, 12), p. 147.

die lini even is ande lini dieder valt uyt des wechs halfmiddellijn, van t'punt M als uyterste des houckmaetpijls D M van 30 tr. soo laet ons E D ansien voor wechs halfmiddellijn, waer in E M sal sijn houckmaet van 60 tr. dats van t'vierendeelrontschil der ghegeven 30 tr. doende die houckmaet 8660: Dit soo sijnde ick segh aldus: Doende de halfmiddellijn E D 10000, soo doet E M 8660, wat sal E M doen wesende E D ghestelt op 92308? comt alsdan voor E M 79939, daer af ghetrocken E F doende deur t'ghegheven 3378, blijft voor F M 76561, waer me de driehouck M F N drie bekende palen heeft, te weten de sijde F M 76561, den houck M F N 2 tr. 43 ①, en den houck M N F recht: Hier me ghesocht de sijde M N, wort bevonden deur het 4 voorstel der platte driehoucken voor t'begheerde van 3629.

Merckt nu dat sulcx als hier gheweest heeft de manier van t'vinden der lini M N tusschen F en D, alsoo salse oock sijn tot allen plaetsen tusschen F en D.

6 VOORBEELT.

Ghenomen ten sesten dat ghevonden moet sijn de lini O P, commende uyt des wechs 89 tr. van t'punt der uyterste afwijcking D ghetelt: Maer want die lini even is ande lini dieder valt uyt des wechs halfmiddellijn E D van t'punt O als uyterste des houckmaetpijls D O van 89 tr. soo laet ons E D ansien voor wechs halfmiddellijn, waer in E O sal sijn houckmaet van 1 tr. dats van t'vierendeelrontschil der ghegeven 89 tr. wesende die houckmaet van 175. Dit soo sijnde ick segh doende de halfmiddellijn E D 10000, soo doet E O 175, wat sal E O doen wesende E D ghestelt op 92308? comt alsdan voor E O 1615, die getrocken van E F doende deur t'ghegheven 3378, blijft voor O F 1763, waer me de driehouck O P F drie bekende palen heeft, te weten de sijde F O 1763, den houck O F P 2 tr. 43 ①, en den houck O P F recht: Hier me ghesocht de sijde O P wort bevonden deur het 4 voorstel der platte driehoucken voor t'begeerde van 84.

Merckt noch dat sulcx als hier gheweest is de manier van t'vinden der lini O P tusschen E en F, alsoo salse oock sijn tot allen plaetsen tusschen E en F.

T'BESLVYT. Wy hebben dan gevonden de langde der lini die van een gegevē punt in Saturnuswech rechthouckich valt opt plat des duysteraers, in sulcke deelen alsser des Eertclootwechs halfmiddellijn 10000 doet, deur wisconstighe werckingh ghegront op stelling eens roerenden Eertcloots, na den eysch.

VERVOLGH.

T'is kennelick hoe datmen om de begheerde lini van Saturnuswech rechthouckich opt plat des duysteraers met lichticheyt te vinden, sal meughen maken een tafel dier linien van trap tot trap, vant verstepunt beginnende, als by voorbeelt om te hebbē de lini vallende alsoo van Saturnuswechs eersten trap, ick neem voor my de form des 21 voorstels, daer in teyckenende t'punt Q, alsoo dat van des wechs verstepunt A tot Q 1 tr. beteyckent, waer uyt volght N Q te doē 51 tr. (want de wechlangde van N doet deur het 23 voorstel 310 tr. wiens rontschil voor N A 50 tr. daer toe A Q 1 tr. comt als boven voor N Q 51 tr.) daerom alsmen deur dit 24 voorstel vindt de hanghende vanden 51 tr. van B afghetelt (ghelijck vooren mette hangende H I van 30 tr. gedaen wiert) men heeft t'begheerde, om dat die lini vallen sal uyt Saturnuswechs 1 tr. en alsoo met alle ander.

Merckt noch dat wanneer dese dinghen niet aldus voorbeeltsche wijse ghedaen en wordē, maer met ernst om op een toecommende tijt dadelick de breebut because this line is equal to the line which extends from the semi-diameter of the orbit, from the point *M* as extremity of the versed sine *DM* of 30°, let us regard *ED* as the semi-diameter of the orbit, in which *EM* shall be the sine of 60°, that is of the complement of the said 30°, said sine making 8,660. This being so, I say as follows: When the semi-diameter *ED* makes 10,000, *EM* makes 8,660. What will *EM* be when *ED* is taken 92,308? *EM* then becomes 79,939. When from this is subtracted *EF,* making by the supposition 3,378, there is left for *FM* 76,561, so that the triangle *MFN* has three known terms, to wit, the side $FM = 76{,}561$, the angle $MFN = 2°43'$, and the angle *MNF* right. When the side *MN* is sought therewith, the required value is found by the 4th proposition of plane triangles to be 3,629.

Now note that such as here has been the method of finding the line *MN* between *F* and *D,* the same it will also be in any place between *F* and *D.*

6th EXAMPLE.

Let it sixthly be assumed that the line *OP* has to be found, which proceeds from 89° of the orbit, taken from the point of the greatest deviation *D.* But because this line is equal to the line which extends from the semi-diameter of the orbit *ED,* from the point *O* as extremity of the versed sine *DO* of 89°, let us regard *ED* as the semi-diameter of the orbit, in which *EO* shall be the sine of 1°, that is of the complement of the given 89°, said sine making 175. This being so, I say: When the semi-diameter *ED* makes 10,000, *EO* makes 175. What will *EO* be when *ED* is taken 92,308? *EO* then becomes 1,615. When this is subtracted from *EF,* making by the supposition 3,378, there is left for *OF* 1,763, so that the triangle *OPF* has three known terms, to wit, the side $FO = 1{,}763$, the angle $OFP = 2°43'$, and the angle *OPF* right. When the side *OP* is sought therewith, the required value is found by the 4th proposition of plane triangles to be 84.

Note also that such as here has been the method of finding the line *OP* between *E* and *F,* the same it will also be in any place between *E* and *F.*

CONCLUSION. We have thus found the length of the line which from a given point in Saturn's orbit is dropped perpendicular to the plane of the ecliptic, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth; as required.

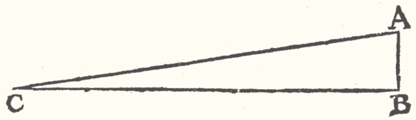
SEQUEL.

It is obvious that in order easily to find the required line from Saturn's orbit perpendicular to the plane of the ecliptic, it is possible to make a table of those lines from degree to degree, starting at the apogee. For example, in order to have the line thus extending from the first degree of Saturn's orbit, I take before me the figure of the 21st proposition and mark therein the point *Q* such that the distance from the orbit's apogee *A* to *Q* denotes 1°, from which it follows that *NQ* makes 51° (for the orbital longitude of *N* by the 23rd proposition makes 310°, whose difference from 360°, *NA,* makes 50°; when to this is added $AQ = 1°$, *NQ* becomes, as above, 51°). If therefore by this 24th proposition the perpendicular at 51° is found, taken from *B* (as has been done above for the perpendicular *HI* at 30°), the required value is obtained, because that line de te vinden, dattet noodich soude sijn deur dadelicke ervaring eerst te vinden waer nu des wechs verstepunts schijnbaer duysteraerlangde is: Ten anderẽ hoe de duysteraersne verloopen mach sijn sichtent *Ptolemeus* tijt (diens ervaringen ick voorbeeltsche wijse ghenomen hebbe om de redenen t'haerder plaets verclaert) want de Beschrijvers der dachtafels nemen al of den loop van Saturnus duysteraersne, even waer met des wechs verstepuntsloop altijt 50 tr. van malcander blijvende, maer daer soude nieuwe dadelicke gagheslaghen ervaring af behooren te wesen, want by aldien den loop des verstepuntsen des duysteraersnees van Saturnus verscheyden sijn, ghelijck vande Maen ghebeurt, soo mocht sulcx oirsaeck wesen waer deur sijn breeden niet dadelick soo bevonden en wierden, ghelijck rekeninghen der dachtafels mebrenghen.

25 VOORSTEL.

Te vinden Saturnus schijnbaer duysteraerbreede op een ghegeven tijt, deur vviſconſtighe vvercking gegront op stelling eens roerenden Eertcloots.

T'GHEGHEVEN. Laet Saturnus op den ghegheven tijt wesen in sijn wechs 340 tr. alwaer de lini van hem rechthouckich opt plat des duysteraers doẽ sal 3949 deur het 2 voorbeelt des 24 voorstels deses 3 boucx: En de verheyt van hem totten Eertcloot die gevonden wort na de manier verclaert int 6 lidt des 15 voorstels deses 3 boucx, wesende daer de lini XV, sy neem ick, van 80000. T'BEGHEERDE. Wy moeten hier me vinden Saturnus schijnbaer duysteraerbreede. T'BEREYTSEL. Ick teycken de rechthouckige driehouck A B C recht an B, doende de sijde A B de ghegheven 3949, en A C 80000.



T' WERCK.

De driehouck A B C heeft drie bekende palen, te weten de sijde A B 3949, de sijde A C 80000, en den houck B recht: Hier me ghesocht den houck A C B, wort bevonden deur het 5 voorstel der platte driehoucken voor de begheerde breede van 2 tr. 50 ①. Maer om nu te weten of die Zuydelick of Noordelick is, dat wijst sijn wechlangde, want deur t'vervolgh des 23 voorstels blijckt, dat die wesende tusschen den 44 tr. 12 ①, ende den 215 tr. 48 ①, sy is Zuydelick, maer inde rest des wechs (daer desen 340 tr. in valt) Noordelick. Waer af t'bewijs openbaer is. T'BESLVYT. Wy hebben dan gevonden Saturnus schijnbaer duysteraerbreede op een ghegheven tijt, deur wisconstighe werckinghe ghegront op stelling eens roerenden Eertcloots, na den eysch.

MERCKT.

Anghesien de reghel der ses voorstellen tot hier toe van Saturnus breede beschreven, gemeen is over de breeden van d'ander vier Dwaelders Iupiter, Mars, Venus en Mercurius, soo en sal ick daer op gheen gheformde voorstellen maken, ghelijck oock verhaelt is int Cortbegrijp deses 5 Onderscheyts, maer beschrijwill extend from the first degree of Saturn's orbit, and the same with all the others.

Note also that when these things are not done thus by way of example, but in earnest, in order to find practically the latitude at future times, it would be necessary first to find by practical experience where the apparent ecliptical longitude of the orbit's apogee is now situated. Secondly, how the line of nodes may have shifted since the days of *Ptolemy* (whose experiences I have taken by way of example for the reasons explained in their place), for the Describers of the ephemerides all assume the motion of Saturn's line of nodes to be equal to that of the orbit's apogee, always remaining 50° apart, but this ought to be ascertained by new direct observational experience, for if the motion of the apogee and that of Saturn's line of nodes are different, as is the case with the Moon, this might be the cause why its latitudes were not found in practice to be such as calculations of the ephemerides imply [1]).

25th PROPOSITION.

To find Saturn's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Let Saturn be at the given time at 340° of its orbit, where the line dropped perpendicular from it to the plane of the ecliptic shall be 3,949 by the 2nd example of the 24th proposition of this 3rd book. And the distance from it to the Earth, which is found in the manner explained in the 6th section of the 15th proposition of this 3rd book, which there is the line *XV*, I assume to be 80,000. WHAT IS REQUIRED. We have to find herewith Saturn's apparent ecliptical latitude. PRELIMINARY. I draw the right-angled triangle *ABC,* right-angled in *B,* the side *AB* making the said 3,949 and *AC* = 80,000.

PROCEDURE.

The triangle *ABC* has three known terms, to wit, the side *AB* = 3,949, the side *AC* = 80,000, and the angle *B* right. When the angle *ACB* is sought therewith, the required latitude is found by the 5th proposition of plane triangles to be 2°50′. But in order to know whether this is Southerly or Northerly, this is indicated by its orbital longitude, for by the sequel of the 23rd proposition it appears that when it is between 44°12′ and 215°48′ [2]), it is Southerly, but in the rest of the orbit (in which falls this 340°) it is Northerly. The proof of which is evident. CONCLUSION. We have thus found Saturn's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a moving Earth; as required.

NOTE.

Since the rule of the six propositions described so far for Saturn's latitude is common to the latitudes of the other four Planets Jupiter, Mars, Venus, and Mercury, I will not make any illustrated propositions thereon, as has also been

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[1]) *Cf.* footnote [3]) to p. 279.

[2]) The provenance of the longitudes 44°12′ and 215°48′ is not clear. The values are obtained when *twice* the angle 2°6′ is added to 40° and subtracted from 220°.

ſchrijven alleenelick by manier van vermaen t'gene breeder verclaring ſchijnt te vereyſſchen, beginnende met Iupiter als volght.

NV VAN IVPITERS BREEDE.

Ptolemeus heeft deur dadelicke gagheſlaghen ervaringhen de gedaenten van Iupiters breedeloop alſins bevonden gelijck van Saturnus int voorgaende geſeyt is, welcke int beſonder duſdanich ſijn:

Ten eerſten ſoo was ſijn meeſte breede op de Noortſijde van 2 tr. 4 Ⓘ, ghebeurende altijt als ſijn inronts middelpunt ſchijnbaerlick was 20 tr. voor ſijn wechs verſtepunt, te weten onder des duyſteraers 141 tr. en Iupiter an des inronts naeſtepunt, maer buyten het naeſtepunt weſende, ſoo was ſijn Noorderſche breede voor datmael cleender, en ten minſten doen hy alſoo an het verſtepunt was.

Ten tweeden ſoo bevant hy op de Zuytſijde de meeſte breede van 2 tr. 8 Ⓘ, ghebeurende altijt als ſijn inronts middelpunt was onder het teghenoverpunt des boveſchrevē 141 tr. dats onder des duyſteraers 321 tr. en Iupiter an ſijn inronts naeſtepunt: Maer buyten het naeſtepunt weſende, ſoo was ſijn Zuyderſche breede voor datmael cleender, en ten minſten doenmen hem an het verſtepunt vandt.

Ten derdē weſende het inronts middelpunt in een der duyſteraerſnees twee uyterſtē, ſo bevant hy Iupiter, gelijck oock van Saturnus geſeyt is, altijt int plat des duyſteraers ſonder breede, tot wat plaets des inronts hy oock weſen mocht.

De ſpiegeling by *Ptolemeus* hier op met ſtelling eens vaſten Eertcloots veroirdent, is van ghedaente teenemael gheweeſt als die van Saturnus, ſulcx dattet onnoodich ſchijnt de ſelve hier te verhalen: Ende want de ſpiegeling met ſtelling eens roerenden Eertcloots ande voorgaende van Saturnus oock ghelijck is, en daer deur verſtaen wort, ſoo en beſchrijf ick die niet int langhe.

NV VAN MARS BREEDE.

Ptolemeus heeft deur dadelicke gagheſlaghen ervaringhen de gedaenten van Mars breedeloop alſins bevonden ghelijck van Saturnus int voorgaende gheſeyt is.

Ten eerſten ſoo was ſijn meeſte breede op de Noortſijde van 4 tr. 21 Ⓘ, gebeurende altijt als ſijn inronts middelpunt was an ſijn wechs verſtepunt (het welcke deur de Byeenvougingh des 13 voorſtels is onder des duyſteraers 115 tr. 30 Ⓘ) en Mars in des inronts naeſtepunt, maer buyten het naeſtepunt weſende, ſoo was ſijn Noorderſche breede voor datmael cleender, en ten minſtē doen hy alſoo ant verſtepunt was.

Ten tweeden ſoo bevant hy op de Zuytſijde de meeſte breede van 7 tr. 7 Ⓘ, gebeurende altijt als ſijn inronts middelpunt was in ſijn wechs naeſtepunt onder het teghenoverpunt des boveſchreven 115 tr. 30 Ⓘ, dats onder des duyſteraers 295 tr. 30 Ⓘ, en Mars an ſijn inronts naeſtepunt: Maer buyten het naeſtepunt weſende, ſoo was ſijn Zuyderſche breede voor dat mael cleender, en ten minſten doemen hem alſoo ant verſtepunt vandt.

Ten derdē weſende het inronts middelpunt an een der duyſteraerſnees twee uyterſten, ſoo bevant hy Mars, ghelijck oock van Saturnus gheſeyt is, altijt int plat said in the Summary of this 5th Chapter, but I will merely mention those things which seem to call for a fuller explanation, starting with Jupiter, as follows.

NOW OF JUPITER'S LATITUDE.

Ptolemy through direct observational experiences has found the character of Jupiter's motion in latitude to be in every respect as has been said of Saturn in the foregoing, which in particular is as follows.

Firstly, its greatest latitude on the North side was 2°4′, which always occurred when its epicycle's centre was apparently at 20° ahead of its orbit's apogee, to wit, at 141° of the ecliptic, and Jupiter at the epicycle's perigee; but when it was outside the perigee, its Northerly latitude was for that case smaller, and smallest when it was thus at the apogee.

Secondly, he found on the South side the greatest latitude of 2°8′, which always occurred when its epicycle's centre was in the point opposite to the above-mentioned 141°, *i.e.* at 321° of the ecliptic, and Jupiter was at its epicycle's perigee. But when it was outside the perigee, its Southerly latitude was for that case smaller, and smallest when at the apogee.

Thirdly, when the epicycle's centre was in one of the two extremities of the line of nodes, he always found Jupiter, as has also been said of Saturn, in the plane of the ecliptic without any latitude, no matter in what place of the epicycle it might be.

The theory that *Ptolemy* based on this, on the assumption of a fixed Earth, was entirely of the same character as that of Saturn, so that it seems unnecessary to relate it here. And because the theory on the assumption of a moving Earth is also equal to the preceding one of Saturn, and is understood therefrom, I shall not describe it in full.

NOW OF MARS' LATITUDE.

Ptolemy through direct observational experiences has found the character of Mars' motion in latitude to be in every respect as has been said of Saturn in the foregoing.

Firstly, its greatest latitude on the North side was 4°21′, which always occurred when its epicycle's centre was at its orbit's apogee (which by the Compilation of the 13th proposition is at 115°30′ of the ecliptic), and Mars in the epicycle's perigee; but when it was outside the perigee, its Northerly latitude was for that case smaller, and smallest when at the apogee.

Secondly, he found on the South side the greatest latitude of 7°7′, which always occurred when its epicycle's centre was at its orbit's perigee, the point opposite to the above-mentioned 115°30′, *i.e.* at 295°30′ of the ecliptic, and Mars at its epicycle's perigee. But when it was outside the perigee, its Southerly latitude was for that case smaller, and smallest when at the apogee.

Thirdly, when the epicycle's centre was at one of the two extremities of the line of nodes, he always found Mars, as has also been said of Saturn, in the plat des duysteraers sonder breede, tot wat plaets des inronts hy oock mocht wesen.

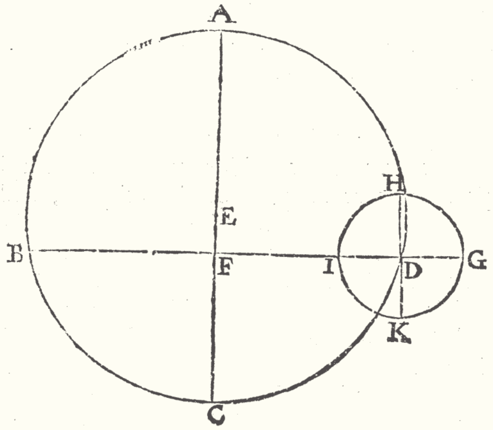
De spiegheling by *Ptolemeus* hier op met stelling eens vasten Eertcloots veroirdent, is van ghedaente teenemael gheweest als die van Saturnus, sulcx dattet onnoodich schijnt de selve hier te verhalen: Ende want de spiegeling met stelling eens roerenden Eertcloots, ande voorgaende van Saturnus oock ghelijck is, en daer deur verstaen wort, soo en beschrijf ick die niet int langhe: Doch sal ick hier verhalen dat in dit werck wat contheyt valt anders als in dat, deur dien Mars uyterste breeden an sijn wechs verstepunt en naestepunt ghebeuren, als gheseyt is, want anghesien de lini tusschen de selve twee punten bekent is deur de Byeenvouging des 13 voorstels deses 3 boucx, alwaer Marswechs halfmiddellijn staet op 15190, welcke tweemael ghenomen comt voor de selve lini 30380, soo en behoufmen die niet te soucken na de manier des 21 voorstels, noch oock als int 23 voorstel de wechlangde vande twee uyterste punten der afwijcking in Marswech, anghesien de selve het verstepunt en naestepunt sijn.

NV VAN MERCVRIVS BREEDE.

1 LIDT.

Inhoudende *Ptolemeus* dadelicke gagheslaghen ervaringhen van Mercuriusloop.

Om beuqamelicker te verclarē *Ptolemeus* dadelicke gageslagē ervaringen der gedaēte vā Mercurius breedeloop mette somme vā sijn spiegeling daer op veroirdent, so laet A B C dē inrontwech beteyckenen, diens middelpunt is E, dē vasten Eertcloot F, deur welcke en oock deur E ghetrocken sy de lini C F E A, soo dat A t'verstepunt is, C t'naestepunt, en deur F ghetrocken wesende de lini B F D rechthouckich op A C, so bedien de twee punten B, D, des wechs 90 tr. en 270 tr. en hoewel dat so veel verschilt als de uytmiddelpunticheyt E F veroirsaeckt, nochtans angesien *Ptolemeus* de selve twee punten den 90 en 270 noemt, wy sullenser voor nemen, daer na sy opt punt D als middelpunt beschreven het inront G H I K, diens verstepunt G, naestepunt I, en middelverheden H, K, tusschen welcke vier punten gheteyckent sijn de twee middellijnen I G, H K.



Dit aldus wesende, soo heeft *Ptolemeus* deur dadelicke ervaring bevonden dat Mercurius meeste breede op de zuytsijde was van 4 tr. 5 ①, ghebeurende altijt plane of the ecliptic without any latitude, no matter in what place of the epicycle it might be.

The theory that *Ptolemy* based on this on the assumption of a fixed Earth was entirely of the same character as that of Saturn, so that it seems unnecessary to describe it here. And because the theory on the assumption of a moving Earth is also equal to the preceding one of Saturn, and is understood therefrom, I shall not describe it in full. But I will relate that in this procedure some abridgement can be made different from that, because Mars' greatest latitudes occur at its orbit's apogee and perigee, as has been said, for since the line between these two points is known by the Compilation of the 13th proposition of this 3rd book, where the semi-diameter of Mars' orbit is given as 15,190, which, when taken twice, gives 30,380 for the said line, it need not be sought after the manner of the 21st proposition, nor, as in the 23rd proposition, the orbital longitudes of the two extremities of the deviation in Mars' orbit, since these are the apogee and the perigee.

NOW OF MERCURY'S LATITUDE.

1st SECTION.

Consisting in *Ptolemy's* practical observational experiences of Mercury's motion.

In order to explain more easily *Ptolemy's* practical observational experiences of the character of Mercury's motion in latitude, together with the summary of his theory based thereon, let *ABC* denote the deferent, whose centre is *E*, the fixed Earth *F*, through which and also through *E* let there be drawn the line *CFEA*, so that *A* is the apogee, *C* the perigee, and when through *F* is drawn the line *BFD* perpendicular to *AC*, the two points *B*, *D* denote 90° and 270° of the orbit, and though this differs as much as amounts to the eccentricity *EF*, yet since *Ptolemy* calls these two points 90° and 270°, we shall take them thus. Thereafter let there be described about the point *D* as centre the epicycle *GHIK*, whose apogee is *G* and perigee *I*, and points of medium distance *H*, *K*, between which four points are drawn the two diameters *IG*, *HK*.

This being so, *Ptolemy* found by direct experience that Mercury's greatest latitude on the South side was 4°5′, which always occurred when its epicycle's als sijn inronts middelpunt was 90 tr. van sijn wechs verstepunt, soo wel ter eender als ter ander sijde, dat is soo wel inden 270 tr. an D, als inden 90 an B (welcke 90 tr. is onder des duysteraers 280 tr. gemerckt deur de Byeenvouging des 13 voorstels deses 3 boucx, het verstepunt onder des duysteraers 190 tr. is) en Mercurius in des inronts naestepunt als an I: Maer buyten het naestepunt wesende, soo was sijn zuydersche breede voor datmael cleender, en ten minsten doen hy alsο ant verstepunt G was. D'oirsaeck waerom de boveschreven breeden even vallē als het inronts middelpunt is an D of B, blijckt inde form, om dat de lini vanden Eertcloot F tot D, even is ande lini van F tot B: Want daer uyt volght dattet inronts middelpunt an B wesende, sijn verstepunt en naestepunt in sulcken ghestalt uyt den Eertcloot F ghesien worden, als wanneert an D is.

Ten tweedē so bevant hy op de noortsijde de meeste breede van 1 tr. 45 ①, ghebeurende altijt als sijn inronts middelpunt was 90 tr. vā sijn wechs verstepunt, soo wel ter eender als ter ander sijde, en Mercurius an sijn inronts naestepunt, maer buyten het naestepunt wesende, soo was sijn noordersche breede voor datmael cleender, en ten minsten doemē hem alsoo ant verstepunt vandt.

2 LIDT.

Inhoudende *Ptolemeus* spiegheling ghetrocken uyt de voorgaende dadelicke ervaringhen des 1 lidts.

Anghesien *Ptolemeus* voorgaende ervaringhen, noch breeder connen verclaert worden deur sijn spieghelinghen die hy daer op verdocht heeft, soo sal ick de somme van dien hier by voughen als volght.

Nadien des inrontwechs twee punten die uyt den vasten Eertcloot F inde meeste breede ghesien worden sijn B en D, soo soude daer uyt volghen dat de ghemeene sne van die wech en den duysteraer, rechthouckich moet commen op B D, ende want *Ptolemeus* sich in aller Dwaelders breedeloopen voorstelde die te gaen deur den Eertcloot F, soo soude volgens die oirden de lini A F C de duysteraersne moeten sijn, en daerom het inronts middelpunt wesende an A of C, en Mercurius ant inronts verste of naestepunt, soo soude hy uyt den Eertcloot F moeten gesien worden int plat des duysteraers sonder breede: Maer de ervaring wees hem anders, want hy doē sijn breede altijt bevant vā 45 ① na t'Zuyden: Sulcx dat hy daerom (benevens noch een ander redē daer inden Byvough des breedeloops met stelling eens vasten Eertcloots af geseyt sal worden) Mercurius spiegheling anders veroirdende als vande drie bovenste Dwaelders, t'welck aldus toeginck: Hy heeft gheseyt de lini B D streckende deur de twee punten der uyterste afwijcking, te wesen de ghemeene sne des inrontwechs en duysteraers, teghen de natuerlicke reghel: Maer weerom ter ander sijde gaf den inrontwech en het inront seker drie seer versierlicke wagghelende roersels, die onderscheyts halven ghenoemt worden * afweging, * afwijcking, en * afkeering, waer me hy tot sijn voornemen gherocht, waer af ick oock van elck int besonder verhael sal doen, dat treckende uyt de bepalinghen die *Purbachius* tot dese plaets van dit roersel verstandelick doet.

Deviatio. *Declinatio.* *Reflectio.*

Deviatio. Om dan mette * afweging te beginnen, soo is te weten dat *Ptolemeus* den inrontwech gheseyt heeft op B D als as een wagghelende roersel te hebben overentweer gaende of waggelende als de balck eens waeghs, ghenoemt afweging, sulcx dattet inronts middelpunt wesende an B of D, soo en isser gheen afweging, centre was at 90° from its orbit's apogee, both on one side and on the other, *i.e.* both at 270° in *D* and at 90° in *B* (which 90° is at 280° of the ecliptic, seeing that by the Compilation of the 13th proposition of this 3rd book the apogee is at 190° of the ecliptic), and Mercury was at the epicycle's perigee, namely at *I*. But when it was outside the perigee, its Southerly latitude was for that case smaller, and smallest when at the apogee *G*. The cause why the above-mentioned latitudes are equal when the epicycle's centre is at *D* or *B* appears from the figure, because the line from the Earth *F* to *D* is equal to the line from *F* to *B*. For from this it follows that when the epicycle's centre is at *B*, its apogee and perigee are seen from the Earth *F* in the same manner as when it is at *D*.

Secondly, he found on the North side the greatest latitude of 1°45′, which always occurred when its epicycle's centre was at 90° from its orbit's apogee, both on one side and on the other, and Mercury at its epicycle's perigee; but when it was outside the perigee, its Northerly latitude was for that case smaller, and smallest when at the apogee.

2nd SECTION.

Consisting in *Ptolemy's* theory derived from the foregoing practical experiences of the 1st section.

Since *Ptolemy's* foregoing experiences can be explained even more fully by means of the theories he has conceived thereon, I will here add the summary thereof, as follows.

Since the two points of the deferent which from the fixed Earth *F* are seen at the greatest latitude are *B* and *D*, it would follow therefrom that the intersection of that deferent and the ecliptic must be perpendicular to *BD*, and because *Ptolemy*, in the case of all the Planets' motions in latitude, imagined them to pass through the Earth *F*, according to this rule the line *AFC* would have to be the line of nodes, and therefore, if the epicycle's centre were at *A* or *C*, and Mercury at the epicycle's apogee or perigee, it would have to be seen from the Earth *F* in the plane of the ecliptic without any latitude. But experience taught him differently, for he then always found its latitude to be 45′ towards the South, so that he therefore (apart from another reason, which is to be dealt with in the Supplement of the motion in latitude on the assumption of a fixed Earth) framed Mercury's theory differently from that of the three upper Planets, in the following way. He said that the line *BD* passing through the two points of the greatest deviation was the intersection of the deferent and the ecliptic, contrary to the natural rule. But on the other hand again he gave the deferent and the epicycle three highly artificial oscillatory motions, which for the sake of distinction are called deviation, declination, and deflection [1]), with which he achieved his intention, of which I will also give an account for each in particular, taking

[1]) The term "deviation" is used here in a more restricted sense than previously, *e.g.* page 75, line 5 and further on, where it was the translation of *afwycking* in general. The name for the second oscillatory motion of Mercury, *afwycking*, has been translated "declination", in accordance with the Latin term in the margin, though this word has here quite a different meaning from that of the later equatorial co-ordinate, which in Dutch books is often denoted by the same word. For the corresponding verb, *afneyghen* is sometimes used; this is translated here by "to diverge". For the Latin *reflectio*, here indicating bending back, we have taken "deflection" as the best English equivalent.

ging, maer is des wechs heel plat int plat des duysteraers: Daer na het inronts middelpunt van B of D vertreckende, soo begint den wech af te weghen, of te neyghen altijt na het Zuyden, en vermeerdert dese afweging gheduerlick tot dattet inronts middelpunt ghecommen is ant verstepunt A, of naestepunt C, en dan is de afweging ten grootsten vande boveschreven 45 ①; Welck daer na weerom vermindert, tot dattet inronts middelpunt ghecommen is an B, of D, alwaer dan weerom gheen afweging en is. En blijckt hier uyt dattet roersel t'welck het inronts middelpunt vande afweging ontfangt nummermeer op de noortsijde en gheschiet.

Maer om nu vande * afwijcking te segghen, soo is te weten dattet plat des inronts deur een waggelende roersel afneyght vant plat sijns wechs op tweederley wijse: Ten eersten met afwijcking op de middellijn H K, streckende deur de middelverhedē H, K: Met dit roersel gebeuret dat de middellijns I G verstepunt G, op d'een sijde des wechs afwijckt, en t'naestepunt I op d'ander. Dese afwijcking houdt dusdanige reghel: Wanneer des inronts middelpunt als D, is an des wechs verstepunt A, de voorschrevē middellijn I G is in t'plat des wechs: Maer het inronts middelpunt van t'verstepunt A vertreckende, so begint des inronts verstepunt af te wijcken na het Zuyden, ende het naestepunt na het Noorden, welcke afwijcking geduerlick vermeerdert tot dat des inronts middelpunt ande middelverheyt B gecommen is, en alsdan gebeurt de middellijns als I G meeste afwijcking, welcke daer na gheduerlick vermindert, tot dattet middelpunt des inronts is an des wechs naestepunt C, alwaer de voorschrevē middellijn als I G wederom geen afwijcking en heeft, maer het inronts middelpunt van C vertreckende na de middelverheyt D, het verstepunt als G begint af te wijcken van t'plat des wechs na het Noorden, en het naestepunt gelijck I na het Zuyden, en vermeerdert die afwijcking geduerlick tot dat des inronts middelpunt ant uyterste D gecommen is, alwaer die afwijcking weerom ten grootsten wort. Van daer voort vermindertse tot dattet inronts middelpunt an t'verstepunt A is, alwaer gelijck int begin de voorschrevē middellijn als I G, weerom int plat des wechs is: En daer na volgt weerom d'eerste gestalt. Vyt t'gene geseyt is blijct dat des wechs afweging tē grootstē sijnde, so heeft het inront geen afwijcking, maer de wech sonder afweging wesende, dat dā des inronts afwijcking tē grootstē is. *Declinatione.*

Nu rester noch de * afkeering te verclaren welcke dusdanich is: Des inronts plat heeft boven de voorschreven waggeling op den as H K, noch een ander op den as I G tusschen des inronts verstepunt en naestepunt (ghenouchsaem toegaende gelijck de tweederley waggeling die het zeecompas ontfangt, met twee assen op malcander rechthouckich commende) deur welck roersel gebeurt dat de middellijn H K het plat des wechs deursnijt inde lini I G als gemeene sne, so dat des inronts slincker helft op d'een sijde des inrontwechs, de rechter helft op d'ander sijde afkeert: Dese middellijns afkeering houdt dusdanige regel: Wanneer des inronts middelpunt is ande middelverheyt D, soo en heeft de voorschreven middellijn H K geen afkeering vande wech A B C D, dan is int plat der selve: Maer het inronts middelpunt van D vertreckende na het verstepunt A, soo begint de middellijns slincker helft H D af te keeren na het Zuyden, de ander helft na het Noorden, welcke afkeering geduerlick vermeerdert tot dattet inronts middelpunt ant verstepunt A ghecommen is, alsdan ten grootsten wesende. Daer na voortgaende na het ander punt B soo vermindertse weerom tot datse an gecommen is, alwaer weerom geen afkeering en ghescheit. Maer het inronts middelpunt van die plaets vertreckende na het naestepunt C, so begint de voorschreven slincker helft ghelijck H D weerom af te keeren na het *Reflectio.*

 Noorthis from the description which *Purbachius* gives of this motion in a comprehensible manner in this place.

To begin with the deviation, it is to be noted that *Ptolemy* has said that the deferent has about *BD* as axis an oscillatory motion, going to and fro or oscillating like the beam of a balance, called deviation, so that when the epicycle's centre is at *B* or *D,* there is no deviation, but the entire plane of the orbit is in the plane of the ecliptic. Thereafter, when the epicycle's centre starts from *B* or *D,* the orbit begins to deviate or incline always to the South, and this deviation increases continuously until the epicycle's centre has arrived at the apogee *A* or perigee *C,* and then the deviation is greatest: the above-mentioned 45′. Thereupon it decreases again until the epicycle's centre has arrived at *B* or *D,* where then again there is no deviation. And from this it is apparent that the motion which the epicycle's centre receives from the deviation never occurs on the North side.

But to speak now of the declination, it is to be noted that the plane of the epicycle through an oscillatory motion diverges from the plane of its orbit in two ways. Firstly, with declination about the diameter *HK,* passing through the points of medium distance *H, K.* With this motion the apogee *G* of the diameter *IG* diverges on one side of the orbit and the perigee *I* on the other side. This declination observes the following rule. When the epicycle's centre (as in the present case *D*) is at the orbit's apogee *A,* the aforesaid diameter *IG* is in the plane of the orbit. But when the epicycle's centre starts from the apogee *A,* the epicycle's apogee begins to decline towards the South and the perigee towards the North, which declination constantly increases until the epicycle's centre has arrived at the point of medium distance *B,* and then there is the greatest declination of the diameter (*IG*), which thereupon decreases continuously until the centre of the epicycle is at the orbit's perigee *C,* where the aforesaid diameter (*IG*) again has no declination; but when the epicycle's centre starts from *C* towards the point of medium distance *D,* the apogee (*G*) begins to decline from the plane of the orbit towards the North, and the perigee (*I*) towards the South, and this declination increases continuously until the epicycle's centre has arrived at the extremity *D,* where this declination again becomes greatest. From this point onwards it decreases until the epicycle's centre is at the apogee *A,* where, just as in the beginning, the aforesaid diameter (*IG*) is again in the plane of the orbit. And this is followed again by the first situation. From what has been said it appears that when the orbit's deviation is greatest, the epicycle has no declination, but when the orbit is without deviation, then the epicycle's declination is greatest.

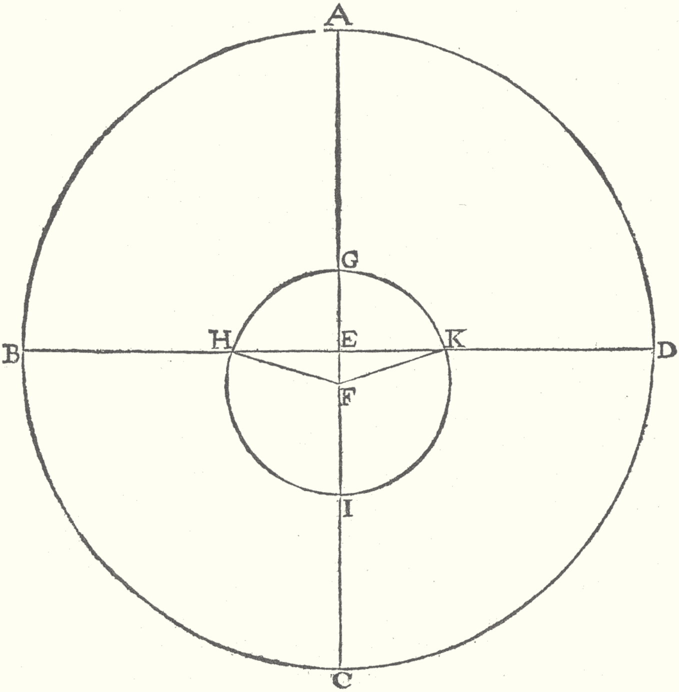
Now it still remains to explain the deflection, which is as follows. The epicycle's plane has, in addition to the aforesaid oscillation about the axis *HK,* yet another oscillation about the axis *IG* between the epicycle's apogee and perigee (which proceeds substantially like the two kinds of oscillation which the mariner's compass receives, with two axes at right angles to one another), in consequence of which motion the diameter *HK* intersects the plane of the orbit in the line *IG* as intersection, so that the epicycle's left half deflects on one side of the deferent, the right half on the other side. This deflection of the diameter observes the following rule. When the epicycle's centre is at the point of medium distance *D,* the aforesaid diameter *HK* has no other deflection from the orbit *ABCD* than that which is in the plane of the latter.

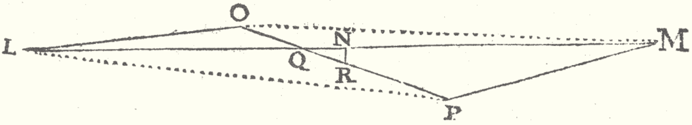
Noorden, en vermeerdert alſoo tot datſe ant naeſtepunt C is, alwaermenſe dan weerom ten grootſten bevint: Van daer vermindertſe geduerlick tot dattet inronts middelpunt comt an d'ander middelverheyt D, alwaer weerom geen afkeering en is, en alſdan begint weerom de voorgaende geſtalt. Hier uyt is kennelick dat ter plaets des inrontwechs daer het inront geen afwijcking en heeft, ſijn meeſte afkeering te ghebeuren.

3 LIDT.

Inhoudende verclaring vā Mercurius breedeloop met ſtelling eens roerenden Eertcloots.

Hier vooren beſchreven ſijnde het 1 lidt inhoudende *Ptolemeus* dadelicke ervaringen (waer by tot breeder verclaring noch vervought wiert het 2 lidt van ſijn ſpiegeling) men ſoude deur vervouging der ſelve ervaring op ſtelling eens roerenden Eertcloots volgens de voorgaende gemeene regel tottet begheerde commen, nochtans angeſien de form vande onderſte Dwaelders binnen den Eertclootwech loopende, wat anders valt als vande bovenſte, en dat Mercurius wech niet deur des Eertclootwechs middelpunt en ſtreƈt, t'welck voor de ſommige verclaring mocht vereyſſchen, ſoo ſal icker wat af ſeggen. Laet A B C D





den

But when the epicycle's centre starts from *D* towards the apogee *A,* the left half of the diameter *HD* begins to deflect towards the South, the other half towards the North, which deflection increases continuously until the epicycle's centre has arrived at the apogee *A,* when it is greatest. Thereupon proceeding towards the other point *B,* it decreases again until it has arrived there, where again there is no deflection. But when the epicycle's centre starts from that place towards the perigee *C,* the aforesaid left half (*HD*) begins to deflect towards the North again, and thus increases until it is at the perigee *C,* where it is then again found greatest. From there onwards it decreases continuously until the epicycle's centre arrives at the other point of medium distance *D,* where again there is no deflection, and then the preceding situation occurs again. From this it is obvious that at the place of the deferent where the epicycle has no declination its greatest deflection occurs.

3rd SECTION.

Consisting in the explanation of Mercury's motion in latitude on the assumption of a moving Earth.

Since above has been described the 1st section consisting of *Ptolemy's* practical experiences (to which, for a fuller explanation, was also added the 2nd section of his theory), by applying this experience to the assumption of a moving Earth one might according to the preceding common rule arrive at what is required. Nevertheless, since the figure of the orbits of the lower Planets within the Earth's orbit is somewhat different from that of the upper Planets, and since Mercury's orbit does not pass through the centre of the Earth's orbit, which for some might require an explanation, I will say something about it. Let *ABCD* denote the Earth's orbit, whose centre is *E,* the centre of Mercury's orbit *F,* about which has been described its orbit *GHIK;* when therafter through *E* and den Eertclootwech beteyckenen, diens middelpunt E, Mercuriuswechs middelpunt F, waer op beschreven is sijn wech G H I K, daer na deur E en F ghetrocken de middellijn A C, en deur t'punt E de lini B D rechthouckich op A C, soo is I t'punt verst van des Eertclootwechs middelpunt E, en G het naestepunt: Voort anghesien *Ptolemeus* bevonden heeft dat Mercurius meeste breeden altijt gebeurden als het inronts middelpunt was by de middelverheden, so volght daer uyt met stelling eens roerendē Eertcloots, dat de selve meeste breedē altijt gesien worden als Mercurius is an sijn wechs middelverheden H en K, en den Eertcloot an B of D: Nu dan H en K wesende de twee puntē welcke in Mercuriuswech de grootste afwijcking crijgē diemē uyt den Eertclootwech sien can, soo seggen wy dier twee platten gemeene sne te moeten commen rechthouckich op B D, dats in G I, of evewijdich mette selve, en niet in H K gelijck *Ptolemeus* die stelt: Maer om nu te vindē waer de selve duysteraersne valt, metsgaders Mercuriuswechs afwijcking van t'plat des duysteraers, ghelijck van Saturnus int 21 en 22 voorstel ghedaen wiert, ick souck voor al de langde der lini H K, tot dien eynde aldus segghende: De driehouck E K F heeft drie bekende palen, te wetē Mercuriuswechs halfmiddellijn F K 3572, en de uytmiddelpunticheytlijn E F 947 deur de Byeenvouging des 13 voorstels deses 3 boucx, en dē houck K E F recht: Hier me gesocht de sijde E K, wort bevonden deur het 5 voorstel der platte driehoucken van 3444.

Daer toe noch soo veel voor E K, comt voor de begheerde H K 6888.

En van E D 10000, ghetrocken E K 3444 eerste in d'oirden, blijft voor de lini K D, oock me voor H B 6556.

Dit aldus bekent sijnde ick teycken een ander form als gedaen wiert met Saturnus int 22 voorstel, treckende ten eersten L M als Eertclootwech overcant ghesien, even an B D 20000, en stel int middel van L M t'punt N als Eertclootwechs middelpunt, ick treck daer na de lini O P van 6888 even an H K tweede in d'oirden, en sniende L M in Q, daer na L O evē met H B 6556 derde in d'oirden, sgelijcx M P even met K D, dats oock doende 6556, en segh den houck O L Q te doen 1 tr. 45 ①, en P M Q 4 tr. 5 ① volghens d'ervaring int 1 lidt: Dit soo wesende de cruysvierhouck L O M P heeft vijf bekende palen, te wetē L M 20000, L O 6556, M P 6556, den houck O L Q 1 tr. 45 ①, en den houck P M Q 4 tr. 5 ① deur t'ghegheven. Merckt noch dat benevens de boveschrevē vijf palen tot meerder gerief bekent sijn drie ander, te wetē O P 6888, P L 13444 als even genouch sijnde met O P 6888 en O L 6556 t'samen, voort O M doende oock soo veel: Hier me ghesocht den houck O Q L, wort bevonden deur het 6 voorstel inde Byvough der platte veelhoucken voor Mercuriuswechs begheerde afwijcking vanden duysteraer 5 tr. 32 ①.

En de lini Q M van 11364, waer af ghetrocken M N 10000, blijft voor de lini vande duysteraersne Q totten Eertclootwechs middelpunt N. 1364.

Tot hier toe is beschreven t'ghene ick van Mercurius int besonder verclaren wilde. Belanghende ander byvallen als inde ses voorstellen van Saturnus breede beschreven sijn, die houden wy als voor ghemeene reghel over Mercurius en d'ander Dwaelders te verstrecken.

Angaende overeencomminghen en verschil deser stelling van Mercurius

F is drawn the diameter *AC,* and through the point *E* the line *BD* perpendicular to *AC, I* is the point furthest from the centre of the Earth's orbit *E,* and *G* is the nearest point. Further, since *Ptolemy* found that Mercury's greatest latitudes always occurred when the epicycle's centre was at the points of medium distance, it follows therefrom on the assumption of a moving Earth that these greatest latitudes are always seen when Mercury is at its orbit's points of medium distance *H* and *K,* and the Earth at *B* or *D. H* and *K* now being the two points which in Mercury's orbit receive the greatest deviation that can be seen from the Earth's orbit, we say that the intersection of these two planes must be perpendicular to *BD, i.e.* in *GI,* or parallel thereto, and not in *HK,* as *Ptolemy* assumes. But in order now to find where this line of nodes falls, as also the deviation of Mercury's orbit from the plane of the ecliptic, as was done with Saturn in the 21st and 22nd propositions, I seek first of all the length of the line *HK,* saying to this end as follows. The triangle *EKF* has three known terms, to wit, the semi-diameter of Mercury's orbit *FK* = 3,572, and the line of eccentricity *EF* = 947 by the Compilation of the 13th proposition of this 3rd book, and the angle *KEF* right; when the side *EK* is sought therewith, this is found by the 5th proposition of plane triangles to be 3,444

When to this is added the same value for *EK,* the required *HK* becomes 6,888

And when from *ED* = 10,000 is subtracted *EK* = 3,444 (the first in the present list), there is left for the line *KD,* and also for *HB,* 6,556

This being known, I draw another figure, as was done with Saturn in the 22nd proposition, first drawing *LM* as the Earth's orbit, seen transversely, equal to *BD* = 20,000, and I mark in the middle of *LM* the point *N,* as centre of the Earth's orbit; I then draw the line *OP* of 6,888 equal to *HK* (the second in the list) and intersecting *LM* in *Q,* then *LO* equal to *HB* = 6,556 (the third in the list), also *MP* equal to *KD, i.e.* also making 6,556, and I say that the angle *OLQ* makes 1°45′, and *PMQ* 4°5′, according to the experience in the 1st section. This being so, the crossed quadrilateral *LOMP* has five known terms, to wit, *LM* = 20,000, *LO* = 6,556, *MP* = 6,556, the angle *OLQ* = 1°45′, and the angle *PMQ* = 4°5′ by the supposition. Note also that besides the above-mentioned five terms for greater convenience three more are known, to wit, *OP* = 6,888, *PL* = 13,444 as being substantially equal to *OP* = 6,888 and *OL* = 6,556 together, further *OM* having the same value. When the angle *OQL* is sought therewith, by the 6th proposition in the Supplement of plane polygons [1]) the required deviation of Mercury's orbit from the ecliptic is found to be 5°32′

And when from the line *QM* of 11,364 is subtracted *MN* = 10,000, there is left for the line from the line of nodes *Q* to the centre of the Earth's orbit *N* 1,364

Up to this point has been described what I wished to explain in particular for Mercury. As to other cases, such as have been described in the six propositions

[1]) See p. 189, note 1.

breedeloop met die van *Ptolemeus*, daer af sal ghesеyt worden inden volgenden Byvough des breedeloops met stelling eens vasten Eertcloots.

NV VAN VENVS BREEDE.

Ptolemeus heeft deur dadelicke gagheslaghen ervaringhen de ghedaenten van Venus breedeloop alsins bevonden ghelijck van Mercurius int voorgaende ghesеyt is, welcke int besonder dusdanich sijn.

Ten eersten soo was haer meeste breede op de noortsijde van 6 tr. 22 Ⓘ, gebeurende altijt als haer inronts middelpunt was 90 tr. van sijn wechs verstepunt soo wel ter eender als ter ander sijde, dat is soo wel inden 270 tr. als inden 90 (welcke 90 tr. is onder des duysteraers 145 tr. ghemerckt deur de Byeenvouging des 13 voorstels deses 3 boucx het verstepunt onder des duysteraers 55 tr. is) en Venus in des inronts naestepunt: Maer buyten het naestepunt wesende, soo was heur noordersche breede voor datmael cleender, en ten minsten doen sy alsoo an t verstepunt was.

Ten tweeden soo bevant hy op de zuytsijde de meeste breede van 1 tr. 2 Ⓘ, ghebeurende altijt als heur inronts middelpunt was 90 tr. van sijn wechs verstepunt soo wel ter eender als ter ander sijde, en Venus an haer inronts naestepunt, maer buyten het naestepunt wesende, soo was haer zuyderсche breede voor datmael cleender, en ten minsten doenmense alsoo ant verstepunt vandt.

De spiegheling by *Ptolemeus* hier op met stelling eens vasten Eertcloots veroirdent is met haer afweging, afwijcking, afkeering, en ander ghedaente, teenemael gheweest als die van Mercurius, sulcx dattet onnoodich schijnt de selve alhier te verhalen: Ende want de spiegeling met stelling eens roerenden Eertcloots ande voorgaende van Saturnus en Mercurius oock ghelijck is, en daer deur verstaen wort, soo en beschrijf ick die niet int langhe.

Angaende overeencomminghen en verschil deser stelling van Venus breedeloop met die van *Ptolemeus*, daer af sal ghesеyt worden inden volghenden Byvough des breedeloops met stelling eens vasten Eertcloots.

of Saturn's latitude, we assume that these apply as a common rule to Mercury and the other Planets.

As regards the correspondences and difference of this theory of Mercury's motion in latitude and that of *Ptolemy,* these are to be discussed in the subsequent Supplement of the motion in latitude on the assumption of a fixed Earth.

NOW OF VENUS' LATITUDE.

Ptolemy through practical observational experiences has found the character of Venus' motion in latitude to be in every respect as has been said of Mercury in the foregoing, which in particular is as follows.

Firstly, its greatest latitude on the North side was 6°22′, which always occurred when its epicycle's centre was at 90° from its orbit's apogee, both on one side and on the other, *i.e.* both at 270° and at 90° (which 90° is at 145° of the ecliptic, seeing that by the Compilation of the 13th proposition of this 3rd book the apogee is at 55° of the ecliptic), and Venus was at the epicycle's perigee. But when it was outside the perigee, its Northerly latitude was for that case smaller, and smallest when at the apogee.

Secondly, he found on the South side the greatest latitude to be 1°2′, which always occurred when its epicycle's centre was at 90° from its orbit's apogee, both on one side and on the other, and Venus was at its epicycle's perigee; but when it was outside the perigee, its Southerly latitude was for that case smaller, and smallest when at the apogee.

The theory that *Ptolemy* on the assumption of a fixed Earth based thereon was, with its deviation, declination, deflection, and other properties, altogether the same as that of Mercury, so that it seems unnecessary here to describe it. And because the theory on the assumption of a moving Earth is also equal to the preceding one of Saturn and Mercury, and is understood therefrom, I will not describe it in full.

As regards the correspondences and difference of this theory of Venus' motion in latitude and that of *Ptolemy,* these are to be discussed in the subsequent Supplement of the motion in latitude on the assumption of a fixed Earth.

BYVOVGH
DES BREEDELOOPS
DER VYF DWAELDERS

Saturnus, Iupiter, Mars, Venus, en
Mercurius, ghegront op ſtelling
eens vaſten Eertcloots.

SUPPLEMENT

OF THE MOTION IN LATITUDE OF THE FIVE PLANETS

Saturn, Jupiter, Mars, Venus, and Mercury,
Based on the Theory of a Fixed Earth

CORTBEGRYP DESES BYVOVGHS.

INt 19 voorstel des 3 boucx is ghesejt, gherievigher te sijn rekeninghen vande langdeloop der Dwaelders te maken op de oneyghen stelling eens vasten Eertcloots, dan op de eygen eens roerenden, en dien volghens soudemen om de selve reden meughen segghen derghelijcke met stelling eens vasten Eertcloots oock oirboirder te sijn vande breedeloop, en dat daerom de natuerlicke oirden soude vereysschen desen Byvough niet hier, maer int tweede bouck met stelling eens vasten Eertcloots behooren vervought te wesen, ghelijck mette Manens breedeloop daer ghedaen is. Om hier op te antwoorden, soo is te weten dat nadien grondelicke kennis deses handels ghetrocken wort uyt de breedeloop met stelling eens roerenden Eertcloots, welcke alsdoen noch niet beschreven en was, noch volghens mijn voorghenomen oirden beschreven en moest wesen, soo en conde dit daer niet bequamelick commen: Maer de selve breedeloop nu int derde bouck verclaert sijnde, soo connen wy daer uyt trecken t'ghene de breedeloop met stelling eens vasten Eertcloots vereyscht, en daerom heb ick die hier beschreven.

Theoremata. Problemata. De selve sal ses voorstellen hebben, wesende d'eerste vier * vertooghen, de laetste twee * werckstucken, te weten:

Het 1 voorstel, dat de ronden der twee onderste Dwaelders Venus en Mercurius die byde stelders eens vasten Eertcloots inrontdragers genoemt worde, inronde sijn, en t'gene sy inronde heeten, inrontdragers te wesen.

Het 2, dattet plat des inronts der vijf Dwaelders Saturnus, Iupiter, Mars, Venus en Mercurius met stelling eens vasten Eertcloots, altijt evewijdich is mettet plat des duysteraers.

Het 3, dat wesende twee even en evewijdege ronden, het een hooger alst ander, de lini tusschen het middelpunt vant leeghste, en een punt inden omtreck vant hooghste, even en evewijdege te sijn mette lini tusschen sijn lijcstandich tegenoverpunt int leeghste, en het middelpunt vant hooghste.

Het 4, dat de Dwaelders met stelling eens vasten Eertcloots de selve schijnbaer duysteraerbreede ontfangen, diese hebben met stelling eens roerenden Eertcloots.

Het 5, wesende gegeven eens Dwaelders meeste noordersche en zuydersche breede, te vindē sijn wechs afwijcking vanden duysteraer: Oock mede hoe verre de duysteraersne vanden Eertcloot valt, deur wisconstighe wercking ghegront op stelling eens vasten Eertcloots.

Het 6, om te vindē eens Dwaelders schijnbaer duysteraerbreede op een gegevē tijt, deur wiscōstige wercking gegrōt op stelling eēs vastē Eertcloots.

SUMMARY OF THIS SUPPLEMENT.

In the 19th proposition of the 3rd book it has been said that it is more convenient to make calculations of the motion in longitude of the Planets on the untrue theory of a fixed Earth than on the true theory of a moving Earth; consequently it might be said for the same reason that similar calculations on the theory of a fixed Earth are also more suitable for the motion in latitude, and that therefore the natural order would require that this Supplement should not be added here, but in the second book, based on the theory of a fixed Earth, as has been done there with the motion in latitude of the Moon. In order to answer this argument, it is to be noted that since thorough knowledge of this subject is derived from the motion in latitude on the theory of a moving Earth, which had not yet been described at that moment, nor should have been described according to the sequence planned by me, this could not be properly inserted there. But now that this motion in latitude has been explained in the third book, we can derive therefrom what is required for the motion in latitude on the theory of a fixed Earth, and that is why I have described it here.

The Supplement is to comprise six propositions, the first four being theorems and the last two being practical problems, to wit:

The 1st proposition, that the circles of the two lower Planets Venus and Mercury, which are called deferents by those who hold the theory of a fixed Earth, are epicycles, and what they call epicycles are deferents.

The 2nd, that the planes of the epicycles of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury on the theory of a fixed Earth are always parallel to the plane of the ecliptic.

The 3rd, that when there are two equal and parallel circles, one higher than the other, the line between the centre of the lower one and a point on the circumference of the higher one is equal and parallel to the line between its homologous opposite point in the lower one and the centre of the higher one.

The 4th, that on the theory of a fixed Earth the Planets acquire the same apparent ecliptical latitude that they have on the theory of a moving Earth.

The 5th, given a Planet's greatest northerly and southerly latitudes, to find its deferent's deviation from the ecliptic; also how far the line of nodes is from the Earth, by mathematical operations based on the theory of a fixed Earth.

The 6th, to find a Planet's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a fixed Earth.

VERTOOCH. 1 VOORSTEL.

De ronden der tvvee onderste Dvvaelders Venus en Mercurius die by de stelders eens vasten Eertcloots ghenoemt vvorden * inrontdragers, sijn inronden: En t'gene sy inronden heeten, sijn inrontdraghers.

Deferens epirycliorum.

T'is deur kennis der voorgaende stelling eens roerenden Eertcloots openbaer, dat de inronden der drie bovenste Dwaelders met stelling eens vasten Eertcloots niet int wesen en bestaen, dan versiert worden even te sijn anden Eertclootwech, maer t'gaet anders toe mette twee onderste, want t'ghene men Venus of Mercurius inront noemt, en is niet versiert noch even anden Eertclootwech, dan om eyghentlick te spreken het is hun wech self daer sy dadelick in loopen, en in stelling eens vasten Eertcloots, volghens d'oirden der drie bovenste, inrontwech behoort te heeten: En daerom alsmen de voorgaende regel der langdeloop vande bovenste met stelling eens vasten Eertcloots, soude willen ghemeen hebben over de twee onderste, men soude daer moeten op de selve wijse spreken, te weten inrontwech noemen het cleenste dat eyghentlick inrontwech is, en inront het grootste dat versiert wort even anden Eertclootwech, want daer me dan ghedaen als mette bovenste, de reghel sal ghemeen sijn. Maer om van sulcke stelling noch claerder te spreken ick segh aldus: Angesien yder Dwaelders inront even is anden Eertclootwech, soo moetense met malcander al evegroot sijn, waer uyt wijder volght, dat de weghen vande leegher Dwaelders teghen haer inronden verleken, minder sullen sijn dan de weghen vande hoogher Dwaelders.

Laet tot voorbeelt in dese eerste volghende form op A als middelpunt, beschrevē worden het rondt B C als Saturnus inrontswech, diens halfmiddellijn A B doet deur de Byeenvouging des 13 voorstels vant 3 bouck 92308, en op B als middelpunt, het inrondt diens halfmiddellijn D B even anden Eertclootwech diens halfmiddellijn doet 10000.

Ten tweeden sy op A als middelpunt, beschreven het rondt E F als Iupiters inrontswech, diens halfmiddellijn A E doet deur de Byeenvouging des 13 voorstels 52174, en op E als middelpunt het inront diens halfmiddellijn E G 10000.

Ten derden sy op A als middelpunt beschreven het rondt H I als Mars inrontswech, diens halfmiddellijn A H doet deur de Byeenvouging des 13 voorstels 15190, en op H als middelpunt het inront diēs halfmiddellijn H K 10000.

Ten vierden sy op L als middelpunt, beschreven het rondt M N als Venus inrontswech, diens halfmiddellijn M L doet deur de Byeenvouging des 13 voorstels 7194, en op M als middelpunt het inront, diens halfmiddellijn M O 10000, welck inront in hem heeft des inrontwechs middelpunt L, anders dan een der drie voorgaende formen: Doch is kennelick dat de reghel des langdeloops voor alle vier ghemeen moet sijn.

Ten vijfden sy op P als middelpunt beschreven het rondt Q R als Mercurius inrontswech, diens halfmiddellijn P Q doet deur de Byeenvouging des 13 voorstels 3572, en op Q als middelpunt het inront, diens halfmiddellijn Q S 10000, welck inront in hem heeft den heelen inrontwech Q R, anders dan in een der vier voorgaende formen, doch is kennelick dat de reghel des langdeloops

THEOREM 1st PROPOSITION.

The circles of the two lower Planets Venus and Mercury, which are called deferents by those who hold the theory of a fixed Earth, are epicycles; and what they call epicycles are deferents.

It is evident from a knowledge of the foregoing theory of a moving Earth that the epicycles of the three upper Planets on the theory of a fixed Earth do not really exist, but are imagined to be equal to the Earth's orbit; but matters are different with the two lower Planets, for what is called Venus' or Mercury's epicycle is neither imagined nor really equal to the Earth's orbit; but, accurately speaking, it is their orbits themselves in which they move in actual practice, and which, on the theory of a fixed Earth, according to the arrangement of the three upper Planets, ought to be called deferents. Therefore, if the foregoing rule about the motion in longitude of the upper Planets on the theory of a fixed Earth should be desired to apply also to the two lower Planets, it would be necessary to speak there in the same way, to wit, call deferent the smallest, which really is a deferent, and epicycle the largest, which is imagined to be equal to the Earth's orbit, for if these are then dealt with in the same way as the upper Planets, the rule will apply generally. But to speak even more clearly of this theory, I say as follows: Since the epicycle of each Planet is equal to the Earth's orbit, they must all be equal to one another, from which it follows further that the orbits of the lower Planets, when compared with their epicycles, will be smaller than the orbits of the upper Planets.

By way of example, in this first subsequent figure let there be described about *A* as centre the circle *BC* as Saturn's deferent, whose semi-diameter *AB* by the Compilation of the 13th proposition of the 3rd book [1]) makes 92,308, and about *B* as centre the epicycle, whose semi-diameter *DB*, equal to the semi-diameter of the Earth's orbit, makes 10,000.

Secondly, let there be described about *A* as centre the circle *EF* as Jupiter's deferent, whose semi-diameter *AE* by the Compilation of the 13th proposition makes 52,174, and about *E* as centre the epicycle, whose semi-diameter *EG* makes 10,000.

Thirdly, let there be described about *A* as centre the circle *HI* as Mars' deferent, whose semi-diameter *AH* by the Compilation of the 13th proposition makes 15,190, and about *H* as centre the epicycle, whose semi-diameter *HK* makes 10,000.

Fourthly, let there be described about *L* as centre the circle *MN* as Venus' deferent, whose semi-diameter *ML* by the Compilation of the 13th proposition makes 7,194, and about *M* as centre the epicycle, whose semi-diameter *MO* makes 10,000, which epicycle has within it the deferent's centre *L*, unlike anyone of the three foregoing figures. It is obvious that the rule of the motion in longitude must be the same for all four Planets in common.

Fifthly, let there be described about *P* as centre the circle *QR* as Mercury's deferent, whose semi-diameter *PQ* by the Compilation of the 13th proposition makes 3,572, and about *Q* as centre the epicycle, whose semi-diameter *QS* makes 10,000, which epicycle has within it the entire deferent *QR*, unlike anyone of the four foregoing figures. It is obvious that the rule of the motion in longitude must be

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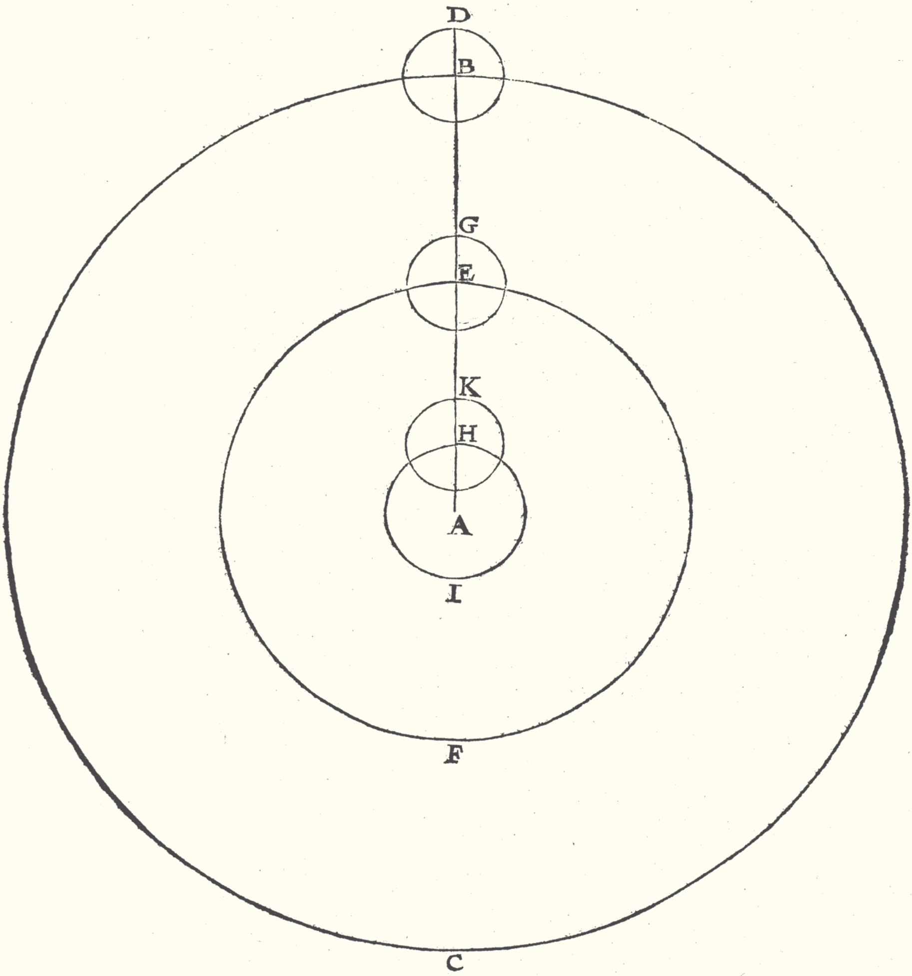
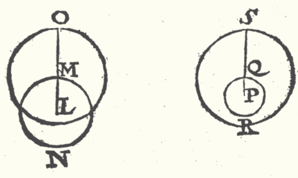
[1]) See p. 173.





loops voor alle vijf gemeen moet sijn, sonder de verkeerde haspeling te vallen diemen ontmoet anders doende. Dit oude misbruyck heeft sijn bekende oirsaeck, want tewijle d'eerste ondersouckers des Hemelloops gheen kennis en hadden vande ghedaente des roerenden Eertcloots, soo en conden sy niet beter schrijven dan van t'ghene voor hemlien uyterlick scheen te wesen. Noch is oock te anmercken dat de regel des breedeloops (soo wel als des langdeloops) met stelling eens vasten Eertcloots, hier deur over allen ghemeen is ghelijck int vol-the same for all five Planets in common, without the occurrence of the tangle, due to inversion, which we meet with if we do otherwise. This ancient abuse has a well-known cause, for since the first investigators of the Heavenly Motions were not acquainted with the character of the moving Earth, they could merely describe things as they appeared to them. It is also to be noted that the rule of the motion in latitude (as well as of the motion in longitude) on the theory of a fixed Earth is thus common to all, as will become apparent in the sequel.

int volghende blijcken sal. T'BESLVYT. De ronden dan der twee onderste Dwaelders Venus en Mercurius die by de stelders eens vasten Eertcloots ghenoemt worden inrontdragers, sijn inronden: En t'ghene sy inronden heeten, sijn inrontdraghers, t'welck wy bewijsen moesten.

VERTOOCH. 2 VOORSTEL.

Het plat des inronts der vijf Dvvaelders Saturnus, Iupiter, Mars, Venus en Mercurius, is met stelling eens vasten Eertcloots altijt evevvijdich mettet plat des duysteraers.

Voor al anghesien het inront versiert is evegroot metten Eertclootwech, en dat den Dwaelder daer in ghenomen wort een langdeloop te hebben even en ghelijck mette langdeloop des Eertcloots in haer wech, soo volght daer ghenouchsaem uyt deser twee ronden evewijdicheyt behooren toegelaten te worden, ghemerckt datter anders gheen volcommen ghelijcke langdeloop sijn en soude, nochtans anghesien de selve evewijdicheyt uyt het voorgaende can bewesen worden, soo sal ick die beschrijven als volght: Ick verkies hier toe de form des 15 voorstels vant 3 bouck, waer me ick aldus segh: Ten eersten soo is deur de ghemeene reghel beschreven int 22 voorstel des 3 boucx, bekent hoe groot dat moet sijn de breede van Mars ter plaets des punts N ghesien uyt den roerenden Eertcloot O, maer soo groot die daer wesen moet, even soo groot moet Mars breede oock sijn ter plaets van I met stelling eens vasten Eertcloots, dats ghesien uyt A: Ende want A I even is met O N, soo volght daer uyt dat de lini van I rechthouckich opt plat des duysteraers even moet sijn ande lini van N rechthouckich opt plat des duysteraers. Ten tweedē segh ick, dat soo lanck als is de lini van N tot opt plat des duysteraers gesien uyt den roerenden Eertcloot an P, soo lanck moet oock sijn de lini van F tot opt plat des duysteraers ghesien uyt den vasten Eertcloot A, om dat A F even is met P N.

Maer elck der twee linien van F en N rechthouckich opt plat des duysteraers, aldus even sijnde ande lini van N tot opt plat des duysteraers, soo moeten die selve twee linien van F en N rechthouckich op t'plat des duysteraers met malcander even sijn, en vervolgens soo is des inronts heele halfmiddellijn I F evewijdich mettet plat des duysteraers. Ende op de selve voet ist openbaer te connen bethoont worden de middellijn deur H rechthouckich op F I soose ghetrocken waer, oock evewijdich te wesen mettet plat des duysteraers, want soo men deur t'punt K treckt een middellijn rechthouckich op O P, en datmen naem den roerenden Eertcloot te wesen ande uytersten van dien (het welck cortheytshalven ghelaten wort) t'bewijs soude daer me sijn als vooren: Nu dan de middellijn F I, met d'ander middellijn deur H, soose als gheseyt is getrocken waer, beyde evewijdich sijnde metten duysteraer, soo volghter uyt het heel plat des inronts F I evewijdich te wesen mettet plat des duysteraers. Ende sulcx als dit bewijs is wesende het inront met sijn middelpunt an H, alsoo sal dergheliijcke bewesen worden tot alle plaetsen. Voort gelijck dit bewijs is geweest mette form des 15 voorstels, dienende voor de drie opperste Dwaelders, alsoo salt oock sijn mette form des 16 voorstels dienende voor de onderste.

T'BESLVYT. Het plat dan des inronts der vijf Dwaelders Saturnus, Iupiter, Mars, Venus en Mercurius, is met stelling eens vasten Eertcloots altijt evewijdich mettet plat des duysteraers, t'welck wy bewijsen moesten.

CONCLUSION. The circles of the two lower Planets Venus and Mercury, which are called deferents by those who hold the theory of a fixed Earth, are therefore epicycles; and what they call epicycles are deferents; which we had to prove.

THEOREM. 2nd PROPOSITION.

The planes of the epicycles of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury on the theory of a fixed Earth are always parallel to the plane of the ecliptic.

First of all, since the epicycle is imagined to be equal to the Earth's orbit and since the Planet is assumed to have therein a motion in longitude equal and similar to the motion in longitude of the Earth in its orbit, it follows sufficiently that the parallelism of these two circles should be admitted, since otherwise there would be no perfectly similar motion in longitude; nevertheless, since this parallelism can be proved from the foregoing, I will describe it as follows. I choose for this the figure of the 15th proposition of the 3rd book, with regard to which I say as follows: Firstly, by the common rule described in the 22nd proposition of the 3rd book it is known how great must be the latitude of Mars at the point *N*, when seen from the moving Earth *O;* but as great as it must be there, so great must also be Mars' latitude at *I* on the theory of a fixed Earth, *i.e.* when seen from *A*. And because *AI* is equal to *ON*, it follows that the line from *I* perpendicular to the plane of the ecliptic must be equal to the line from *N* perpendicular to the plane of the ecliptic. Secondly, I say that as long as is the line from *N* to the plane of the ecliptic, when seen from the moving Earth at *P*, so long must also be the line from *F* to the plane of the ecliptic, when seen from the fixed Earth *A*, because *AF* is equal to *PN*.

But each of the two lines from *F* and *I* [1]) perpendicular to the plane of the ecliptic thus being equal to the line from *N* to the plane of the ecliptic, those two lines from *F* and *I* [1]) perpendicular to the plane of the ecliptic must be equal to one another, and consequently the epicycle's entire semi-diameter *IF* is parallel to the plane of the ecliptic. And on the same grounds it is evident that it can be proved that the diameter through *H* perpendicular to *FI*, if it were drawn, would also be parallel to the plane of the ecliptic, for if through the point *K* is drawn a diameter perpendicular to *OP*, and if the moving Earth were assumed to be at the extremities thereof (which is omitted for brevity's sake), the proof would be the same as above. The diameter *FI*, with the other diameter through *H*, if — as has been said — it were drawn, thus both being parallel to the ecliptic, it follows that the entire plane of the epicycle *FI* is parallel to the plane of the ecliptic. And such as is this proof when the epicycle is with its centre at *H*, a similar proof can be given for any place. Further, as this proof has been with the figure of the 15th proposition, serving for the three upper Planets, so will it also be with the figure of the 16th proposition, serving for the lower Planets.

CONCLUSION. The planes of the epicycles of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury on the theory of a fixed Earth are thus always parallel to the plane of the ecliptic; which we had to prove.

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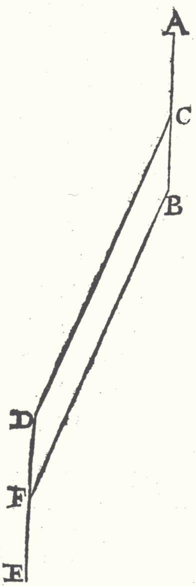
[1]) For *N* in the Dutch text read *I*.

VERTOOCH. 3 VOORSTEL.

Weſende tvvee even en evevvijdeghe ronden het een hoogher als t'ander, de lini tuſſchen het middelpunt vant leeghſte, en een punt inden omtreck vant hoochſte, is even en evevvijdege mette lini tuſſchen ſijn lijckſtandich teghenoverpunt int leeghſte, en het middelpunt van het hoochſte.

T'GHEGHEVEN. Laet A B het hoochſte rondt ſijn overcant gheſien diens middelpunt C, opperſte punt A, onderſte punt B, en D E ſy het leeghſte rondt, even en evewijdich met A B, diens middelpunt F, opperſte punt D, onderſte punt E, en F B ſy de lini tuſſchē het middelpunt F vant leeghſte rondt, en eenich punt inden omtreck vant hooghſte rondt, daer ick hier toe neem het onderſte punt B: Sghelijcx ſy D C de lini tuſſchen ſijn lijckſtandich tegenoverpunt int leeghſte, dat is het opperſte punt D, en het middelpunt C vant hooghſte rondt. Ende is te weten dat ick D ghenoemt hebbe lijckſtandich tegenoverpunt van B, uyt oirſaeck dat E een lijckſtandich punt van B is (als weſende in ſijn rondt ſoo het onderſte ghelijck D int ſijne) en D teghenoverpunt van E.



T'BEGHEERDE. Wy moeten bewijſen dat F B even en evewijdeghe is met D C. T'BEWYS. Angheſien C B en D F even en evewijdeghe halfmiddellijnen ſijn deur t'ghegheven, ſoo ſijn F B en D C twee linien tuſſchen de uyterſten van twee even en evewijdeghe, en daerom ſijnſe oock ſelf even en evewijdeghe: En ſulcx als hier beweſen is mettet lijckſtandich tegenoverpunt vant onderſte punt B, alſoo is derghelijcke openbaer mettet lijckſtandich teghenoverpunt van alle voorgheſtelt punt des omtrecx.

T'BESLVYT. Weſende dan twee even en evewijdeghe ronden het een hoogher als t'ander, de lini tuſſchen het middelpunt vant leeghſte, en een punt inden omtreck vant hooghſte is even en evewijdeghe mette lini tuſſchen ſijn lijckſtandich teghenoverpunt int leeghſte, en het middelpunt vant hooghſte, t'welck wy bewijſen moeſten.

VERTOOCH. 4 VOORSTEL.

De Dvvaelders ontfanghen met ſtelling eens vaſten Eertcloots de ſelve ſchijnbaer duyſteraerbreede, dieſe hebben met ſtelling eens roerenden Eertcloots.

Angheſien de Son of Eertcloot nummermeer breede en hebben, ſoo en valter niet af te ſegghen. Angaende de breede der Maen, het is int voorgaen-

deghe-

THEOREM. 3rd PROPOSITION

When there are two equal and parallel circles, one higher than the other, the line between the centre of the lower one and a point on the circumference of the higher one is equal and parallel to the line between its homologous opposite point in the lower one and the centre of the higher one.

SUPPOSITION. Let *AB* be the higher circle seen transversely, its centre being *C*, the uppermost point *A*, the lowermost point *B*, and let *DE* be the lower circle, equal and parallel to *AB*, its centre being *F*, the uppermost point *D*, the lowermost point *E*, and let *FB* be the line between the centre *F* of the lower circle and some point on the circumference of the higher circle, for which I here take the lowermost point *B*. Similarly, let *DC* be the line between its homologous opposite point in the lower circle, *i.e.* the uppermost point *D*, and the centre *C* of the higher circle. And it is to be noted that I have called *D* homologous opposite point to *B* because *E* is a point homologous to *B* (as being in its circle the lowermost point just as *D* is in its circle) and *D* is the point opposite *E*.

WHAT IS REQUIRED. We have to prove that *FB* is equal and parallel to *DC*.

PROOF. Since *CB* and *DF* are equal and parallel semi-diameters by the supposition, *FB* and *DC* are two lines between the extremities of two equal and parallel lines, and for this reason they are also equal and parallel themselves. And such as has here been proved for the homologous point opposite the lowermost point *B*, so the same is evident for the homologous point opposite any given point of the circumference.

CONCLUSION. When there are two equal and parallel circles, one higher than the other, the line between the centre of the lower one and a point on the circumference of the higher one is thus equal and parallel to the line between its homologous opposite point in the lower one and the centre of the higher one; which we had to prove.

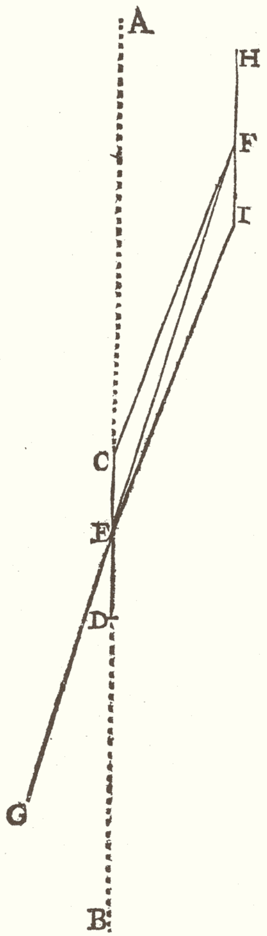
THEOREM. 4th PROPOSITION.

On the theory of a fixed Earth the Planets acquire the same apparent ecliptical latitude that they have on the theory of a moving Earth.

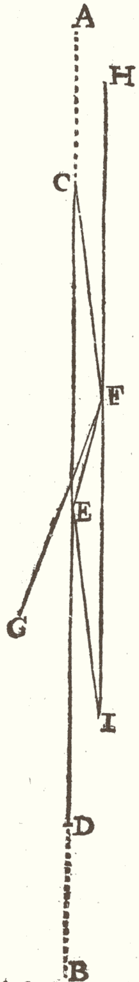
Since the Sun or Earth never have any latitude, there is nothing to be said about them. As to the latitude of the Moon, it has appeared in the foregoing de ghebleken die in d'een en d'ander ſtelling een ſelve te weſen, ſulcx datter alleenelick vande vijf ander verclaring behouft.

T'GHEGHEVEN. Laet A B den duyſteraer bedien overcant gheſien ghelijck al d'ander ronden, C D ſy den Eertclootwech, diens middelpunt E, waer deur als inde eerſte form diēede voor de drie bovēſte Dwaelders, of waer buytē als inde 2 form weſende Mercurius teyckening dienende voor de twee onderſte, getrockē den Dwaelderwech F G, in welcke ick neem den Dwaelder te weſen ant opperſte punt F, en den Eertcloot an haer wechs opperſte pūt C. Tot hier toe de teyckening gedaē ſijnde vande ſtelling eēs roerendē Eertcloots, ick ſal nu die eens vaſten beginnen, tot welckē eynde ick des Eertclootwechs middelpūt E voor vaſten Eertcloot neem, want dat in die ſtelling daer tegen ſulcke beteyckening heeft deur het 15 voorſtel des 3 boucx, en F G die eerſt was Dwaelderwech, neem ick nu voor inrontwech: Voort angheſien den Dwaelder ghenomen is te weſen an des wechs opperſte punt F, ſoo moet het inrondt t'welck H I ſy, met ſijn middelpunt weſen an F, oock even en evewijdich mettē Eertclootwech C D deur het 2 voorſtel deſes Byvoughs. Voort want den roerenden Eertcloot ghenomen wiert an C, ſoo moet den Dwaelder ghenomen ſijn an I als lijckſtandich tegenoverpunt van C, waer af de reden blijckt int 15 voorſtel des 3 boucx.

1 FORM.



2 FORM.



T'BEGHEERDE. Wy moeten bewijſen dat den Dwaelder an I, met ſtelling des vaſten Eertcloots an E, de ſelve ſchijnbaer duyſteraerbreede ontfangt, die hy heeft an F met ſtelling des roerenden Eertcloots an C.

T'BEREYTSEL. Laet getrockē wordē de twee linien E I, C F. T'BEWYS. Angheſien C D even en evewijdeghe is met H I, ſoo ſijn haer helſten E C, F I oock even en evewijdeghe, en de twee linien daer tuſſchen E I, C F moeten oock evewijdege ſijn, en daerom oock den houck A E I, weſende des Dwaelthat it is the same in one theory as well as the other, so that an explanation is required only for the five other Planets.

SUPPOSITION. Let *AB* denote the ecliptic, seen transversely like all the other circles; let *CD* be the Earth's orbit, its centre being *E,* through which — as in the first figure serving for the three upper Planets — or alongside of which — as in the 2nd figure, which is a drawing of Mercury, serving for the two lower Planets — is drawn the Planet's orbit *FG,* in which I take the Planet to be at the uppermost point *F,* and the Earth at its orbit's uppermost point *C.* The drawing so far having been made for the theory of a moving Earth, I will now start on one for the theory of a fixed Earth, to which end I take the centre of the Earth's orbit *E* for the fixed Earth, for in this theory this has the said significance by the 15th proposition of the 3rd book, and *FG,* which first was the Planet's orbit, I now take for the deferent. Further, since the Planet is assumed to be at the orbit's uppermost point *F,* the epicycle, which shall be *HI,* must be with its centre at *F,* also equal and parallel to the Earth's orbit *CD* by the 2nd proposition of this Supplement. Further, because the moving Earth was taken at *C,* the Planet must be taken at *I* as homologous point opposite *C,* the reason of which appears from the 15th proposition of the 3rd book. WHAT IS REQUIRED. We have to prove that the Planet at *I,* on the theory of the fixed Earth at *E,* acquires the same apparent ecliptical latitude that it has at *F* on the theory of the moving Earth at *C.*

PRELIMINARY. Let the two lines *EI, CF* be drawn. PROOF. Since *CD* is equal and parallel to *HI,* their halves *EC, FI* are also equal and parallel, and the two lines in between, *EI* and *CF,* must also be parallel, and therefore also the ders schijnbaer duysteraerbreede ghesien uyt den vasten Eertcloot E, is even anden houck A C F wesende des Dwaelders schijnbaer duysteraerbreede ghesien uyt den roerenden Eertcloot C: Maer sulcx als hier bethoont is wesende den roerenden Eertcloot en den Dwaelder tot dier rondē hooghste en leeghste punt, sal oock alsoo blijcken tot alle plaetsen, om dat ghelijck in dese twee evewijdege ronden C D, H I, de lini E I tusschen het middelpunt E vant leeghste rondt, en het punt I inden omtreck vant hooghste, even en evewijdeghe is mette lini C F, tusschen sijn lijckstandich teghenoverpunt C int leeghste, en het middelpunt F vant hooghste, soo sijn alle sulcke linien overal evewijdeghe deur het 3 voorstel deses Byvoughs. T'BESLVYT. De Dwaelders dan ontfanghen met stelling eens vasten Eertcloots de selve schijnbaer duysteraerbreede diese hebben met stelling eens roerenden Eertcloots, t'welck wy bewijsen moesten.

5 VOORSTEL.

Wesende ghegheven eens Dvvaelders meeste noordersche en zuydersche breede, te vinden sijn vvechs afvvijcking vanden duysteraer: Oock mede hoe verre de duysteraersne vanden Eertcloot valt, deur vvisconstighe vvercking ghegront op stelling eens vasten Eertcloots.

Angesien den inrontwech der drie bovenste Dwaelders deur den Eertcloot streckt, of ghenomen wort voor na ghenouch daer deur te strecken, maer vande twee onderste daer buyten, soo sal ick daer af twee voorbeelden beschrijven.

1 Voorbeelt vande drie bovenste Dvvaelders.

T'GHEGHEVEN. Laet A B den duysteraer beteyckenen overcant ghesien, diens middelpunt dats den vasten eertcloot C, waer deur ghetrocken is Saturnus inrontswech D E, streckende C D na de noortsijde, C E na de zuytsijde, sulcx dat de twee houcken A C D, B C E, des inrontwechs afwijcking bedien, en by sijn uyterste punt D na t'Noorden als middelpunt is beschreven het inront F G, evewijdich metten duysteraer A B, diens verstepunt F, naestepunt G, van t'welck getrocken sy de lini G C, en den houck A C G, doet als bevonden wiert inde ervaring des 20 voorstels des 3 boucx 3 tr. 2 ①: Sghelijcx is opt uyterste punt E na het Zuyden als middelpunt, beschreven het inront H I, evewijdich metten duysteraer A B, diens verstepunt I, naestepunt H, van t'welck getrocken sy de lini H C, en den houck B C H doet als bevonden wiert inde ervaring des voorschreven 20 voorstels 3 tr 5 ①. T'BEGHEERDE. Wy moeten vinden des inrontwechs D E afwijcking vanden duysteraer, dats den houck A C D.

T'WERCK.

De driehouck C D G heeft drie bekende palen, te weten D G 10000 deur t'gestelde, C G soo veel als ten tijde der ervaring dede de lini vanden Eertcloot tottet inronts naestepunt, t'welck deur t'vervolgh van het 13 voorstel des 3 boucx int ghemeen bekent wort, en besonderlick bevonden is int 21 voorstel des 3 boucx van 88718, of anders om gherieviger wercking machmen nemen de lini angle *AEI,* being the Planet's apparent ecliptical latitude when seen from the fixed Earth *E,* is equal to the angle *ACF,* being the Planet's apparent ecliptical latitude when seen from the moving Earth *C*. But such as has here been proved when the moving Earth and the Planet are at their circles' uppermost and lowermost points, the same will also be apparent in any place, because just as in these two parallel circles *CD, HI* the line *EI* between the centre *E* of the lower circle and the point *I* on the circumference of the higher circle is equal and parallel to the line *CF* between its homologous opposite point *C* in the lower and the centre *F* of the higher circle, thus all such lines are always parallel by the 3rd proposition of this Supplement. CONCLUSION. On the theory of a fixed Earth the Planets thus acquire the same apparent ecliptical latitude that they have on the theory of a moving Earth; which we had to prove.

5th PROPOSITION.

Given a Planet's greatest northerly and southerly latitudes, to find its deferent's deviation from the ecliptic; also how far the line of nodes is from the Earth, by mathematical operations based on the theory of a fixed Earth.

Since the deferent for the three upper Planets passes through the Earth, or is assumed to pass through it approximately, but for the two lower Planets outside it, I will describe two examples thereof.

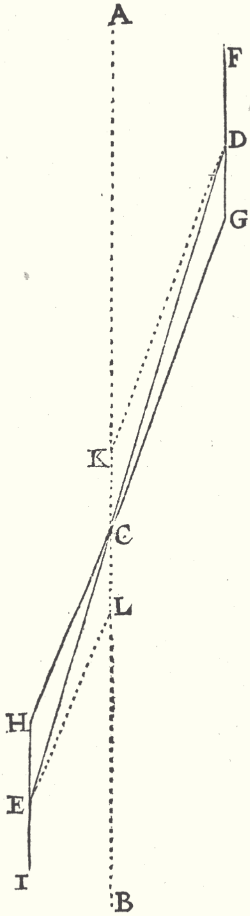
1st Example of the Three Upper Planets.

SUPPOSITION. Let *AB* denote the ecliptic, seen transversely, whose centre is the fixed Earth *C,* through which is drawn Saturn's deferent *DE, CD* tending towards the north side, *CE* towards the south side, so that the two angles *ACD, BCE* denote the deferent's deviation, and about its extremity *D* towards the North as centre has been described the epicycle *FG,* parallel to the ecliptic *AB,* its apogee being *F,* its perigee *G,* from which let there be drawn the line *GC;* then the angle *ACG,* as was found in the experience of the 20th proposition of the 3rd book, makes 3°2′. Similarly, about the extremity *E* towards the South as centre has been described the epicycle *HI,* parallel to the ecliptic *AB,* its apogee being *I,* its perigee *H,* from which let there be drawn the line *HC;* then the angle *BCH,* as was found in the experience of the aforesaid 20th proposition, makes 3°5′. WHAT IS REQUIRED. We have to find the deviation of the deferent *DE* from the ecliptic, *i.e.* the angle *ACD.*

PROCEDURE.

The triangle *CDG* has three known terms, to wit, *DG* = 10,000 by the supposition, *CG* as much as the line from the Earth to the epicycle's perigee was at the time of the experience, which by the sequel of the 13th proposition of the 3rd book becomes known generally and has been found in particular in the 21st proposition of the 3rd book to be 88,718 [1]), or otherwise, with a view

[1]) This value is not to be found in Proposition 21 of the Third Book.



de lini C D, doende na ghenouch 10000 meer, dats 98718, en den houck D G C 176 tr. 58 ①, als even ſijnde metten houck G C B, die halfrontſchil is der bekende grootſte breede A C G 3 tr. 2 ①: Hier me geſocht den houck C D G, wort bevonden deur het 5 voorſtel der platte drichoucken van 2 tr. 43 ①, twelck oock is voor den houck A C D begheerde afwijcking des wechs vanden duyſteraer, overeencommende mette 2 tr. 43 ① die int 22 voorſtel des 3 boucx ghevonden wierden. Ick doe daer na derghelijcke wercking over d'ander ſijde, ſouckende den houck B C E, tot dien eynde aldus ſegghende: De driehouck C H E heeft drie bekende palen, te weten E H 10000 deur t'gheſtelde, C H 75210 deur t'vervolgh van het 13 voorſtel des 3 boucx, en den houck C H E 176 tr. 55 ①, als even ſijnde mettē houck A C H, die halfrontſchil is vande bekende grootſte breede 3 tr. 5 ①: Hier me gheſocht den houck H E C, wort bevonden deur het 6 voorſtel der platte driehouckē van 2 tr. 43 ①, t'welck oock is voor den houck B C E, even vallende mettē boveſchreven houck A C D oock van 2 tr. 43 ①, ſulcx dat volghens deſe rekening G H deur den vaſten Eertcloot C ſtreckt. Maer by aldien ſoodanighe evenheyt niet ghecommen en waer, ghelijckt mette twee onderſte Dwaelders ghebeurt, men ſoude dan volghen de wijſe des nabeſchreven 2 voorbeelts.

1 MERCK

Noch is te weten dat Saturnus inrontwechs afwijcking vanden duyſteraer met ſtelling eens vaſten Eertcloots can gevonden worden op een ander wijſe, welcke alſooſe noch opentlicker verclaert de ghemeenſchap der twee ſtellinghen eens vaſten en roerenden Eertcloots, ſoo ſal ick die met een beſchrijven. Laet als by manier van bereytſel ghetrocken worden D K, even en evewijdeghe met G C: Sghelijcx E L, even en evewijdeghe met H C, t'welck ſoo ſijnde de cruyſvierhouck K D E L, is even en ghelijck metten cruyſvierhouck A D E B des 22 voorſtels vant 3 bouck, daerom hier me gheſocht des inrontwechs C D afwijcking vanden duyſteraer na de manier des ſelven 22 voorſtels, t'ghene daer uyt comt is t'begeerde, en moet nootſakelick even ſijn mettet beſluyt vande voorgaende eerſte wercking. En blijckt hier me ſichtbaerlick hoe de form der ſtelling eens roerenden Eertcloots, te weten den cruyſvierhouck K D E L, metten Eertclootwech K L, een ſelve beſluyt voortbrengt als de form der ſtelling eens vaſtē Eertcloots C, en haer inront ter eender ſijde G D F, ter ander H E I, want den driehouck C D G is even en gelijck metten driehouck D C K, alſoo oock is C E H met E C L, waer uyt volght dat alſulcken afwijcking als van C D bevonden wort deur de bekende palen des driehoucx D C K, ſoodanighe moeter oock bevonden worden deur de bekende palen des driehoucx C D G. Oock is to a more convenient procedure, the line *CD* may be taken, which is approximately 10,000 more, *i.e.* 98,718; and the angle *DGC* = 176°58′ as being equal to the angle *GCB,* which is the supplement of the known greatest latitude *ACG* = 3°2′. When the angle *CDG* is sought therewith, it is found by the 5th proposition of plane triangles to be 2°43′, which is also the value of the angle *ACD,* the required deviation of the deferent from the ecliptic, corresponding to the 2°43′ found in the 22nd proposition of the 3rd book. I then perform a similar operation on the other side, seeking the angle *BCE,* saying to this end as follows. The triangle *CHE* has three known terms, to wit, *EH* = 10,000 by the supposition, *CH* = 75,210 by the sequel of the 13th proposition of the 3rd book, and the angle *CHE* = 176°55′, as being equal to the angle *ACH,* which is the supplement of the known greatest latitude 3°5′. When the angle *HEC* is sought therewith, it is found by the 6th proposition of plane triangles [1]) to be 2°43′, which is also the value of the angle *BCE,* which is equal to the above-mentioned angle *ACD,* also 2°43′, so that by this calculation *GH* passes through the fixed Earth *C.* But if such equality had not resulted, as is the case with the two lower Planets, the method of the following (2nd) example would have to be used.

NOTE.

It is also to be noted that the deviation of Saturn's deferent from the ecliptic on the theory of a fixed Earth can be found in a different manner, and since this makes the similarity of the two theories of a fixed and a moving Earth even clearer, I will describe it at the same time. By way of preliminary let there be drawn *DK,* equal and parallel to *GC.* Similarly *EL,* equal and parallel to *HC.* This being so, the crossed quadrilateral *KDEL* is equal and similar to the crossed quadrilateral *ADEB* of the 22nd proposition of the 3rd book. Hence, when the deviation of the deferent *CD* from the ecliptic is sought therewith in the manner of the said 22nd proposition, the result is the required value and must needs be identical with the result of the foregoing first procedure. And thus it is clearly evident that the figure of the theory of a moving Earth, to wit, the crossed quadrilateral *KDEL,* with the Earth's orbit *KL,* leads to the same result as the figure of the theory of a fixed Earth *C,* and its epicycle on one side *GDF,* on the other side *HEI,* for the triangle *CDG* is equal and similar to the triangle *DCK,* and so is *CEH* to *ECL,* from which it follows that the same deviation that is found for *CD,* from the known terms of the triangle *DCK,* must also be found

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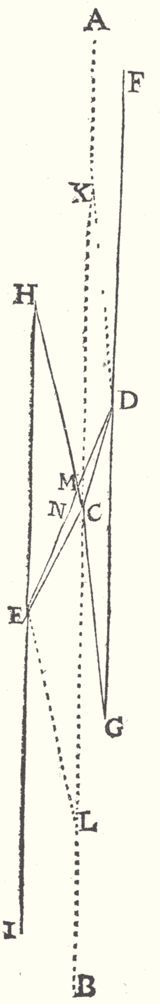
[1]) Stevin's *Trigonometry* (Work XI; i, 12) p. 156.

ſghelijcx te verſtaen met d'ander twee driehoucken. En ſulcx als hier gheweeſt is het voorbeelt met Saturnus, alſo iſt kennelick te ſullen ſijn met d'ander twee bovenſte Iupiter en Mars.

2 *Voorbeelt vande tvvee onderſte Dvvaelders.*

Hoewel al de letters inde form deſes 2 voorbeelts van ſulcke beteyckening ſijn als die des eerſten voorbeelts, en dat yemant achten mocht de meyning van dit deur t'eerſte genouch bekent te weſen, nochtans inſiende de verſcheydenheyt der uyterlicke form, te weten dat hier den inrontwech veel cleender is dan het inront, daer int 1 voorbeelt de wech veel grooter was, en dat bovẽ dien deſe wech niet deur dẽ Eertcloot en ſtreſt, ſo beſchrijf ick noch dit 2 voorbeelt.

T'GEGEVEN. Laet de deelen der volgende form A B C D E F G H I K L ſijn van Mercurius, met ſulcke beteyckening als de deelen der form des eerſten voorbeelts van Saturnus, doch daer in verſchillende, dat deſen inrontswech D E die ghevonden ſy gheweeſt na de manier vant werck beſchreven int 1 voorbeelt, niet en ſtrecke deur den Eertcloot C, maer ſniende den duyſteraer int punt M: Ten anderen daer het inront van Saturnus veel cleender viel dan ſijn wech, in die placts iſt iſt hier veel grooter, te weten F G in ſulckẽ reden tot D E, als inde form des 1 voorſtels deſes Byvoughs de lini P Q tot Q R, voort doet den houck B C G 1 tr. 45 ①, en A C H 4 tr. 5 ① als bevondẽ wiert inde ervaring van Mercurius breede achter het 25 voorſtel des 3 boucx. T'BEGHEERDE. Wy moeten vinden des inrontwechs D E afwijcking vanden duyſteraer, dats den houck A M D.



T'WERCK.

Deur des driehoucx D C G drie bekende palen, wort als op de wijſe des 1 voorbeelts ghevonden den houck — C D G.
En den houck — D C G.
Daer toe den ghegheven houck — B C G.
Comt den houck — B C D.
Die ghetrocken van 180 tr. blijft den houck — D C K.
Daer toe vergaert den ghegheven houck — K C H.
Comt den houck — D C H.
Deur des driehoucx E C H drie bekende palen wort als op de wijſe des 1 voorbeelts, ghevonden den houck — H C E.
Die vergaert tot D C H ſevende in d'oirdẽ, comt den houck — D C E.
De driehouck D C E heeft drie bekende palen, te weten dien houck D C E neghende in d'oirden, en de twee linien C D, C E, vanden Eertcloot tot des inronts middelpunt, die bekent worden deur

from the known terms of the triangle *CDG*. The same is also to be understood for the other two triangles. And such as has here been the example with Saturn, the same will it obviously also be with the other two upper Planets Jupiter and Mars.

2nd Example of the Two Lower Planets.

Although all the letters in the figure of this 2nd example have the same denotation as those of the first example, and it may be considered that the meaning of this example is sufficiently clear from the first, nevertheless, recognizing the difference of outward appearance, to wit, that here the deferent is much smaller than the epicycle, whereas in the 1st example the deferent was much larger, while moreover this deferent does not pass through the Earth, I will also describe this 2nd example.

SUPPOSITION. Let the parts of the following figure *ABCDEFGHIKL* be of Mercury, with the same denotation as the parts of the figure of the first example of Saturn, but differing in that this deferent *DE*, which shall have been found in the manner of the procedure described in the 1st example, does not pass through the Earth *C*, but intersects the ecliptic in the point *M*. Secondly, whereas the epicycle of Saturn was much smaller than its deferent, it is much larger here instead, to wit, *FG* in the same ratio to *DE* as in the figure of the 1st proposition of this Supplement the line *PQ* to *QR*. Further the angle *BCG* makes $1°45'$ and *ACH* $4°5'$, as was found in the experience of Mercury's latitude, after the 25th proposition of the 3rd book. WHAT IS REQUIRED. We have to find the deviation of the deferent *DE* from the ecliptic, *i.e.* the angle *AMD*.

PROCEDURE.

From the three known terms of the triangle *DCG* is found, in the same manner as in the 1st example, the angle	*CDG*.
And the angle	*DCG*.
When to this is added the angle	*BCG*,
We get the angle	*BCD*.
When this is subtracted from 180°, there is left the angle	*DCK*.
When to this is added the given angle	*KCH*,
We get the angle	*BCD*.
From the three known terms of the triangle *ECH* there is found, in the manner of the 1st example, the angle	*HCE*.
When this is added to *DCH* (the seventh in this list), we get the angle	*DCE*.

The triangle *DCE* has three known terms, to wit, that angle *DCE* (the ninth in the list) and the two lines *CD, CE,* from the Earth to the deur t'vervolgh van het 13 voorstel des 3 boucx: Hier me wort ghevonden den houck CDE.

Daer toe vergaert dē houck C D G eerste in d'oirden, comt dē houck E D G, dats oock dē houck M D G, welcke evē sijnde met A M D, om dat M D is tusschen de twee evewijdege K M, D G, soo is daer bekent de begheerde afwijcking AMD.

T'is oock kennelick hoe ghevonden sal worden de langde der lini M C, dat is soo verre de duysteraersne vanden Eertcloot valt, want den driehouck D C M heeft drie bekende palē, te weten dē houck C D M, als wesende den houck C D E achtste in d'oirden, voort den houck M C D, als even sijnde metten houck C D G eerste in d'oirden, om dat C D is tusschen de twee evewijdeghe K C, D G, ten derden de sijde C D, met welcke drie palen als gheseyt is bekent wort de lini MC.

Noch is te weten dat Mercurius inrontwechs afwijcking vanden duysteraer met stelling eens vasten Eertcloots can gevonden worden op een ander wijse, welcke alsoose noch opentlicker verclaert de gemeenschap der twee stellinghen eens vasten en roerenden Eertcloots, soo sal ick die met een beschrijven. Laet als by manier van bereytsel ghetrocken worden D K, even en evewijdeghe met G C: Sghelijcx E L even en evewijdeghe met H C, t'welck soo sijnde, de cruysvierhouck K D E L, is even en ghelijck metten cruysvierhouck L O M P int 3 lidt van Mercurius breede achter het 25 voorstel des 3 boucx, daerom hier me ghesocht des inrontwechs M D afwijcking vanden duysteraer na de manier des selven 3 lidts, t'ghene daer uyt comt is t'begheerde, en moet nootsakelick even sijn mettet besluyt vande voorgaende eerste werking. En blijckt hier me sichtbaerlick hoe de form der stelling eens roerenden Eertcloots, een selve besluyt voortbrengt als de form der stelling eens vastē Eertcloots, also van dergelijcke wat breeder geseyt is achter het 1 voorbeelt.

En sulcx als hier gheweest is het voorbeelt met Mercurius, alsoo ist kennelick te sullen sijn met dander onderste Dwaelster Venus. T'BESLVYT. Wesende dan ghegheven eens Dwaelders meeste noordersche en zuydersche breede, wy hebben gevonden sijn wechs afwijcking vanden duysteraer: Oock mede hoe verre de duysteraersne vanden Eertcloot valt, deur wisconstighe werking ghegront op stelling eens vasten Eertcloots, na den eysch.

6 VOORSTEL.

Te vinden eens Dvvaelders schijnbaer duysteraerbreede op een ghegheven tijt, deur vvisconstige vverckinge gegront op stelling eens vasten Eertcloots.

Om t'begheerde te crijghen, wy sullen eerst vinden de lini vanden Dwaelder int inront rechthouckich op den duysteraer. Als by gelijckenis inde form des 5 voorstels deses Byvoughs, de verdochte lini van F rechthouckich opt plat des duysteraers A B: Maer sulcke lini is tot allen plaetsen des inronts, even ande lini van des inronts middelpunt als D, rechthouckich op A B, deur diē D F evewijdeghe is met A B, en daerom salmen dese altijt vinden in plaets van die, t'welck doende soo en sal de manier van t'vinden der selve geen verschil hebben, mette vinding van dergelijcke lini in stelling eens roerenden Eertcloots epicycle's centre, which become known from the sequel of the 13th proposition of the 3rd book.

Herewith is found the angle CDE.

When to this is added the angle *CDG* (the first in the list), we get the angle *EDG, i.e.* also the angle *MDG,* and since this is equal to *AMD,* because *MD* is between the two parallel lines *KM, DG,* there is known the required deviation *AMD.*

It is also obvious how the length of the line *MC* has to be found, *i.e.* as far as the line of nodes is from the Earth, for the triangle *DCM* has three known terms, to wit, the angle *CDM,* which is also the angle *CDE* (the eighth in the list); further the angle *MCD,* which is equal to the angle *CDG* (the first in the list) because *CD* is between the two parallel lines *KC, DG;* thirdly, the side *CD,* by means of which three terms, as has been said, is found the line *MC.*

It is also to be noted that the deviation of Mercury's deferent from the ecliptic on the theory of a fixed Earth can be found in a different manner, and since this makes the similarity of the two theories of a fixed and a moving Earth even clearer, I will describe it at the same time. By way of preliminary let there be drawn *DK,* equal and parallel to *GC.* Similarly *EL* equal and parallel to *HC.* This being so, the crossed quadrilateral *KDEL* is equal and similar to the crossed quadrilateral *LOMP* in the 3rd section of Mercury's latitude, after the 25th proposition of the 3rd book [1]). Hence, when the deviation of the deferent *MD* from the ecliptic is sought therewith after the manner of the said 3rd section, the result is the required value and must needs be identical with the result of the foregoing first procedure. And from this it is clearly evident that the figure of the theory of a moving Earth leads to the same result as the figure of the theory of a fixed Earth, as has been said somewhat more fully in a similar case after the 1st example.

And such as has here been the example with Mercury, the same will it obviously be with the other lower Planet Venus. CONCLUSION. Given a Planet's most northerly and southerly latitudes, we have thus found the deviation of its deferent from the ecliptic; also how far the line of nodes is from the Earth, by mathematical operations based on the theory of a fixed Earth; as required.

6th PROPOSITION.

To find a Planet's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a fixed Earth.

In order to get the required value, we first have to find the line from the Planet in the epicycle perpendicular to the ecliptic; for example, in the figure of the 5th proposition of this Supplement, the imagined line from *F* perpendicular to the plane of the ecliptic *AB.* But such a line is in any place of the epicycle equal to the line from the epicycle's centre (*D*) perpendicular to *AB,* because *DF* is parallel to *AB,* and therefore the one will always be found instead of the other. When doing so, the method of finding it will not be different from the finding of a similar line, described in the theory of a moving Earth in the 24th proposition of the 3rd book. The cause of this similarity is even more

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[1]) In the figure on page 248.

beſchreven int 24 voorſtel des 3 boucx: De oirſaeck deſer ghelijckheyt is daer deur noch kennelicker, dat den Dwaelder met ſtelling eens roerenden Eertcloots, comt ter plaets van des inronts middelpunt met ſtelling eens vaſten Eertcloots. Nu dan ſulcke lini alſoo bekent wordende, en daer benevens deur t'vervolgh vant 13 voorſtel des 3 boucx de lini vanden vaſten Eertcloot totten Dwaelder, ſoo heeft den rechthouckighen driehouck begrepen onder de ſelve twee linien, en de derde* ſchoenſche drie bekende palen, waer me opnbaerlick ghevonden wort de begheerde breede als int 25 voorſtel des 3 boucx met ſtelling eens roerenden Eertcloots, ſulcx dat my onnoodich ghedocht heeft het ſelve hier andermael int langhe te beſchrijven. T'BESLVYT. Wy hebben dan ghevonden eens Dwaelders ſchijnbaer duyſteraerbreede op een ghegheven tijt, deur wiſconſtighe wercking ghegront op ſtelling eens vaſten Eertcloots, na den eyſch.

Hypothenuſa

SAMING

Van ettelicke overeencommingen en verſchillen tuſſchen de beſchrijving des breedeloops deſes Byvoughs, mette breedeloop van *Ptolemeus*.

Alſoo my inde voorgaende beſchrijving deſer ſtof des breedeloops, ſomwijlen voorvielen eenighe verſcheydenheden tuſſchen de ſelve en de breedeloop by *Ptolemeus* beſchreven, ſoo heeft my bequamer ghedocht die hier by den anderen te verſamen, dan inde voorgaende leering te vermenghen, ſulcx dat ick daer af ſtellen ſal de volghende leden.

1 LIDT.

Angheſien d'oirſaeck des breedeloops der Dwaelders ghetrocken wort uyt ſtelling eens roerenden Eertcloots die tot *Ptolemeus* tijt onbekent ſcheen, ſoo heeft hy nochtans int veroirdenen der ſpiegheling vande drie bovenſte, de ſake al ſeer na ghecommen, ſtellende het inront te wijle het loopt altijt bycans evewijdich vanden duyſteraer te blijven, want ſoo hy dat gelijck de ſake vereyſcht, en int 2 voorſtel des Byvoughs beweſen is te moeten ſijn, voor heel evewijdich ghenomen hadde (t'ſchilde als blijckt int 3 Hoofſtick ſijns 13 boucx in Saturnusloop alleenelick 2 tr. 4 Ⓘ, dat den houck gemaeckt vant inront mette wech, ten alderhoochſten grooter was dan den houck gemaeckt vande wech metten duyſteraer: In Iupiter was ſulck verſchil alleenelick van 1 tr. 6 Ⓘ, in Mars van 1 tr. 15 Ⓘ, ja het inront anden duyſteraer ſijnde, t'waſſer volgens ſijn ſtelling teenemael in ſonder ſnyen, ghelijck na de volcommen ſpiegheling ſijn moet) ſoo en waer de taſtende onſeker wercking int ſoucken vande afwijcking der inrontswegen beſchreven int ſelve 3 Hoofſtick niet noodich gheweeſt, en ſouden d'ander rekeninghen van der Dwaelders breeden, dan deurgaens overeenghecommen hebben met deſe, ſonder daer af inde reſt ſoo te verſchillen als int volghende 2 lidt gheſeyt ſal worden.

2 LIDT.

Ptolemeus tafel van Saturnus breedeloop int 5 voorſtel ſijns 13 boucx, en ſchijnt geen genouchſaem overeencomming te hebben met ſijn ſpiegheling. Om van t'welck breeder verclaring te doen, ſoo is te weten dat hy int 4 voorſtel

vant

obvious from the fact that on the theory of a moving Earth the Planet comes at the place of the epicycle's centre on the theory of a fixed Earth. Hence, this line thus becoming known, and in addition, from the sequel of the 13th proposition of the 3rd book, also the line from the fixed Earth to the Planet, the right-angled triangle contained between these two lines and the third (the hypotenuse) has three known terms, with which the required latitude is evidently found, as in the 25th proposition of the 3rd book on the theory of a moving Earth, so that I deemed it unnecessary to describe this here once more at length. CONCLUSION. We have thus found a Planet's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a fixed Earth; as required.

COLLECTION.

Of some similarities and differences between the description of the motion in latitude according to this Supplement and the motion in latitude according to *Ptolemy.*

Since in the foregoing description of this subject matter of the motion in latitude I sometimes came across some differences between it and the motion in latitude described by *Ptolemy,* it seemed more suitable to me to collect them here with the others than to mix them with the foregoing subjects, so that I will give thereof the following sections.

1st SECTION.

Since the cause of the motion in latitude of the Planets is derived from the theory of a moving Earth, which seemed unknown in *Ptolemy's* time, nevertheless in framing the theory of the three upper Planets he came very near to the matter, assuming the epicycle, while moving, to remain always nearly parallel to the ecliptic; for if — as the matter requires and has been proved to be true in the 2nd proposition of the Supplement — he had taken it to be perfectly parallel (as appears in the 3rd Chapter of his 13th book, the difference for Saturn's motion was only 2°4′ by which amount the angle made by the epicycle with the deferent was greater at most than the angle made by the deferent with the ecliptic; with Jupiter this difference was only 1°6′, with Mars 1°15′, nay, when the epicycle was in the ecliptic, according to his assumption it fell altogether in it, without intersecting it, as it must be according to the perfect theory), the tentative, uncertain procedure in the seeking of the deviation of the deferents, described in the said 3rd Chapter, would not have been necessary, and the other calculations of the Planets' latitudes would generally have agreed with the present calculations, without further differing therefrom as much as is to be mentioned in the following (2nd) section.

2nd SECTION.

Ptolemy's table of Saturn's motion in latitude in the 5th chapter [1]) of his 13th book seems not to be sufficiently in agreement with his theory. To explain this more fully, it is to be noted that in the 4th chapter of the said 13th book he takes the deferent's deviation from

[1]) Stevin designates Ptolemy's chapters by the same name of *Voorstel* which he also uses for the propositions.

vant selfde 13 bouck, des inrontwechs afwijcking vanden duysteraer, die inde eerste form vant 5 voorstel deses Byvoughs beteyckent sy metten houck A C D neemt op 2 tr. 26 Ⓘ.

Den houck C D G op 4 tr. 30 Ⓘ.

Het halfrontschil van dien doet voor den houck C D F 175 tr. 30 Ⓘ.

By aldien nu vanden Eertcloot C, tot des inronts verstepunt F, wesende mettet middelpunt an sijn wechs verstepunt D, getrocken waer de lini C F, sy soude doen deur de Byeenvouging des 13 voorstels vant 3 bouck 108718.

Waer me sulcke driehouck C D F drie bekende palen soude hebbē, te weten dē houck C D F 175 tr. 30, derde in d'oirden, D F 10000, en C F 108718 vierde in d'oirden: Hier me ghesocht den houck D C F, wort bevondē deur het 5 voorstel der platte driehouckē vā 25 Ⓘ.

Die ghetrocken vanden houck A C D 2 tr. 26 Ⓘ eerste in d'oirden, blijft voor den houck A C F 2 tr. 1 Ⓘ.

En soo veel moet volghens *Ptolemeus* spiegeling wesen Saturnus breede als hy is an des inronts verstepunt F, met des inronts middelpunt an sijn wechs verstepunt D. Maer volgens t'gebruyck der boveschrevē tafels, so is hy dan sonder breede, waer deur die tafels soo veel van haer spiegheling verschillen.

En doēde dergelijcke op d'ander sijde als Saturnus is ant inrōts verstepunt I, men sal sijn breede bevinden van 1 tr. 58 Ⓘ, in welcker plaets hy volghens de tafels sonder breede bevonden wort. Inder vougen dat, volghens t'ghene ick voorgenomen hadde te verclarē, *Ptolemeus* tafel van Saturnus breedeloop geē genouchsaem overeencomming en schijnt te hebbē met sijn spiegelingen. Eñ dergelijcke is oock te verstaen vande tafels en spiegelingē der Dwaelders Iupiter en Mars. Maer hoe groot soodanighe breeden vallen met stelling eens roerenden Eertcloots, dats openbaer deur het 25 voorstel des 3 boucx.

3 LIDT.

Int 1 lidt is geseyt dat *Ptolemeus* niet verre vande rechte spiegheling en was, stellende de inronden vande drie bovenste Dwaelders altijt bycans evewijdich vanden duysteraer te blijven, maer sulcx en is mette twee onderste soo na niet geluckt, waer af hy uyt sijn gageslagen ervaringen niet sulcken wijse van spiegeling trecken en conde, als hy uyt de drie bovenste gedaen hadde: De voornamelicste oorsaeck die hem verhinderde schijnt tweederley, d'eerste dusdanich: T'gene hy Mercurius inront noemde en daer voor gebruycte en wast niet deur het 1 voorstel deses Byvoughs, maer veel cleender dan na t'behooren, sulcx dat als hy t'selve al prouvende annam voor altijt bycans evewijdich vanden duysteraer te blijven, gelijck hy mette inronden der drie bovenste dede, dat en hadde mette ervaringen geen gemeenschap, want het was t'ander groot ront datmen daer toe nemen moest. D'ander oirsaeck te weten die hem de waggeling des inrontwechs dede besluyten, en de duysteraersne op een verkeerde sijde nemen, schijnt dat dusdanich was: Hy heeft sich in gebeelt alle gemeene sneen der inrontwegen en des duysteraers deur den Eertcloot te strecken: Als by voorbeelt dē inrontwech D E, die inde form des 5 voorstels deses Byvoughs dē duysteraer eygentlick in M deursnijt, heeft hy ghemeent te strecken deur den vasten Eertcloot C, waer op hy sijn rekeningē makende, soo isser uyt gevolght dat doen hy meende het inronts middelpunt gecommē te wesen van D tot C, en alsdan behoiren inden duysteraer A B te wesen sonder breede, soo bevant hijt metter the ecliptic, which in the first figure of the 5th proposition of this Supplement [1]) is denoted by the angle *ACD*, to be 2°26′

The angle *CDG* to be 4°30′

The supplement thereof (the angle *CDF*) makes 175°30′

Hence, if from the Earth *C* to the epicycle's apogee *F*, being with the centre at its deferent's apogee *D*, the line *CF* were drawn, by the Compilation of the 13th proposition of the 3rd book it would make [2]) 108,718

Thus, this triangle *CDF* would have three known terms, to wit, the angle *CDF* = 175°30′ (the third in the present list), *DF* = 10,000, and *CF* = 108,718 (the fourth in the list). When the angle *DCF* is sought therewith, by the 5th proposition of plane triangles it is found to be 25′

When this is subtracted from the angle *ACD* = 2°26′ (the first in the list), there is left for the angle *ACF* 2°1′.

And this, according to *Ptolemy's* theory, must be the amount of Saturn's latitude when it is at the epicycle's apogee *F*, with the epicycle's centre at its deferent's apogee *D*. But according to the use of the above-mentioned tables it is then without latitude [3]), owing to which these tables differ so much from the theory.

And when we do the same on the other side, when Saturn is at the epicycle's apogee *I*, its latitude will be found to be 1°58′, whereas according to the tables it is found without latitude, so that, in accordance with what I had intended to explain, *Ptolemy's* table of Saturn's motion in latitude seems not to be sufficiently in agreement with his theories [4]). And the same is also to be understood for the tables and theories concerning the Planets Jupiter and Mars. But the amounts of these latitudes on the theory of a moving Earth are evident from the 25th proposition of the 3rd book.

3rd SECTION.

In the 1st section it has been said that *Ptolemy* was not far from the true theory in assuming the epicycles of the three upper Planets to remain always nearly parallel to the ecliptic, but he did not succeed as well with the two lower Planets, for which he could not deduce from his observational experiences the same kind of theory as he had done for the three upper Planets. The chief cause that prevented him from doing so seems to be of a twofold nature, the first being as follows. What he called Mercury's epicycle (and used as such) was not so by the 1st proposition of the present Supplement, but much smaller than it ought to be, so that when, testing it, he assumed this to remain always nearly parallel to the ecliptic, as he did with the epicycles of the three upper Planets, this did not agree with the experiences, because it was the other greater circle that ought to be taken for it. The other cause, to wit, the one that made him deduce the oscillation of the deferent and take the line of nodes on the wrong side, seems to have been as follows. He imagined all the intersections of the deferents and the ecliptic to pass through the Earth. For example, he thought

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[1]) Page 270.

[2]) This value is not found in the 13th proposition. Compare footnote to page 269.

[3]) [4]) Stevin is wrong here, because he overlooks Ptolemy's remark that before entering the table of the sixtieths the argument has to be increased by 50° (for Saturn; decreased by 20° for Jupiter).

daet daer buyten an N (welverstaende dat D N even geteyckent is an D C) met soo veel breede als veroirsaeckt wort deur de verheyt van C tot N, welcke misgrijping hem heeft doē des inrontwechs waggeling versieren, die beschreven is int 2 lidt van Mercurius breede achter het 25 voorstel des 3 boucx.

4 LIDT.

Maer angesien *Ptolemeus* de voorschreven waggeling des Dwaelders van C tot N, seyde deur ervaring in Mercurius bevondē te hebben van 45 ①, soo sullē wy nu ondersouckē hoe sulcx hier me overeencomt: Tot dien einde segh ick dat de verheyt van C tot N, anghewesen wort mette lini N R inde form des 3 lidts van Mercurius loop achter het 25 voorstel des 3 boucx, diens langde ick aldus vinde: De driehouck Q N R heeft drie bekende palē, te weten dē houck N Q R 5 tr. 32 ①, als even sijnde met O Q L 5 tr. 32 ① deur des voorschrevē 3 lidts vierde in d'oirden, Q N 1364 deur des selfden lidts vijfde in d'oirden, en den houck Q N R recht: Hier me ghesocht de lini N R, wort bevonden deur het 4 voorstel der platte driehoucken van 132.

By aldien my *Ptolemeus* verclaert hadde, tot wat plaets de Son was ten tijde van sijn dadelicke ervaringē doen hy de afweging van 45 ① vandt, waer deur ons met stelling eens roerenden Eertcloots bekent soude sijn des selven Eertcloots plaets, soo mochten wy na die overeencommingen met meerder sekerheyt trachten, maer dat niet gheschiet sijnde, wy sullen de twee uyterste afwijckinghen soucken om te sien ofter de 45 ① oock tusschen vallen, als volght:

Doende de lini N R als vooren 132, en datmen daer an vervought na de manier des 25 voorstels int 3 bouck de meeste verheyt van Mercurius totten Eertcloot, doende deur de Byeenvouging des 13 voorstels vant 3 bouck 14519, soo wort deur de gemeene regel des selfden 25 voorstels Mercurius breede daer me bevonden van 31 ①: Maer van 1 tr. 21 ① op de minste verheyt 5481, tusschen welcke 1 tr. 21 ① en 31 ①, de boveschreven 45 ① der ervaring sijn. Soo nu bekent waer de lini vanden Eertcloot tot Mercurius ten tijde der ervaring, mē soude als geseyt is van dese overeencomming noch eygentlicker connē sprekē.

Angaende Venus afweging welcke *Ptolemeus* stelt op 10 ①, die soude na de voorgaende wijse van Mercurius ten cleensten bevonden worden van 20 ①: Watter eyghentlick af is, daer soudemen deur nieuwe dadelicke gaslaginghen nauwer af connen oordeelen.

BESLVYT DES BREEDELOOPS.

Tot hier toe is vande breedeloop der vijf Dwaelders Saturnus, Iupiter, Mars, Venus en Mercurius, beschreven t'ghene ick voor my ghenomen hadde, waer me ick oock meyne de selve niet meer voor soo een onbekent roersel behooren ghenomen te worden ghelijckmen ghedaen heeft, als blijckende deur des Eertcloots loop sulcke verscheydenheden nootsakelick te moeten vallen, en elck Dwaelders hemel eenvoudelick te draeyen op haer as, sonder datmen behoeft eenich onnatuerlick roersel daer by te versieren, en datmen al die voorgaende tijtslijtighe haspeling sal moghen verlaten, oock met oirsakelicker kennis sich voortaen daer in oeffenen.

that the deferent *DE,* which in the figure of the 5th proposition of the present Supplement really intersects the ecliptic in *M,* passes through the fixed Earth *C,* and when he based his calculations thereon, it followed that when he thought the epicycle's centre had moved from *D* to *C,* and then ought to be in the ecliptic *AB,* without any latitude, in actual fact he found it outside, at *N* (it being understood that *DN* has been drawn to be equal to *DC*), with so much latitude as is caused by the distance from *C* to *N;* this misconception made him imagine the deferent's oscillation, described in the 2nd section of Mercury's latitude, after the 25th proposition of the 3rd book.

4th SECTION.

But since *Ptolemy* said that for Mercury he had found the aforesaid oscillation of the Planet from *C* to *N* by experience to be 45′, we will now examine how this agrees therewith. To this end I say that the distance from *C* to *N* is denoted by the line *NR* in the figure of the 3rd section of Mercury's motion, after the 25th proposition of the 3rd book, whose length I find as follows. The triangle *QNR* has three known terms, to wit, the angle $NQR = 5°32'$, as being equal to $OQL = 5°32'$, by the fourth in the list of the aforesaid 3rd section, $QN = 1{,}364$ by the fifth in the list of the said section, and the angle *QNR* being a right angle. When the line *NR* is sought therewith, by the 4th proposition of plane triangles it is found to be 132.

If *Ptolemy* had told me at what place the Sun was at the time of his practical experiences, when he found the deviation of 45′, from which on the theory of a moving Earth we should know the place of the said Earth, we might try with greater certainty to find those correspondences, but since this has not happened, we have to seek the two extreme deviations, to see whether the 45′ fall in between, as follows: When the line *NR* makes, as above, 132 and to this is added, after the manner of the 25th proposition in the 3rd book, the greatest distance from Mercury to the Earth, which by the Compilation of the 13th proposition of the 3rd book makes 14,519, by the common rule of the said 25th proposition Mercury's latitude is therewith found to be 31′. But this would be found to be 1°21′ at the smallest distance 5,481, and between this 1°21′ and 31′ lie the aforesaid 45′ of the experience. If now the line from the Earth to Mercury at the time of the experience were known, it would be possible, as has been said, to speak more truly of this correspondence.

As to Venus' deviation, which *Ptolemy* takes to be 10′, by the foregoing method for Mercury this would be found to be at least 20′. The true state of affairs might be judged more accurately by means of new practical experiences.

CONCLUSION OF THE MOTION IN LATITUDE.

Up to this point that part of the motion in latitude of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury which I had intended has been described, and therefore I am also of opinion that this should no longer be considered such an unknown motion as it has been done, since it appears that from the Earth's motion such differences are inevitable, and that the Heaven of each Planet simply turns about its axis, without our having to invent any unnatural motion, and that all the foregoing time-devouring complications can be abandoned, and the matter can henceforth be practised with better knowledge of the causes.

ANHANGH DES HEMELLOOPS VAN DER DWAELDERS

onbekende roerſels by *Ptolemeus* gageſlagen: En vande ſpieghelingen by hem en *Copernicus* daer uyt beſchreven.

CORTBEGRYP DESES ANHANGS.

NAdien Ptolemeus *ter handt ghecommen vvas het Hemelloopſchrift met ſtelling eens vaſten Eertcloots, eenvoudich gelijckt int voorgaende verclaert is, ſoo heeft hy der Dvvaelders plaetſen en loopen ſeer neerſtelicken gageſlagen en onderſocht, om te ſien hoeſe daer me overeen quamē: De beſchrijving des loops der Son (daer* Hypparchus *an tvvijffelde) docht hem recht, maer niet van d'ander ſes Dvvaelders, vvant hoevvel hemlien middelloop op ſeer langhe tijt effen genouch uyt quam, ſoo oordeelde hy nochtans in de beſonder keeren verſcheydenheyt te vveſen, ſulcx dat hy daer af ſijn ſpiegheling beſchreef, en die vermengde onder de voornoemde eenvoudige loop mette ſtelling eens vaſten Eertcloots. En dergelijcke vermenging heeft daer na* Copernicus *van ſijn ſpiegeling oock gedaen mette ſtelling eens roerendē Eertcloots. Maer vvant de plaetſen der Dvvaelders daer na gagheſlagen, niet bevonden en vvorden met die regels overeen te commen, en dat daerom die roerſels als noch onbekent ſchijnen, ſoo heb ickſe int voorgaende in d'een en d'ander ſtelling vveerom uyt be bekende gheſcheyden: Maer op datmen eyghentlicker vveten ſoude hoedanich die uytgheſcheyden byvouging vvas, ſoo is om deſe redenen en noch ander die int voorgaende breeder verclaert ſijn, daer af deſen Anhang gemaect, die derthien voorſtellen ſal hebben, vveſende de ſeven van* Ptolemeus *bygevoughde ſpiegheling, te vveten d'eerſte vijf vande Maen. Het ſeſte van Saturnus, Jupiter, Mars en Venus. Het ſevende van Mercurius. D'ander vijf ſijn van* Copernicus *bygevoughde ſpiegeling te vveten het 8 vande Maen. Het 9 van Saturnus, Iupiter en Mars. Het 10 van Venus. Het 11 en 12 van Mercurius. Het 13 een verhael op der ſterren onbekende loop, en des duyſteraers onbekende afvvijcking vanden evenaer.*

EERST

APPENDIX

TO THE HEAVENLY MOTIONS

OF THE PLANETS' UNKNOWN MOTIONS

observed by Ptolemy; and the Theories derived therefrom by him and Copernicus

SUMMARY OF THIS APPENDIX

After the description of the Heavenly Motions on the assumption of a fixed Earth had come into the hands of *Ptolemy,* in the simple form in which it has been set forth in the foregoing, he very industriously observed and investigated the Planets' positions and motions, so as to see in how far they agreed therewith. The description of the motion of the Sun (which *Hipparchus* doubted) appeared to him to be correct, but not that of the other six Planets, for though their mean motion accorded well enough in a very long time, yet he was of opinion that there was some difference in the separate revolutions, so that he described his theory about this and combined it with the aforesaid simple motion on the assumption of a fixed Earth. And a similar combination was also made thereafter by *Copernicus* for his theory on the assumption of a moving Earth. But because the positions of the Planets observed thereafter are not found to agree with those rules, so that those motions still seem unknown, I have separated them again from the known motions in the foregoing, on one as well as the other assumption. But in order that the reader might know more truly of what nature those separated additions were, for this reason as well as for others, which have been set forth more fully in the foregoing, this Appendix has been made about them. It is to comprise thirteen propositions, seven of them relating to the theory added by *Ptolemy,* to wit, the first five of the Moon, the sixth of Saturn, Jupiter, Mars, and Venus, the seventh of Mercury. The other five relate to the theory added by *Copernicus,* to wit, the 8th of the Moon, the 9th of Saturn, Jupiter, and Mars. The 10th of Venus. The 11th and 12th of Mercury. The 13th is an account of the stars' unknown motions and the ecliptic's unknown deviation from the equator.

EERST PTOLEMEVS BYGHEvoughde spiegheling des Dvvaelders met stelling eens vasten Eertcloots.

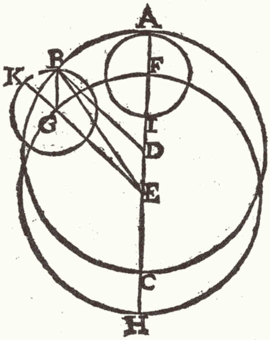
1 VOORSTEL.

Wesende een Dvvaelder gestelt in een uytmiddelpuntichront, of anders in een inront diẽs halfmiddellijn even is an dese uytmiddelpuntichronts uytmiddelpuntichey tlijn, en dat inront in een middelpuntichront, even in grootheyt en loop ant uytmiddelpuntich, en des Dvvaelders loop int inront ghelijck metten loop des inronts int middelpuntichront, doch op een verkeerde sijde: Die tvvee stellinghen gheven den Dvvaelder een selve plaets.

Anghesien de stelling der Maen in een uytmiddelpuntighe wech na de manier inde voorgaende twee boucken beschreven, en volghens t'ghene inde natuer schijnt te bestaen, heur de selve plaets gheeft diese crijcht met een middelpuntich inront op de wijse deses voorstels, die *Ptolemeus* vercoos om sijn ghevonden tweede oneventheden bequamelicker te verclaren, en dat ick de selve oneventheden hier beschrijven wil, soo heeft my voughelick ghedocht eerst te bewijsen (ghelijck *Ptolemeus* oock gedaen heeft int 5 Hoofdstick sijns 4 boucx) sulcke twee stellingen alsoo te overcommen, t'welck d'oirsaeck is der beschrijvinghen van dit voorstel tot dese plaets.

T'GHEGHEVEN. Laet voor eerste stelling het uytmiddelpuntichront A B C de Dwaelderwech beteyckenen, diens middelpunt D, en E den vasten Eertcloot, t'punt A sy den Dwaelder ten eersten ant verstepunt, welcke daer na ghedaen hebbe een loop van A tot B, dats oock den houck A D B.

Laet nu voor tweede stelling op E als middelpunt, beschreven worden het middelpuntichront F G H, even an A B C, en op F mette halfmiddellijn F A (die even moet vallen met E D) het inront A I, diẽs verstepunt A, daer na sy het inronts A I middelpunt F ghecommen an G, sulcx dat sijn loop F G, of houck F E G, even sy anden loop der eerster stelling A D B, en het inront beschrevẽ opt middelpunt G, sy B K, waer in van E deur G ghetrocken sy E G K, soo dat K des inronts verstepunt beteyckent, van t'welck den Dwaelder daerentusschen gheloopen heb na B (tegen d'eerste loop van F na G) een booch ghelijck met F G, of anders geseyt sulcx dat den houck begrepen tusschen de lini K G, en de lini van G na B tot inden omtreck des inronts, welcke lini men crijcht treckende van G een evewijdeghe met E F.



T'BEGHEERDE. Wy moeten bewijsen dat dẽ Dwaelder in dese tweede stelling int selve punt B valt, daer

FIRST THE THEORY OF THE PLANETS, ADDED BY PTOLEMY, ON THE ASSUMPTION OF A FIXED EARTH

1st PROPOSITION.

When a Planet is placed on an eccentric circle or otherwise on an epicycle whose semi-diameter is equal to the line of eccentricity of this eccentric circle, while that epicycle is on a centric [1]) circle, equal in magnitude and motion to the eccentric circle, and the Planet's motion on the epicycle is equal to the motion of the epicycle on the centric circle, but in an opposite direction, those two locations give the Planet the same place.

Since the location of the Moon in an eccentric orbit, after the manner described in the foregoing two books and in accordance with what seems to exist in nature, gives it the same position it gets with a centric epicycle [2]) in the manner of the present proposition, which *Ptolemy* chose in order to explain more conveniently the second inequalities found by him, and since I here wish to describe these inequalities, it seemed appropriate to me first to prove (as *Ptolemy* has also done in the 5th Chapter of his 4th book) that these two locations agree, which is the cause of the descriptions of this proposition in this place.

SUPPOSITION. For the first location let the eccentric circle *ABC* denote the Planet's orbit, its centre being *D,* and *E* the fixed Earth; let the point *A* be the Planet first at the apogee, which thereafter shall have moved from *A* to *B,* that is also the angle *ADB.*

Now for the second location let there be described about *E* as centre the centric circle *FGH,* equal to *ABC,* and about *F* with the semi-diameter *FA* (which has to be equal to *ED)* the epicycle *AI,* its apogee being *A.* Thereafter let the centre *F* of the epicycle *AI* have reached *G,* so that its motion *FG,* or the angle *FEG,* be equal to the motion of *ADB* in the first location. And let the epicycle described about the centre *G* be *BK,* in which let there be drawn from *E* through *G* the line *EGK,* so that *K* denotes the epicycle's apogee, from which let the Planet meanwhile have moved to *B* (contrary to the first motion from *F* to *G)* through an arc equal to *FG,* or, in other words, equal to the angle contained between the line *KG* and the line from *G* to *B* as far as the circumference of the epicycle, which line is obtained by drawing from *G* a line parallel to *EF.* WHAT IS REQUIRED. We have to prove that in this second location the Planet falls

[1]) As contrasted with an eccentric circle.
[2]) Meaning an epicycle whose centre describes a centric circle.

daer hy in d'eerste stelling was. T'BEWYS. Anghesien D B, E G twee even en evewijdeghe halfmiddellijnen sijn, tusschen welcke E D comt, en dat de lini van G na B even en evewijdeghe met E D is, deur t'gegeven, soo moet de form begrepen tusschen de vier linien B D, D E, E G, en de lini van G na B evewijdeghe met E D, dats de form B D E G een * evewijdich vierhouck sijn, en vervolghens t'punt B, te weten den Dwaelder, is soo wel uyterste der lini G B int inront K B na d'eerste stelling, als uyterste der lini D B int uytmiddelpuntichront A B na d'eerste stelling, en vervolgens den Dwaelder gesien vanden Eertcloot E an B int inront sonder uytmiddelpuntichront, of an B int uytmiddelpuntichront sonder inront, het is hem al tot een selve plaets ghesien. T'BESLVYT. Wesende dan een Dwaelder ghestelt &c. *Parallelogrammum.*

MERCKT.

Hoewel dese manier de Manens ware plaets oock anwijst, soo schijnt nochtans datmense niet en behoort te ghebruycken, eensdeels om dat haer duyster plecken die altijt na den Eertcloot ghekeert staen, betuyghen datse in geen inront en draeyt, ten anderen om datmen met meer haspeling twee ronden stelt daert deur een can ghedaen worden: En daerom is by *Ptolemeus* int stellen des Sonloops, en by *Copernicus* des Eertclootloops, met reden het uytmiddelpuntichront vercoren, t'welck *Ptolemeus* int stellen des Maenloops oock soude genomen hebben, ten waer dat, soo hy seght int 5 Hooftstick sijns 4 boucx, de stelling des inronts hem bequamer viel om daer deur sijn voornemen der navolghende tweede oneventheden te verclaren.

2 VOORSTEL.

Te verclaren d'oirsaeck die *Ptolemeus* bevveeghde tottet ondersoucken van sijn tvveede oneventheden der Maen, metsgaders de ghedaente van sijn bygevoughde spiegheling int ghemeen.

Alsoo *Ptolemeus* gheduerlick en seer ernstelick gaslouch de Manens schijnbaer duysteraerlangden, heeft dickwils bevonden die te verschillen mette regelen hier vooren beschreven, en hem van sijn voorganghers naghelaten, welck hy d'eerste oneventheden noemt, sulcx dat hy daer af na sijn goetduncken verbetering ghedaen heeft, ende een tweede onevenheyt daer by vervought. Om hier af sijn meyning te verclaren ick segh aldus: Hy heeft bevonden dat haer uyterste voorosachtringhen in saminghen en tegestanden der Son altijt waren van 5 tr. ghelijck int 30 voorstel des 2 boucx d'eerste oneventheyt mebrengt, maer daer buyten vielder verandering, en dat ten grootsten in vierdeschijn, alwaerse conde vallen van 7 tr. 40 ①, dats 2 tr. 40 ① meer als d'ander, en soo veel mochte int oirdeel der toecommende Maenplaetsen feyl vallen, alsmē int rekenen alleenelick sach na d'eerste oneventheyt. Om nu te verclaren d'oirsaeck die hy stelt van dit verschil, soo is voor al te weten dat hy totte bequaemste uytlegging sijns voornemens (ghelijck hy seght int 5 Hooftstick sijns 4 boucx) eerstelick ghenomen heeft de stelling der Maen niet in een uytmiddelpuntighe wech, ghelijck wy die hier vooren beschreven hebben, maer in een * middelpuntichinront, dat is te loopen in een inront, en t'selve in een middelpuntighe wech, na de manier beschreven int 1 voorstel deses Anhangs. *Concentrepiryclo.*

Ef Laet

in the same point *B* where it was in the first location. PROOF. Since *DB* and *EG* are two equal and parallel semi-diameters, between which *ED* comes, and since the line from *G* to *B* is equal and parallel to *ED*, by the supposition, the figure contained between the four lines *BD, DE, EG,* and the line from *G* to *B* parallel to *ED, i.e.* the figure *BDEG,* must be a parallelogram, and consequently the point *B,* to wit, the Planet, is the extremity of the line *GB* on the epicycle *KB* according to the second [1]) location as well as the extremity of the line *DB* on the eccentric circle *AB* according to the first location, and consequently whether the Planet is seen from the Earth *E* at *B* on the epicycle without eccentric circle or at *B* on the eccentric circle without epicycle, it is always seen in the same place. CONCLUSION. When therefore a Planet is placed, etc.

NOTE.

Although this method also indicates the Moon's true position, nevertheless it seems it ought not to be used, in the first place because the Moon's obscure portions, which are always turned towards the Earth, show that it does not revolve in an epicycle, secondly because it is a greater complication to assume two circles if it can be done with one. And for this reason, when *Ptolemy* framed the theory of the Sun's motion and *Copernicus* that of the Earth's motion, the eccentric circle was rightly chosen, which *Ptolemy* would also have taken when framing the theory of the Moon's motion if it were not for the fact, which he mentions in the 5th Chapter of his 4th book, that the assumption of the epicycle was more convenient to him for explaining by this means his intention with regard to the subsequent second inequalities.

[*The propositions 2 - 5, not reproduced here, concern the Moon's motion*]

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[1]) For *eerste* in the Dutch text read *tweede*.

De driehouck E B F heeft drie bekende palen, te weten E B 48 deel 31 ① eerste in d'oirden, E F 10 deel 19 ① deur het 5 voorstel deses Anhangs, en dē houck B E F 89 tr. 30 ①, als halfrontschil des gegeven houcx D E B 90 tr. 30 ①: Hier me ghesocht den houck E B F, wort bevonden deur het 6 voorstel der platte driehoucken van 12 tr. 1.

Vande booch H I G K doende deur t'bereytsel 333 tr. 12 ①, getrocken de booch H G 180 tr. blijft de booch G K, of houck G B K 153 tr. 12.

Daer toe den houck E B F 12 tr. 1 ① tweede in d'oirden, comt voor den houck E B K 165 tr. 13.

De driehouck E B K heeft drie bekende palen, te weten den houck E B K 165 tr. 13 ① vierde in d'oirdē, E B 48 deel 31 ① eerste in d'oirden, en des inronts halfmiddellijn B K 5 deelen 15 ① : Hier me ghesocht den begheerden houck der voordering B E K, wort bevonden deur het 6 voorstel der platte driehoucken van 1 tr. 25 ①, latet sijn 1 tr. 26.

T'BESLVYT. Wy hebben dan ghevonden op een ghegeven tijt de Manens voorofachtring, deur wercking gegront op stelling van *Ptolemeus* bygevoughde spiegheling, na den eysch.

VERVOLGH.

Deur dit vinden der voorofachtring is openbaer hoe bekent sal worden de Manens schijnbaer duysteraerlangde op een ghegeven tijt, want totte Middelmanens duysteraerlangde vervought de voorofachtring men heeft t'begeerde. Merckt noch dat deur *Ptolemeus* en anderen verscheyden tafels ghemaeckt sijn, om met lichticheyt te vinden t'ghene hier boven deur rekeninghen der platte driehoucken met meerder moeyte ghevonden wort, welcke tafels wy hier onbeschreven laten, als totte kennis vande gedaente deser tweede oneventheden of byghevoughde spiegheling der Maen van *Ptolemeus* niet voorderlick. Dit dan t'ghene sijnde t'welck wy vande selve tweede oneventheden voorghenomen hadden te segghen, sullen nu commen tot sijn onbekende oneventheden van d'ander Dwaelders.

6 VOORSTEL.

Te verclaren de somme van *Ptolemeus* byghevoughde spiegheling des langdeloops van Saturnus, Iupiter, Mars en Venus.

Anghemerckt wy deser Dwaelders eenvoudighe loop ghelijckse *Ptolemeus* ter handt quam voor bekent nemen, als int eerste en tweede bouck beschreven sijnde, soo sal t'verclaren van sijn byvouging hier cort en licht vallen.

Voor al is te weten dat hy deser vier Dwaelders schijnbaer duysteraerlangden, in saminghen en teghestanden der Middelson als des inronts middelpunt was an sijn wechs verstepunt of naestepunt, altijt bevant ghelijck me brocht de rekening des eenvoudighen loops soose hem eerst ter handt ghecommen was, want hy in yder Dwaelder op sulcke stelling vant en berekende de langde der uytmiddelpunticheytlijn, en des inronts halfmiddellijn, maer daer buyten vielder verandering.

Om dan

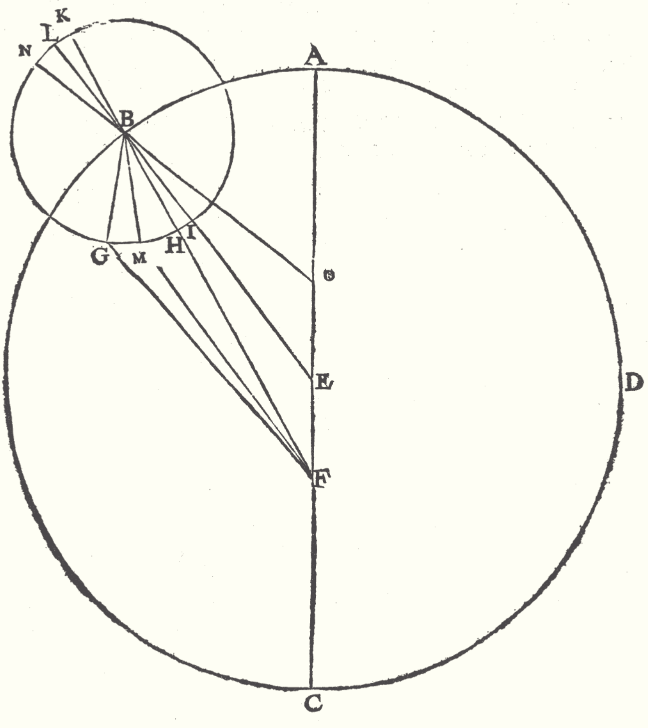
6th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Saturn, Jupiter, Mars, and Venus, added by *Ptolemy.*

Since we assume the simple motion of these Planets, as it was handed down to *Ptolemy,* to be known, as having been described in the first and the second book, the explanation of his addition will here be short and easy.

First of all it is to be noted that he always found the apparent ecliptical longitudes of these four planets, in conjunctions with and oppositions to the Mean Sun, when the epicycle's centre was at its deferent's apogee or perigee, to be as resulted from the calculation of the simple motion as it had first been handed down to him; for he found and calculated for each Planet, in such a location, the length of the line of eccentricity and of the epicycle's semi-diameter; but for other points he found differences.

Om dan totte saecke te commen laet A B C D den inrontswech sijn, diens middelpunt E, verstepunt A, uytmiddelpuntichpunt of den Eertcloot F, des inronts middelpunt B, waer op beschreven is het inront G H I K, daer na van E en F deur B ghetrocken sijnde de twee rechte linien E L, F K, sy snyen het inront in H en I, soo dat H is t'naestepunt, K t'verstepunt, I middelnaestepunt, L middelverstepunt: Voort neem ick dat den Dwaelder na uytwijsen der rekening des tweeden boucx, dat is volghens den eenvoudigen loop soose *Ptolemeus* eerst



ter handt quam, moest sijn an G, soo dat sijn inronts middellangde L G doet 150 tr. en sijn schijnbaer verheyt van t'verstepunt A ghesien uyt den Eertcloot F, soude hebben moeten sijn den houck A F G: Maer hy bevant sulckē houck metter daet cleender, als neem ick A F M, inder voughen dat den Dwaelder int inront wesentlick was an M, en niet volghens de rekening an G.

Hier uyt heeft hy aldus gedocht, nadien vanden Dwaelder tottet middelverstepunt moet wesen 150 tr. gelijck eerst gestelt wiert de booch L G, en dat den Dwaelder int inront soo veel voorder is dan G, als van G tot M, soo moet het middelverstepunt oock even soo veel voorder sijn van L, t'welck sy an N, en hier me is N M van 150 tr. ghelijck L G.

Nu dan N ghenomen sijnde als voor middelverstepunt, van daermen des Dwaelders middelloop begint te tellen, hy heeft van N deur B ghetrocken een rechte lini snyende de middellijn A C in O: Heeft daer na ghesocht hoe lanck de lini E O viel, en want alle noodige palen hem daer toe bekent waren, heeftse even bevonden met E F, en dat niet alleen in dit voorbeelt, maer in allen ande-

ren

Therefore, to come to the matter, let *ABCD* be the deferent, its centre being *E,* its apogee *A,* the point of eccentricity or the Earth *F,* the epicycle's centre *B,* about which has been described the epicycle *GHIK.* If thereafter from *E* and *F* through *B* there are drawn the two straight lines *EL, FK,* they intersect the epicycle in *H* and *I,* so that *H* is the perigee, *K* the apogee, *I* the mean perigee, *L* the mean apogee. Further I assume that, as shown by the calculation of the second book, *i.e.* according to the simple motion as it was first handed down to *Ptolemy,* the Planet should be at *G,* so that its epicycle's mean longitude *LG* makes 150°, while its apparent distance from the apogee *A,* as seen from the Earth *F,* ought to have been the angle *AFG.* But he found this angle in reality to be smaller — I take *AFM* — so that the Planet on the epicycle was in reality at *M* and not, as according to the calculation, at *G.*

From this he concluded as follows: Since from the Planet to the mean apogee there must be 150°, as the arc *LG* was first assumed to be, and since the Planet on the epicycle is as much in advance of *G* as the distance from *G* to *M,* the mean apogee must also be as much in advance of *L;* let this be *N,* then herewith *NM* is 150°, like *LG.*

If therefore *N* is assumed to be the mean apogee, from which we begin to count the Planet's mean motion, *Ptolemy* drew from *N* through *B* a straight line intersecting the diameter *AC* in *O.* Thereafter he sought the length of the line *EO,* and because all the necessary terms were known to him, he found it to be equal to *EF,* such not only in the present example, but in all others, in whatever ren tot wat plaets het inronts middelpunt in ſijn wech, en den Dwaelder int inront weſen mochten: Sulcx dat hy hier af willende ſijn ſpiegheling beſchrijven, heeft O genoemt het * onevenheyts punt, an t'welck genomen ſijnde het oogh geſtelt te weſen, men ſiet het inronts middelpunt B inden inrontswech A B C D, en den Dwaelder an G oirdentlick draeyen, waer uyt volght de voornomde twee punten B en M onoirdentlick te draeyen gheſien uyt des inrontwechs middelpunt E, of anders geſeyt dattet inronts middelpunt B in ſijn wech, en den Dwaelder M int inront, d'een tijt raſſcher als d'ander moetē loopen, teghen de ghemeene natuerlicke oirden, en volghens ſulcke ſtelling ghevonden weſende der Dwaelders voorofachtringen, en ſchijnbaer duyſteraerlangde, ſoo beſtaet hier in *Ptolemeus* voorſchreven vondt en ſpiegeling die hy vermengt heeft by den loop hem van ſijn voorgangers ter handt gecommen.

Punctum inæqualitatis. Purbachius centrum æquantis.

T'BESLVYT. Wy hebbē dan verclaert de ſomme van *Ptolemeus* bygevoughde ſpiegheling des langdeloops van Saturnus, Iupiter, Mars en Venus, na den eyſch.

7 VOORSTEL.

Te verclaren de ſomme van *Ptolemeus* byghevoughde ſpiegheling des langdeloops van Mercurius.

Alſoo *Ptolemeus* dickwils gaſlouch Mercurius plaetſen, en daer benevens noch acht nam op de gaſlaginghen ſijnder voorganghers, heeft bevonden dat hy in elcken keer tweemael ten naeſten by den Eertcloot quam (ghelijck de Maen, die in elck Maenſchijn na ſijn ſegghen tweemael ten naeſten comt) het welcke t'elckens ghebeurde doen ſijn inronts middelpunt ſchijnbaerlick was 120 tr. over weder ſijden vant ſchijnbaer verſtepunt, dat hy bevant in des duyſteraers 190 tr. D'oirſaeck hier af ſtelde hy te weſen dattet inronts middelpunt totte twee voorſchreven plaetſen altijt den Eertcloot ten naeſten was. Dit int ghemeen gheſeyt ſijnde, wy ſullen nu totte beſonder verclaring van ſijn meyning commen.

Laet in de volgende eerſte form A B C D dē wech des inronts ſijn, diēs middelpunt is E, verſtepunt A, uytmiddelpunticheytpunt of den Eertcloot F, des inronts middelpunt A, waer op beſchrevē is het inront G H I K, diens verſtepunt G, naeſtepunt I, voort ſy Mercurius ant verſtepunt G, en dit alles ghenomen na den eenvoudigen loop ſooſe *Ptolemeus* ter handt quam, en beſchreven is int 2 bouck, met welcke gheſtalt *Ptolemeus* gheen feyl en vandt, want hier me heeft hy int 8 Hoofſtick ſijns 9 boucx gheſocht de reden der uytmiddelpunticheytlijn, en des inronts halfmiddellijn tot des inrontwechs halfmiddellijn: Maer Mercurius buyten des inronts verſtepunt of naeſtepunt weſende, of des inronts middelpunt buyten ſijn wechs verſtepunt of naeſtepunt, ſoo vielder verſchil, t'welck *Ptolemeus* na verſcheyden taſtinghen en onderſouckinghen int 9 Hoofſtick ſijns 9 boucx verbeterde, en in form van ſpiegheling brocht als volght: Hy heeft opt middelpunt E des wechs in deſe geſtalt ſijnde, beſchreven een rondeken L M N, ſnyende A C in L en N, weſende des ſelfden rondekens halfmiddellijn E N den helft der uytmiddelpunticheytlijn E F, hier in heeft hy t'punt L angheſien als voor middelpunt des inrontwechs, hoe wel het in dees form eyghentlick E is, en t'ſelve middelpunt L gheſeyt te draeyen int rondeken van L tegē het vervolgh der trappē na M, doende daer in place the epicycle's centre might be on its deferent and the Planet on the epicycle. Thus, wishing to describe hereof his theory, he called *O* the point of inequality; and if the eye is taken to be situated in this point, the epicycle's centre *B* is seen to move regularly on the deferent *ABCD* and the Planet to move regularly at *G*, from which it follows that the aforesaid two points *B* and *M* are seen from the deferent's centre *E* to move irregularly or, in other words, that the epicycle's centre *B* on its deferent and the Planet *M* on the epicycle must move faster at one time than at another, contrary to the common natural order of things; and when according to this assumption the Planets' advance-or-lag and apparent ecliptical longitudes are found, this constitutes *Ptolemy's* aforesaid discovery and theory, which he combined with the motion that was handed down to him by his predecessors. CONCLUSION. We have thus expounded the sum of the theory of the motion in longitude of Saturn, Jupiter, Mars, and Venus, added by *Ptolemy;* as required.

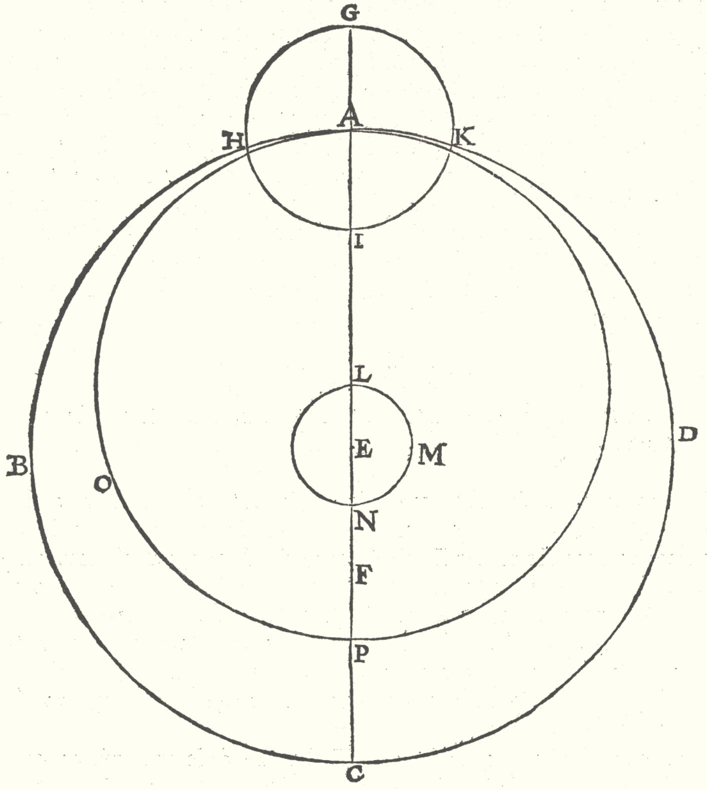
7th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Mercury, added by *Ptolemy.*

Since *Ptolemy* frequently observed Mercury's positions, and in addition also noted the observations of his predecessors, he found that twice in each revolution it came as near as possible to the Earth (just as the Moon, which according to him comes as near as possible twice in each lunation), which happened whenever its epicycle's centre was apparently at 120° on either side of the apparent apogee, which he found to be at 190° of the ecliptic. He assumed that the cause of this was that the epicycle's centre was always as near as possible to the Earth in the two aforesaid places. This having been said in a general way, we shall now come to the separate exposition of his view.

In the subsequent first figure let *ABCD* be the deferent, whose centre is *E*, its apogee *A*, the point of eccentricity or the Earth *F*, the epicycle's centre *A*, about which has been described the epicycle *GHIK*, whose apogee shall be *G*, its perigee *I*. Further let Mercury be at the apogee *G*, all this being taken according to the simple motion as it was handed down to *Ptolemy* and has been described in the 2nd book. With this disposition *Ptolemy* found no fault, for herewith he sought in the 8th Chapter of his 9th book the ratio of the line of eccentricity, and of the epicycle's semi-diameter, to the deferent's semi-diameter. But when Mercury was outside the epicycle's apogee or perigee, or the epicycle's centre was outside its deferent's apogee or perigee, there was a difference, which *Ptolemy* after several tentative efforts and investigations corrected in the 9th Chapter of his 9th book and cast into the form of a theory, as follows. About the centre *E* of the deferent in this disposition he described a small circle *LMN*, intersecting *AC* in *L* and *N*, the semi-diameter of this small circle *EN* being one half of the line of eccentricity *EF;* in this he took the point *L* for the centre of the deferent, though in this figure it is really *E*, and said that this centre *L* moved on the small circle from *L*, against the order of the degrees, to *M*, performing therein one

1 FORM.



een keer, op den selven tijt dattet inronts middelpunt in sijn wech een keer doet, op d'ander sijde na t'vervolgh der trappen, dats van A na B, en alsoo met hem draghende den heelen inrontwech.

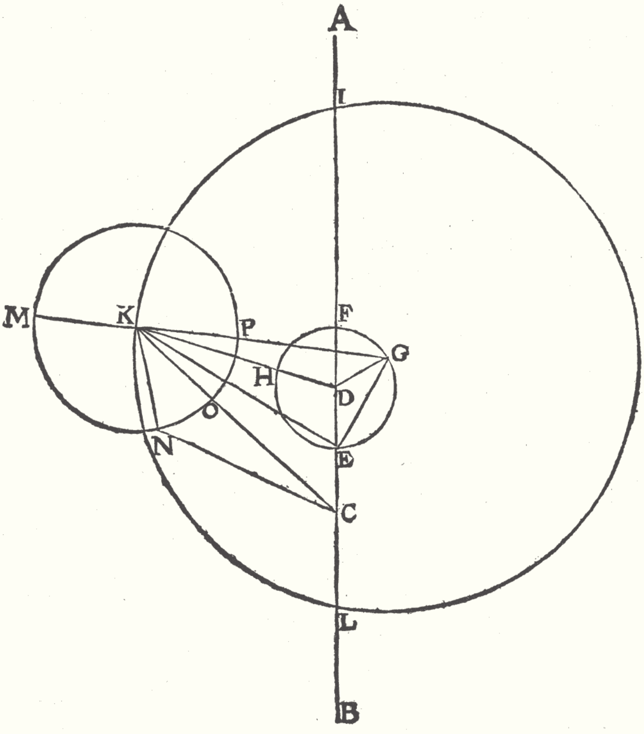
Maer anghesien A L hier genomen is voor wechs halfmiddellijn deser bygevoughde spiegeling van *Ptolemeus*, soo moet des selven wechs omtreck cleender sijn dan A B C D na stelling des eenvoudighen loops soose *Ptolemeus* ter handt quam, daerom laet ons op L als middelpunt beschrijven den inrontwech der spiegheling van *Ptolemeus* A O P: En hoe wel die nu cleender is dan d'eerste, soo blijft hier nochtans de verheyt vandē Eertcloot F tot A de selve, en vervolghens soo blijft oock des inronts grijphouck de selve: En als des wechs middelpunt L ghecommen sal sijn van L over M tot N, soo sal volghens dit gestelde des inronts middelpunt A ghecommen sijn over d'ander sijde an C, en sal alsdan de verheyt vanden Eertcloot F, tot des inronts middelpunt, en vervolghens des inronts grijphouck oock de selve sijn als na de eenvoudige loop soose *Ptolemeus* ter handt quam: Sulcx dat dese spiegeling van *Ptolemeus* geen veranderinge en gheeft int voorschreven soucken der reden vande uytmiddelpunticheytlijn, en inronts halfmiddellijn, totte lini tusschen den Eertcloot en t'verstepunt.

Dese ghedaente aldus verclaert sijnde, wesende des inronts middelpunt an t'punt revolution in the same time in which the epicycle's centre performs one revolution on its deferent — to the other side, in the order of the degrees, *i.e.* from *A* to *B* — while carrying with it the whole deferent.

But since *AL* has here been taken for the deferent's semi-diameter according to this additional theory of *Ptolemy,* this deferent's circumference must be smaller than *ABCD* (which followed from the simple motion as it was handed down to *Ptolemy*); therefore let us describe about *L* as centre the deferent of the theory of *Ptolemy* AOP. And though this is now smaller than the first, nevertheless the distance from the Earth *F* to *A* remains the same here, and consequently the epicycle's angular diameter also remains the same. And when the deferent's centre *L* has moved from *L* via *M* to *N,* according to this supposition the epicycle's centre *A* will have moved on the other side to *C,* and then the distance from the Earth *F* to the epicycle's centre, and consequently the epicycle's angular diameter, will also be the same as according to the simple motion as it was handed down to *Ptolemy,* so that this theory of *Ptolemy* does not bring about any change in the aforesaid seeking of the ratio of the line of eccentricity and the epicycle's semi-diameter to the line between the Earth and the apogee.

This disposition thus having been explained with the epicycle's centre at the t'punt A, verst vanden Eertcloot F, en oock an sijn teghenoverpunt C, soo sullen wy nu noch voorbeelt stellen wesende des inronts middelpunt tot een ander plaets, als neem ick 60 tr. middelloops vant verstepunt A, en Mercurius met 100 tr. inrontlangde. Laet tot dien eynde in dese tweede form de lini A B strecken deur t'verstepunt A (t'welck *Ptolemeus* vandt onder des duysteraers 190 tr.) en sijn teghenoverpunt sy B, den Eertcloot C, des rondekens middelpunt D, waer op beschreven is t'selve rondeken F G E H (in plaets vant ronde-

2 FORM.



ken L M N der eerste form) wesende des selfden verstepunt F, naestepunt E, en halfmiddellijn D E even met E C: Voort sy gheteyckent t'punt G, soo dat de booch van F tot G teghen t'vervolgh der trappen doe de ghegeven 60 tr. en op G als middelpunt sy beschreven den inrontwech I K L, daer na treck ick D K, soo dat den houck A D K, even is met F D G 60 tr. welverstaende dat van I tot K is na t'vervolgh der trappen, ghelijckt van F tot G daer tegen is, alles soo de boveschreven stelling vereyscht: Ick beschrijf daer na op K als middelpunt het inront M N O P, treck voort van G deur K de lini G K M, so dat M des inronts middelverstepunt beteyckent, ick neem daer na N te sijn Mercurius plaets, en de booch M N te doen de ghegheven 100 tr. en treck K N, N C, C K, snyende het inront in O, daer na E K, E G, D K, D G, en neem het inront van G K gesneen te worden in P als middelnaestepunt.

Merckt nu noch dat *Ptolemeus* des inronts middelpunt K neemt oirdentlick te draeyen gesien uyt E, en daerom loopet onoirdentlick gesien uyt sijn wechs point *A,* furthest away from the Earth *F,* and also at its opposite point *C,* we will now also give an example with the epicycle's centre at another place — I assume at 60° of the mean motion from the apogee *A* — and Mercury at a longitude in the epicycle of 100°. To this end, in this second figure let the line *AB* pass through the apogee *A* (which *Ptolemy* found at 190° of the ecliptic), and let its opposite point be *B,* the Earth *C,* the centre of the small circle *D,* about which has been described this small circle *FGEH* (instead of the small circle *LMN* of the first figure), the apogee thereof being *F,* the perigee *E,* and the semi-diameter *DE* being equal to *EC.* Further let there be marked the point *G* such that the arc from *F* to *G,* against the order of the degrees, makes the given 60°, and about *G* as centre let there be described the deferent *IKL.* Thereafter I draw *DK* so that the angle *ADK* is equal to *FDG* = 60°, it being understood that from *I* to *K* it is, in the order of the degrees, just as it is from *F* to *G* against this order, all this as required by the above supposition. I describe thereafter about *K* as centre the epicycle *MNOP;* further I draw from *G* through *K* the line *GKM,* so that *M* denotes the epicycle's mean apogee. Then I take *N* to be Mercury's position and the arc *MN* to make the given 100°, and I draw *KN, NC, CK,* intersecting the epicycle in *O,* thereafter *EK, EG, DK, DG,* and I take the epicycle to be intersected by *GK* in *P* as the mean perigee.

Now it is also to be noted that *Ptolemy* assumes the epicycle's centre *K* to move regularly when seen from *E,* and for that reason it moves irregularly when middelpunt G, dat is inde selve wech d'een tijt rasscher als d'ander. Maer he oogh ghestelt an des wechs middelpunt G, soo neemt hy Mercurius van daer ghesien oirdentlick te loopen in sijn inront, en daerom loopt hy self int inront oock oirdentlick altijt even ras. En volgens sulcke stelling gevonden wesende Mercurius voorosachtring en schijnbaer duysteraerlangde, soo bestaet hier in *Ptolemeus* spiegheling. T'BESLVYT. Wy hebben dan verclaert de somme van *Ptolemeus* byghevoughde spiegheling des langdeloops van Mercurius, na den eysch.

Tot hier toe is ghefeyt van *Ptolemeus* byghevoughde spiegheling des langdeloops die hy vermengt heeft by den eenvoudighen loop hem van sijn voorgangers ter handt gecommen. Angaende sijn spiegheling des breedeloops, de somme daer af schijnt inde voorgaende breedeloop ghenouch verclaert.

NV VAN COPERNICVS BYGHEvoughde spiegheling der Dvvaelders met stelling eens roerenden Eertcloots.

8 VOORSTEL.

Te verclaren de somme van *Copernicus* byghevoughde spiegheling der Maen.

Nemende *Copernicus* dat *Ptolemeus* ervaringhen vande Manens gageslagen schijnbaer plaetsen recht waren, om daer op als vaste gront een spiegheling te stichten, en siende dat hy in sijn byghevoughde spiegheling haer loop int inront, en des inronts loop in sijn wech oneven stelde, d'een tijt rasscher als d'ander, ghelijck vooren geseyt is in deses Anhangs 2 voorstel, sulcx heeft hem onghescickt ghedocht, alsoo oock dede het stellen van haer groote naerdering totten Eertcloot, strijdende teghen d'ervaring, deur welcke men bevint haer grijphouck de verandering niet te crijghen dieder uyt soude moeten volghen, inder vougen dat hy in die plaets een ander wijse beschreef: Om welcke te verclaren, soo is te weten dat hy lettende op de 5 tr. grootste voorosachtring der Maen in middeltegheſtant en middelsaming, maer van 7 tr. 40 ① in ettelicke middelvierdeschijnen door *Ptolemeus* gaghe slaghen, als vooren gheseyt is, heeft ghenomen die reghel vast te gaen, en d'oirsaeck ghestelt dusdanich te wesen: Laet A B den inrontwech der Maen beteyckenen, diens middelpunt dats den Eertcloot C, en D E F het inront daer de Maen in loopt, volghens de eenvoudighe stelling soose *Ptolemeus* eerst ter handt quam, diens middelpunt A, en getrocken de rechte D A F C, oock E C gerakende het inront an E, soo doet den houck A C E de bovefchreven 5 tr. daer na sy ghetrocken de lini C G, soo dat den houck A C G doe de 7 tr. 40 ① achtering, en op H als middelpunt sy beschreven het rondeken E G, gherakende de linien C E, C G, daer na sy getrocken van A deur t'punt des bovefchreven raeckſels E de lini A E H G: En heeft de Maen in dit rondeken E G een loop dobbel ande middelmaenwinst, en des rondekens middelpunt H heeft sijn loop ghelijck mette loop die de Maen gheseyt wort te hebben int inront, volghens de eenvoudighe stelling ghelijckse

Ptoleseen from the deferent's centre *G,* that is in the same orbit faster at one time than at another. But when the eye is situated at the deferent's centre *G,* he assumes Mercury, when seen from there, to move regularly on its epicycle, and for that reason it also moves regularly on the epicycle, always with the same speed. And when according to this supposition Mercury's advance-or-lag and apparent ecliptical longitude have been found, this constitutes *Ptolemy's* theory. CONCLUSION. We have thus expounded the sum of the theory of the motion in longitude of Mercury, added by *Ptolemy;* as required.

Up to this point the theory of the motion in longitude added by *Ptolemy,* which he combined with the simple motion handed down to him by his predecessors, has been discussed. As to his theory of the motion in latitude, the summary of this seems to have been explained sufficiently in the foregoing motion in latitude.

NOW OF THE THEORY OF THE PLANETS, ADDED BY COPERNICUS, ON THE ASSUMPTION OF A MOVING EARTH

[*The 8th proposition, concerning the motion of the Moon, has not been reproduced here*].

gheſocht des inronts halfmiddellijn A E, wort bevonden deur het 4 voorſtel der platte driehoucken van 872, latet ſijn ſoo *Copernicus* ſtelt 860.

De driehouck E C G heeft drie bekende palen, te weten den houck E C G 2 tr. 40 ①, de ſijde E C 10000, als even genouch ſijnde met A C, en den houck G E C recht: Hier me gheſocht des rondekens middellijn E G, wort bevondē deur het 4 voorſtel der platte driehoucken van 466, latet ſijn ſoo *Copernicus* ſtelt 474.

Diens helft voor de halfmiddellijn E H 237.

Tot C A 10000, vergaert A D 860, als even ſijnde met A E eerſte in d'oirdē, en daer toe noch D K 474, als even ſijnde met E G tweede in d'oirden, comt voor C K meeſte verheyt vande Maen tottē Eertcloot 11334.

Van C A 10000, getrocken A F 860, als even ſijnde met A E eerſte in d'oirden, en noch 474 voor de middellijn vant rondeken alſt an F is, blijft voor de minſte verheyt vande Maen totten Eertcloot 8666.

Sulcx dat de meeſte verheyt tot de minſte in ſulckē reden is als 11334, tot 8666, t'welck ſoo groot verſchil der Manens grijphoucken niet en geeft als *Ptolemeus* palen int 3 voorſtel deſes Anhangs van 65 deelē 13 ① tot 34 deelen 9 ①, want ſegghende 34 deelen 9 ①, geven 65 deelen 13 ①, wat de Manens minſte grijphouck 30 ①? Comt deur het 6 voorſtel des 2 boucx de Manens meeſte grijphouck na *Ptolemeus* byghevoughde ſpiegheling 57 ①, maer na *Copernicus* ſtelling alleenelick 39 ①, want ſegghende 8666, gheven 11334, wat de Manens minſte grijphouck 30 ①? comt als vooren 39 ①: Doch iſt 3 ① meer dan na de eenvoudighe ſtelling ſooſe *Ptolemeus* eerſt ter handt quam, welcke dede 36 ① (overeencommende ſoo men ſeght mette dadelicke ervaringhen) want als geſeyt is inde Byeenvouging vant 13 voorſtel des 3 boucx, de meeſte verheyt doet 531, de minſte 445, daerom ſeggende 445 gheeft 531, wat de Manens minſte grijphouck 30 ①? comt als vooren 36 ①, ſulcx dat *Copernicus* de ſake daer in naerder ghecommen is dan *Ptolemeus*.

TEN anderen ſoo en heeft *Copernicus* mettet rondeken gheen ongheregelde loop d'een mael raſſcher als d'ander, ghelijck *Ptolemeus* ſtelling inhoudt.

Angaende d'overeencomminghen mette dadelicke ervaringhen, *Copernicus* en *Ptolemeus* ſtellinghen gheven een ſelve beſluyt in alle middelſaminghen en middeltegeſtanden, en oock in alle middelvierdeſchijnen in welcke de Maen by des inronts middelverheden is, want alſdan de voorofachtring na d'een en d'ander wijſe 7 tr. 40 ① doet, maer buyten die plaetſen valter tuſſchen d'een en d'ander ſtelling verſchil. T'BESLVYT. Wy hebben dan verclaert de ſomme van *Copernicus* byghevoughde ſpiegheling der Maen, na den eyſch.

9 VOORSTEL.

Te verclaren de ſomme van *Copernicus* byghevoughde ſpiegeling des langdeloops van Saturnus, Iupiter en Mars.

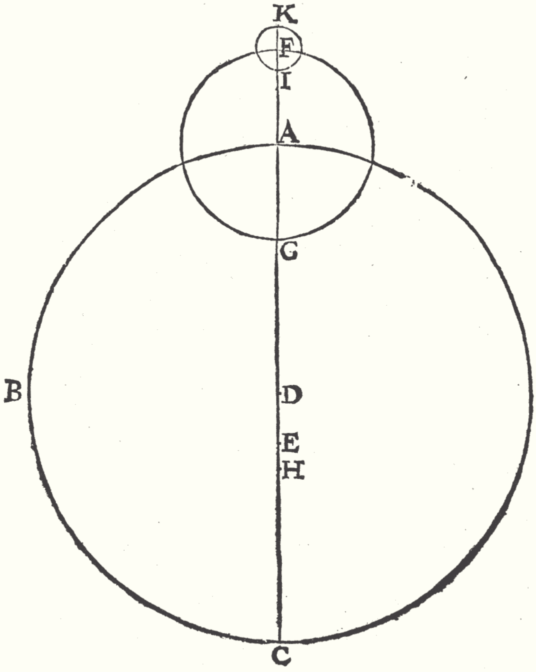
Nemende *Copernicus* dat *Ptolemeus* ervaringhen van Saturnus, Iupiters en Mars gagheſlaghen plaetſen recht waren, om daer op als vaſte gront een ſpiegheling te ſtichten, en ſiende dat hy in ſijn byghevoughde ſpieghelingen haer loop int inront, en des inronts loop in ſijn wech oneven ſtelde, d'een tijt raſ-

ſcher

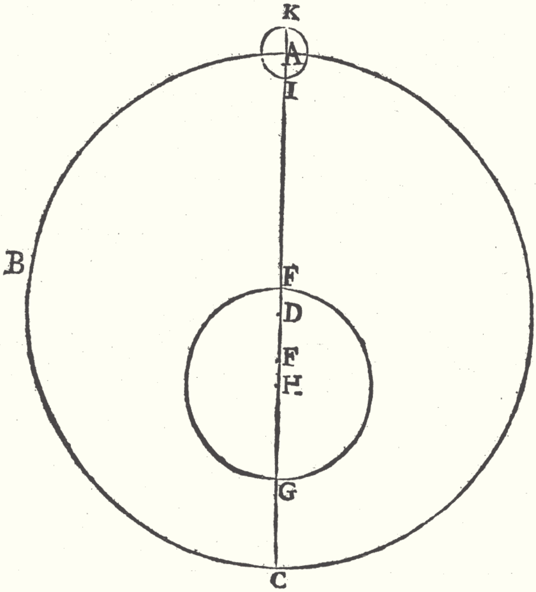
9th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Saturn, Jupiter, and Mars, added by *Copernicus*.

When *Copernicus* assumed that *Ptolemy's* experiences of the observed positions of Saturn, Jupiter, and Mars were right, to base thereon a theory as a firm foundation, and he saw that in his additional theories he assumed their motion on the epicycle and the epicycle's motion on its deferent to be unequal, at one ſcher als d'ander, ghelijck vooren gheſeyt is in deſes Anhangs 6 voorſtel, ſulcx heeft hem ongheſchickt ghedocht, inder voughen dat hy in die plaets een ander wijſe beſchreef aldus toegaende: Laet A B C den inrontwech beteyckenen van een der drie bovenſte Dwaelders, ick neem Saturnus, diens middelpunt D, den vaſten Eertcloot E, deur welcke ghetrocken is de rechte lini A C, ſoo dat A t'verſtepunt bediet, C t'naeſtepunt, D E de uytmiddelpunticheytlijn, doende na *Ptolemeus* rekening 3 deelen 25 ①, ſulcke alſſer D A 60 doet, daer na ſy op A als middelpunt beſchreven het inront F G, diens verſtepunt F, naeſtepunt G, waer in den Dwaelder ſy ant verſtepunt F. Dit is tot hier toe de form der eenvoudighe ſtelling met een vaſten Eertcloot, ſoose *Ptolemeus* eerſt ter handt quam, waer op hy ſijn boveſchreven ſpiegheling veroirdende: Maer de ſelve an *Copernicus* als gheſeyt is niet bevallende, heeft gedocht ofmen ſijn verſierde roerſels niet bequamelicker en ſoude te wege brengen, met den Dwaelder te doen loopen in een rondeken beſchreven int inront: En na verſcheyden taſtinghen quam tot duſdanich beſluyt: Hy nam voor ſich de lini tuſſchen den Eertcloot en het onevenheyts puntby *Ptolemeus* in ſijn byghevoughde ſpiegeling int 5 Hooftſtick ſijns 11 boucx bevonden van 6 deelen 50 ①, ſulcke alſſer des inrontwechs halfmiddellijn 60 doet, waer voor *Copernicus* int 5 Hooftſtick ſijns 5 boucx ghebruyckt 1139 en 10000: Van deſe 1139 der lini tuſſchen den Eertcloot en het onevenheytpunt, trock hy het vierēdeel doende 285, en bleef 854, die heeft hy gheteyckent van D tot H, nemende t'ſelve punt H nu voor Eertcloot, die eerſt deur E hadde, beteyckent geweeſt: Nam daer na het derdendeel van D H 854, dats E H doende 285, en teyckende die langde van F tot I, beſchreef met F I als halfmiddellijn het rondeken I K, nemende den Dwaelder ten tijde van ſijn ſaming mette middelſon te weſen an het naeſtepunt I, hebbende eē loop mettet vervolgh der trappen van I na K, even metten loop des inrondts middelpunt A na B: Maer weſende den Dwaelder in deſe middelſaming alſoo an het rondekens naeſtepunt I, t'is kennelick dat hy in alle middelſaminghen daer an moet weſen, in alle middeltegheſtant an het verſtepunt K, en in alle middelvierdeſchijn an een middelverheyt, en quamen hier me



 over-time faster than at another — as has been said previously in the 6th proposition of this Appendix — this appeared objectionable to him, so that instead of it he described another method, as follows. Let *ABC* denote the deferent of one of the three upper Planets — I assume Saturn — its centre being *D,* the fixed Earth *E,* through which has been drawn the straight line *AC* such that *A* denotes the apogee, *C* the perigee, *DE* the line of eccentricity, making according to *Ptolemy's* calculation $3\frac{25}{60}$ parts such as *DA* makes 60. Thereafter let there be described about *A* as centre the epicycle *FG,* its apogee being *F,* its perigee *G,* on which let the Planet be at the apogee *F.* This is, up to this point, the figure of the simple assumption with a fixed Earth, as it was first handed down to *Ptolemy,* on which he framed his above-mentioned theory. But since, as has been said, this did not satisfy *Copernicus,* he considered whether his imagined motions could not be brought about more conveniently by having the Planet move on a small circle described upon the epicycle. And after several tentative efforts he arrived at the following conclusion. He took before him the line between the Earth and the point of inequality, found by *Ptolemy* in his additional theory in the 5th Chapter of his 11th book to be $6\frac{50}{60}$ parts such as the deferent's semi-diameter makes 60, for which *Copernicus* in the 5th Chapter of his 5th book uses 1,139 and 10,000. From these 1,139 of the line between the Earth and the point of inequality he subtracted one fourth, making 285, upon which there were left 854, which he drew from *D* to *H,* now taking this point *H* for the Earth, which had first been denoted by *E.* Thereafter he took one third of *DH* = 854, *i.e. EH,* making 285, and drew this length from *F* to *I,* described with *FI* as semi-diameter the small circle *IK,* assuming the Planet at the time of its conjunction with the mean sun to be at the perigee *I,* moving in the order of the degrees from *I* to *K,* equally to the motion of the epicycle's centre *A* to *B.* But if the Planet is in this mean conjunction at the perigee of the small circle *I,* it is obvious that it must be there in any mean conjunction, in any mean opposition at the apogee *K,* and in any mean quadrature at a point of medium overeen Saturnus dadelicke langden na ſijn meyning. Doch want ſijn voornemen niet en was deſe beſchrijving aldus te doen met ſtelling eens vaſten Eertcloots, maer met ſtelling eens roerenden, hy heeft die form verandert, beſchrijvende op t'punt ghelijck H als middelpunt, een Eertclootwech even ant boveſchreven inrondt, en opt verſtepunt des Dwaelderwechs een rondeken even an t'ander, nemende den Dwaelder daer in te loopen. Laet tot opentlicker verclaring van dien A B C andermael het uytmiddelpuntichront beteyckenen, diens middelpunt D, en t'punt E ſy hier in ſulckē verheyt van D, alſt in de eerſte form van D was, daer na ſy beſchreven den Eertclootwech F G op t'punt H als middelpunt, weſende t'ſelve punt H in ſulckē verheyt van D als t'punt H in d'eerſte form vā D was, te weten 854 deelen, en des Eertclootwechs halfmiddellijn F G, doe ſoo veel als daer des inronts halfmiddellijn E G 1083, daer na ſy mette halfmiddellijn evē an H E, beſchreven op A als middelpunt het rondeken I K, even ant rondeken I K der eerſte form, waer in Saturnus ſy ant naeſtepunt I, met een loop als in d'eerſte form vā I na K: Deſe form met ſtelling eens roerenden Eertcloots, gheeft den Dwaelder al de ſelve ſchijnbaer duyſteraerlangden die uyt d'eerſte form met ſtelling eens vaſten Eertcloots volgen, waer af ick eerſt beſonder bewijs gedaen hadde, maer denckende daer na de ſake verſtanelick ghenouch te connen ſijn deur het ghene van dergelijcke vertoont is int 15 voorſtel des 3 boucx, ten anderen lettende op de onſekerheyt des gronts daer deſe ſtelling op gebout ſchijnt, ick heb dat bewijs cortheytshalven onbeſchreven ghelaten.



Merćt noch hier vooren geſeyt te weſen dat *Copernicus* het rondeken eerſt verdocht int inront met ſtelling eens vaſten Eertcloots, ende hoewel ſulcx in ſijn ſchriften niet en blijckt, ick hebt nochtans ſoo gheſeyt om mijn meyning int corte beter te verclaren, en dattet metter daet ſoo ſchijnt toeghegaen te hebben.

Deur deſe voorgaende ſpiegheling ſeght *Copernicus* den Dwaelder altijt tot ſijn behoirlicke plaets ghevonden te worden.

Merckt noch int boveſchreven te blijcken, dat *Copernicus* byghevoughde ſpiegeling een vermenging is van ſijn vondt met wat uyt *Ptolemeus* byvouging ghetrocken, ghemerckt de lini daer hy de drievierendeel af neemt by *Ptolemeus* ghebruyckt wiert.

Angaende het ſoucken van des Dwaelders ſchijnbaer duyſteraerlangde volghens distance, and in his view Saturn's practical longitudes agreed with this. But because it was not his intention to make this description thus on the assumption of a fixed Earth, but on the assumption of a moving Earth, he changed that figure, describing about the point *H* as centre an Earth's orbit equal to the above epicycle, and about the apogee of the Planet's deferent a small circle equal to the other, assuming the Planet to move thereon. For a fuller explanation of this, let *ABC* again denote the eccentric circle, its centre being *D,* and let the point *E* be here at the same distance from *D* as it was from *D* in the first figure. Thereafter let there be described the Earth's orbit *FG* about the point *H* as centre, this point *H* being at the same distance from *D* as the point *H* was from *D* in the first figure, to wit, 854 parts, and let the semi-diameter of the Earth's orbit *FG* make the same as there the epicycle's semi-diameter *EG* = 1,083. Thereafter let there be described, with the semi-diameter equal to *HE,* about *A* as centre the small circle *IK,* equal to the small circle *IK* of the first figure, on which let Saturn be at the perigee *I,* with a motion as in the first figure from *I* to *K*. This figure on the assumption of a moving Earth gives the Planet quite the same apparent ecliptical longitudes as follow from the first figure on the assumption of a fixed Earth, of which I had first given a separate proof, but considering afterwards that the matter could be understood easily enough from what has been shown for a similar case in the 15th proposition of the 3rd book, and secondly taking into account the uncertainty of the foundation on which this assumption seems to be built, for brevity's sake I have left this proof unwritten.

Note also that it has been said above that *Copernicus* first imagined the small circle on the epicycle on the assumption of a fixed Earth, and though this does not appear from his writings, I have nevertheless said it so in order better to explain my view in brief, and because it seems really to have taken place like this.

Copernicus says that by the foregoing theory the Planet is always found at its appropriate place.

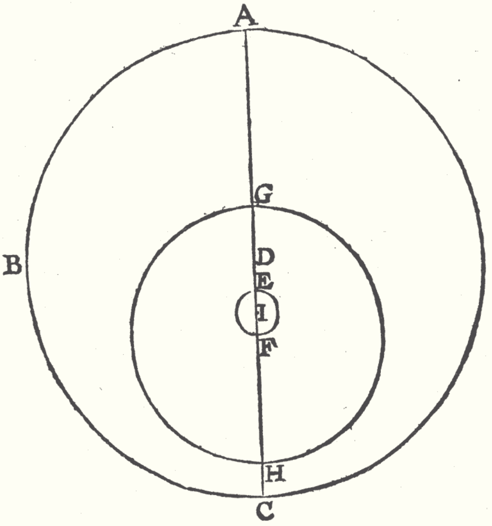
Note also that it appears from the above that *Copernicus'* additional theory is a combination of what he found and what was deduced from *Ptolemy's* addition, since the line of which he takes three fourths was used by *Ptolemy.*

gens dese spiegeling, t'is kennelick datmen inde tweede form sal vindē des rondekens middelpunts schijnbaer duysteraerlangde op de wijse ghelijck int 6 lidt vant 15 voorstel des 3 boucx ghevonden wiert des Dwaelders schijnbaer duysteraerlangde, en daer noch by voughen de voorofachtring die den Dwaelder deur het rondeken crijcht, want datter uyt comt is t'begeerde. Deur t'ghene wy tot hier toe van Saturnus geseyt hebben, is dergheliȷcke te verstaen van Iupiter en Mars. T'BESLVYT. Wy hebben dan verclaert de somme van *Copernicus* byghevoughde spiegheling van Saturnus, Iupiter en Mars, na den eysch.

10 VOORSTEL.

Te verclaren de somme van *Copernicus* byghevoughde spiegheling des langdeloops van Venus.

Angesien Venus met stelling eens roerenden Eertcloots alsoo loopt in haer wech die binnen den eertclootwech is, ghelijck de drie opperste Dwaelders in haer weghen loopen die boven den Eertclootwech sijn, soo soude mijns bedunckens *Copernicus* meyning uyt het voorgaende bekent sijn, alsmen het rondeken beschreef op Venuswech binnen den Eertclootwech: Doch te wijle hy achte de sake claerder te wesen, met Venuswechs middelpunt te doen draeyen in een rondeken even an t'ghene men anders op de wech als gheseyt is mocht beschrijven, soo sal ick dat verclaren. Laet A B C dē Eertclootwech sijn, diens



middelpunt D, halfmiddellijn D A doende 10000, en E is de plaets daer Venus wechs middelpunt bevonden wiert volghens de rekening gemaeckt op de eenvoudighe stelling soose *Ptolemeus* eerst ter handt quam, alwaer de uytmiddelpunticheytlijn D E dede 208: Maer na *Copernicus* spiegeling sy geteyckent het punt F, soo dat E F even is met E D, beduydende t'selve punt F Venuswechs middelpunt, waer op mette halfmiddellijn F G doende 7194, beschrevē is Venuswech G H in dese ghestalt: Daer na sy van E F ghenomen het middel I, en

 daer

As to the computation of the Planet's apparent ecliptical longitude according to this theory, it is obvious that in the second figure the apparent ecliptical longitude of the small circle's centre has to be found in the manner in which in the 6th section of the 15th proposition of the 3rd book the Planet's apparent ecliptical longitude was found, to which has to be added the advance-or-lag which the Planet gets from the small circle, for the result is the required value. From what we have hitherto said about Saturn the same is to be understood for Jupiter and Mars. CONCLUSION. We have thus expounded the sum of the theory of Saturn, Jupiter, and Mars, added by *Copernicus;* as required.

10th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Venus, added by *Copernicus.*

Since on the assumption of a moving Earth Venus moves in its orbit, which is within the Earth's orbit, just as the three upper Planets move in their orbits, which are above the Earth's orbit, in my opinion *Copernicus'* view would be known from the foregoing if the small circle were described on Venus' orbit within the Earth's orbit. But since he deemed the matter to be clearer if the centre of Venus' orbit were made to move on a small circle equal to the one that might otherwise — as has been said — be described on the orbit, I will explain this. Let *ABC* be the Earth's orbit, its centre being *D,* its semi-diameter *DA,* making 10,000; and *E* is the place where the centre of Venus' orbit was found according to the calculation made on the simple assumption, as it was first handed down to *Ptolemy,* where the line of eccentricity *DE* made 208. But according to *Copernicus'* theory let there be marked the point *F* such that *EF* is equal to *ED,* this point *F* denoting the centre of Venus' orbit, about which, with the semi-diameter *FG* making 7,194, has been described Venus' orbit *GH* in this disposition. Thereafter let there be taken of *EF* the middle point *I,* and daer op als middelpunt beschreven het rondeken E F, daer des wechs G H middelpunt in draeyt, tweemael so ras als den Eertcloot, en oock na t'vervolgh der trappen, sulcx dat wanneer den Eertcloot is an t'verstepunt A of naestepunt C, soo is t'middelpunt des wechs G H altijt an E, maer den Eertcloot an een der middelverhedē sijnde, als B, soo is dat middelpunt altijt an F. En hier me seght *Copernicus* dat Venus altijt tot haer behoorlicke langde ghevonden wort.

T'BESLVYT. Wy hebben dan verclaert de somme van *Copernicus* byghevoughde spiegheling des langdeloops van Venus, na den eysch.

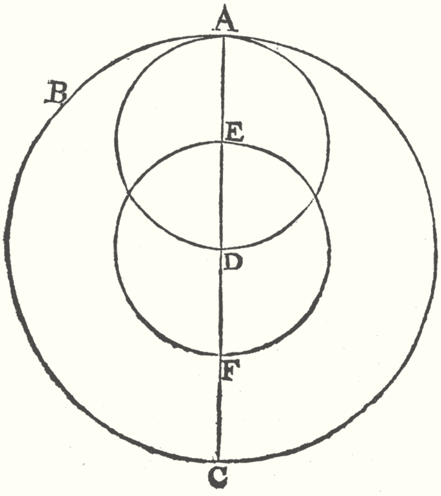
MERCKT.

Alsoo *Copernicus* int 25 Hoofdstick des 5 boucx, tot sijn spiegeling noodich verstont Mercurius te doen overentweer loopē in een rechte lini, en nochtans niet willende toelaten eenighe ongheschickte loop van ronden d'eenmael rasscher als d'ander, soo heeft hy int 4 Hoofdstick sijns 3 boucx beschrevē een vertooch waer deur sulcx met even roersel van ronden te weghe can ghebrocht worden t'selve vertooch (daer oock af ghehandelt wort deur *Proclus* uytlegger der beginselen van *Euclides*) sal ick hier by setten als 11 voorstel, om te dienen tot bewijs des volghenden 12 voorstels.

VERTOOCH. 11 VOORSTEL.

Wesende opt middelpunt eens eerste ronts beschreven een tvveede, diens halfmiddellijn even is ande helft der halfmiddellijn van t'eerste, en op een punt indē omtreck vant tvveede beschrevē een derde, even ant tvveede, hebbende een loop dobbel anden loop vant tvveede, en op een ander sijde. Yder punt des omtrecx vant derde beschrijft een rechte halfmiddellijn vant eerste.

1 FORM.



T'GHEGEVEN. Laet in dees eerste form A B C het eerste rondt sijn, diens middelpunt D, halfmiddellijn A D, en op D sy beschreven een tweede rondt E F, diens halfmiddellijn D E even is anden helft vā A D des eersteronts A B C, daer na sy op een punt als E indē omtreck vant tweede ront E F, beschreven een derde A D even ant tweede E F, en hebbende een loop van A na de rechter sijde, dobbel anden loop vant tweede rondt E F na de slijncker: Laet voort A eenich about this as centre let there be described the small circle *EF,* on which the centre of the orbit *GH* moves, twice as fast as the Earth, and also in the order of the degrees, so that when the Earth is at the apogee *A* or the perigee *C,* the centre of the orbit *GH* is always at *E,* but when the Earth is at one of the points of medium distance such as *B,* that centre is always at *F.* And *Copernicus* says that herewith Venus is always found at its appropriate longitude.

CONCLUSION. We have thus expounded the sum of the theory of the motion in longitude of Venus, added by *Copernicus;* as required.

NOTE.

Since *Copernicus* in the 25th Chapter of the 5th book deemed it necessary for his theory to make Mercury move to and fro in a straight line and yet was not prepared to admit any objectionable motion of circles, at one time faster than at another, in the 4th Chapter of his 3rd book he gave a demonstration by means of which this can be brought about with a uniform motion of circles; I will here add this demonstration (which is also dealt with by *Proclus,* the commentator of the elements of *Euclid*) as the 11th proposition, to serve as a proof of the subsequent 12th proposition.

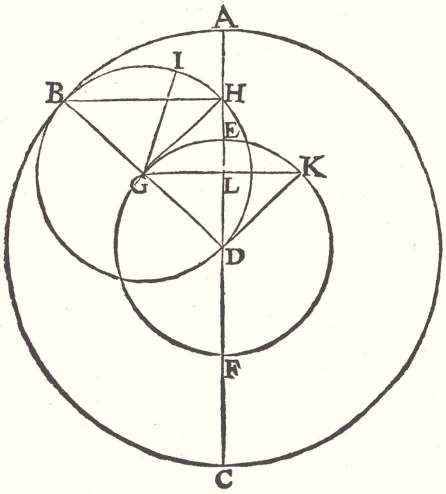
THEOREM. 11th PROPOSITION.

When about the centre of a first circle a second is described, whose semi-diameter is equal to one half of the semi-diameter of the first, and when about a point on the circumference of the second a third is described, equal to the second, having a motion that is twice the motion of the second and in an opposite direction, every point of the circumference of the third describes a straight semi-diameter of the first.

SUPPOSITION. In this first figure let *ABC* be the first circle, its centre being *D,* its semi-diameter *AD,* and about *D* let there be described a second circle *EF,* whose semi-diameter *DE* is equal to one half of *AD* of the first circle *ABC.* Thereafter in a point such as *E* on the circumference of the second circle *EF* let there be described a third circle *AD,* equal to the second *EF* and having a motion from *A* to the right that is twice the motion of the second circle *EF* to eenich genomē punt sijn des omtrecx vant derde rondt A D. T'BEGEERDE. Wy moeten hier me bewijsen dattet punt A des omtrecx A D vant derde ront, een rechte middellijn A C beschrijft des eersten rondts A B C.

T'BEREYTSEL. Latet punt E des tweede rondts E F inde eerste form gecommen sijn na de slijncker sijde tot G als in dees tweede geloopen hebbende dē booch E G, of houck A D B, sulcx dattet verstepunt A des derde ronts A D inde eerste form moet gecommē sijn vā A tot B in dees tweede: Hierentusschē

2 FORM.



moettet genomen roerende punt in het derde rondt ghecommen sijn volgende t'gestelde van B na H, te weten na de rechter sijde, sulcx dat den booch B H dobbel is an E G, of den houck B G H, dobbel anden houck B D H, latet sijn tot H. Om nu op dese derde form het begeerde noch opentlicker te verclaren, wy moeten bewijsen dat H valt inde lini A C. T'BEWYS. Angesien den houck B G H opt middelpunt G int rondt B H D dobbel is anden houck B D H opt punt des omtrecx D int selve ront B H D deur t'bereytsel, en ooc deur het 20 voorstel des 3 boucx van *Euclides*, so segh ick hierom t'punt H nootsakelick te vallen in A C, want by aldient daer buyten viel, ick neem an I, soo en soude dan den houck B G I niet het dobbel connen sijn des houcx E D G, het welck teghen t'gestelde waer. T'BESLVYT. Wesende dan opt middelpunt eens rondts, beschreven &c.

VERVOLGH.

De lini B H valt rechthouckich op A C om dese reden: Laet gheteyckent worden inden omtreck E F het punt K, soo dat de booch E K even is met E G, en daerom oock de booch B H met G K, daer na sy ghetrocken de lini D K en G K: Twelck soo wesende A D snijt de rechte G K in haer middel L, sulcx datse op malcander rechthouckich commen: Maer den driehouck B G H is even en ghelijck metten driehouck G D K, en de sijde B G light met G D in een rechte lini, waer deur B H evewijdeghe is met G K, en comt ghelijck de selve G K oock rechthouckich op A C. Dit soo sijnde ick segh aldus: Want op een gegeven tijt bekent is de booch A B, soo wort ghevonden de houckmaetpijl H A des houckmaets H B, of houcx A D B, in sulcke deelen alsmen A D toeschrijft deur het 14 voorstel vant Houtmaetmaecksel, maer mettet punt H wort Mercurius beteyckent, waer uyt blijckt sijn plaets inde middellijn A C bekent te worden.

the left. Further let *A* be any point taken on the circumference of the third circle *AD*. WHAT IS REQUIRED. We have to prove herewith that the point *A* of the circumference *AD* of the third circle describes a straight diameter *AC* of the first circle *ABC*.

PRELIMINARY. Let the point *E* of the second circle *EF* in the first figure have moved to the left to *G*, as having moved on this second circle through the arc *EG*, or angle *ADB*, so that the apogee *A* of the third circle *AD* in the first figure must have moved from *A* to *B* on this second circle. In the mean time the moving point taken in the third circle must, according to the supposition, have moved from *B* to *H*, to wit, to the right, so that the arc *BH* is twice *EG*, or the angle *BGH* twice the angle *BDH*, let it be as far as *H*. In order now to explain in this third figure more clearly what is required, we have to prove that *H* falls in the line *AC*. PROOF. Since the angle *BGH* at the centre *G* in the circle *BHD* is twice the angle *BDH* at the point of the circumference *D* in the said circle *BHD*, by the preliminary and also by the 20th proposition of the 3rd book of *Euclid*, I say that for this reason the point *H* necessarily falls on *AC*, for if it fell outside it — I assume at *I* — the angle *BGI* could not be twice the angle *EDG*, which would be contrary to the supposition. CONCLUSION. When thus about the centre of a circle is described, etc.

SEQUEL.

The line *BH* is perpendicular to *AC*, for the following reason: Let there be marked on the circumference *EF* the point *K* such that the arc *EK* is equal to *EG*, and consequently also the arc *BH* to *GK*. Thereafter let there be drawn the lines *DK* and *GK*. This being so, *AD* intersects the straight line *GK* in its middle point *L*, so that they come perpendicular to one another. But the triangle *BGH* is equal and similar to the triangle *GDK*, and the side *BG* lies with *GD* in a straight line, owing to which *BH* is parallel to *GK*, and like this *GK* also comes perpendicular to *AC*. This being so, I say as follows: Because at a given time the arc *AB* is known, the versed sine *HA* of the sine *HB* or of angle *ADB* is found, in such parts as are ascribed to *AD* by the 14th proposition of Trigonometry [1]), but by the point *H* Mercury is denoted, from which its position in the diameter *AC* appears to become known.

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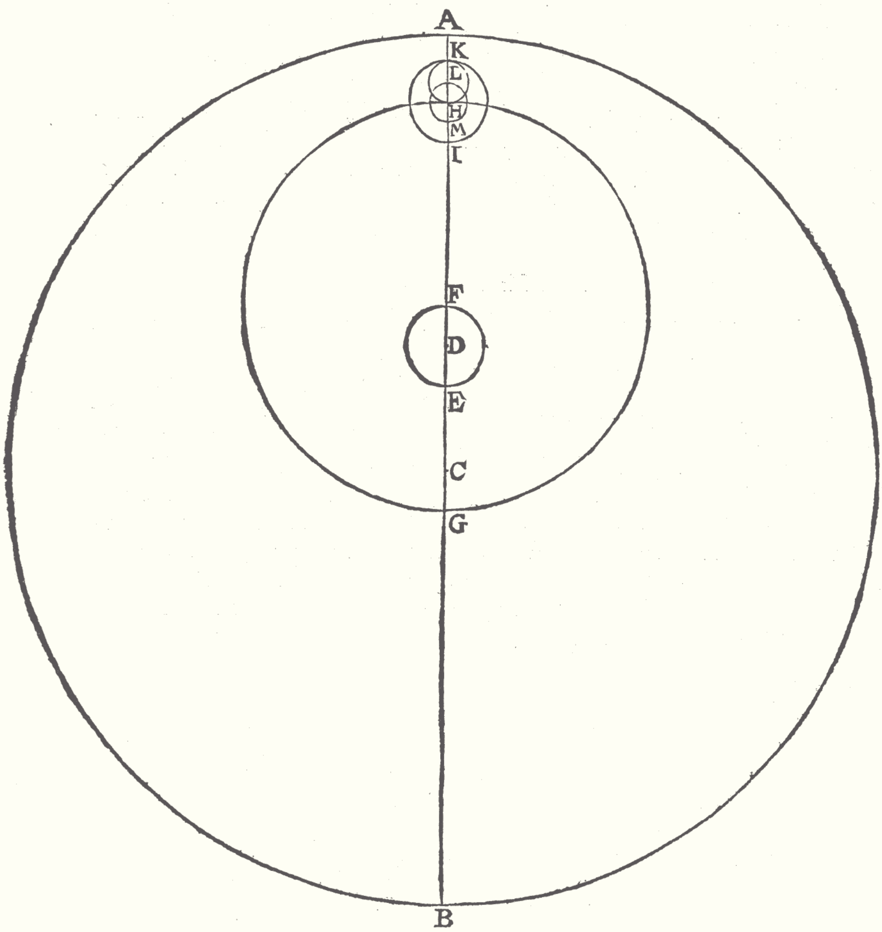
[1]) For *Houtmaetmaecksel* in the Dutch text read *Houckmaetmaecksel*. See Work XI; i, 11, p. 60.

12 VOORSTEL.

Te verclaren de ſomme van *Copernicus* byghevoughde ſpiegheling des langdeloops van Mercurius.

Laet A B den Eertclootwech beteyckenen, diens middelpunt C, middellijn A B, uytmiddelpunticheytlijn D C, ghevonden na de wijſe der eenvoudighe ſtelling ſooſe *Ptolemeus* eerſt ter handt quam van 947 deelen, ſulcke alſſer des Eertclootwechs halſmiddellijn A C 10000 doet, ende op D als middelpunt, en



mettet derdendeel van D C, t'welck ſy D E, is beſchreven een rondeken E F, ſulcx dat F t'verſtepunt beteyckent vā C, en E t'naeſtepunt, daer na op F als middelpunt ſy beſchrevē den inrontwech G H, op wiens verſtepunt H als middelpunt ick het inront I K teycken, en opt ſelve middelpunt H noch een cleender rondt L M, diens halſmiddellijn H M, even is an den helft der halſmiddellijn

H I, en

12th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Mercury, added by *Copernicus.*

Let *AB* denote the Earth's orbit, its centre being *C,* its diameter *AB,* the line of eccentricity *DC,* found by the method of the simple assumption, as it was first handed down to *Ptolemy,* to be 947 parts, such as the semi-diameter of the Earth's orbit *AC* makes 10,000, and about *D* as centre and with one third of *DC,* which shall be *DE,* there is described a small circle *EF* such that *F* denotes the point furthest from *C* (apogee) and *E* the nearest point (perigee). Thereafter about *F* as centre let there be described the deferent *GH,* about whose apogee *H* as centre I draw the epicycle *IK,* and about this centre *H* also a smaller circle *LM,* whose semi-diameter *HM* is equal to one half of the semi-diameter *HI* and intersects *AB* in *L* and *M;* thereafter about *L* as centre the small circle *KH,* whose semi-diameter *LH* is the same as that of the circle described about *HL;* in this small circle *KH* Mercury is carried. This figure thus being completed, we will explain its significance. *F,* the centre of the circle *HG,* performs two revolutions a year, one which it receives from the Earth's orbit, the other being its proper motion, and both in the order of the degrees. The centre *L* of the circle *KH* is fixed on the circumference of the circle *LM* and is carried therein against the order of the degrees, performing one revolution a year. The motion of Mercury on the circle *KH* is in the order of the degrees, equal to the above-mentioned motion of *F,* to wit, two revolutions a year, from which it follows that Mercury always moves to and fro on the diameter *KI* without leaving it, by the 11th proposition of this Appendix. Further it follows that when the Earth is at *A* or *B,* the centre of the circle *HG* must always be at *F,* that is the point of the circle *EF* furthest from *C.* But when the Earth is at the points of medium distance, *i.e.* 90° from *A* or *B,* the aforesaid centre of the circle *HG* must always be at *E,* that is the point of the circle *EF* nearest to *C,* which takes place in the opposite way to what happened with Venus. Further it follows that if, as has been said, Mercury moves to and fro on the diameter *IK,* the nearest point will always be at *I* when the Earth is at *A* or *B,* but at the furthest point *K* when the Earth is at the points of medium distance. In this way it occurs that the centre of *HG* (*F*) on the circle *EF,* and Mercury on the diameter *IK,* each perform two revolutions a year. And meanwhile the proper motion of the epicycle *IK* or the line *FH* in the circle *HG* is uniformly about 88 days for one revolution. And herewith *Copernicus* says that Mercury is found at its appropriate place.

CONCLUSION. We have thus expounded the sum of the theory of the motion in longitude of Mercury, added by *Copernicus;* as required.

HI, en sniende AB in L en M, daer na op L als middelpunt het rondekē KH, diens halfmiddellijn LH de selve is des ronts beschreven op HL, in dit rondeken KH wort Mercurius ghedreghen. Dese form aldus voldaen sijnde, soo sullen wy haer beteyckening verclaren: F middelpunt des rondts HG doet des jaers twee keeren, d'een diet vanden Eertclootwech ontfangt, d'ander sijn eyghen, en beyde na t'vervolgh der trappen: T'middelpunt L des rondts KH, is vast inden omtreck des rondts LM, en wort daer in ghedreghen teghen het vervolgh der trappen doende des jaers een keer: Den loop van Mercurius int rondt KH, is na t'vervolgh der trappen, even anden boveschreven loop van F, te weten des jaers twee keeren, waer uyt volght dat Mercurius altijt overentweer loopt inde middellijn KI sonder daer uyt te commen, deur het 11 voorstel deses Anhangs. Wijder volght dat wesende den Eertcloot an A of B, soo moet het middelpunt des rondts HG dan altijt sijn an F, dats des rondts EF verstepunt van C: Maer den Eertcloot ande middelverheden sijnde, dats 90 tr. van A of B, soo moet het voorschreven middelpunt des rondts HG dan altijt sijn an E, dats des rondts EF naestepunt van C, het welck op verkeerde wijse toegaet van t'ghene met Venus ghebeurde. Wijder volght dat Mercurius de middellijn IK overentweer loopende soo ghesëyt is, altijt het naestepunt an I sal sijn wesende den Eertcloot an A of B, maer an t'verstepunt K den Eertcloot ande middelverheden sijnde: Hier me ghebeuret dat het middelpunt van HG als F, int rondt EF, en Mercurius inde middellijn IK des jaers elck twee keeren doen: En hierentusschen is den eyghen loop des inrondts IK, of de lini FH int rondt HG eenvaerdelick ontrent de 88 daghen over een keer. En hier me segt *Copernicus* dat Mercurius tot sijn behoirlicke plaets ghevonden wort.

T'BESLVYT. Wy hebben dan verclaert de somme van *Copernicus* bygevoughde spiegheling des langdeloops van Mercurius, na den eysch.

13 VOORSTEL.

Verhael op der sterren onbekende loop: En des duysteraers onbekende afvvijcking vanden * evenaer. *Aequatore.*

De vaste sterren worden ghesëyt een onbekende loop te hebben d'een tijt rasscher als d'ander, want hoewelmense altijt eveverre van malcander bevint, nochtans heeft den heelen Hemelscloot een roersel van Westen int Oosten, sulcx dat haer duysteraerlangde die ande lentsne begint, gheduerlick grooter wort, en van *Ptolemeus* tijt tot nu toe over de 21 ① vermeerdert is: Maer volghens de ervaringhen sedert gheschiet, soo wort dit roersel seer onghereghelt gheacht, d'eenmael slapper als d'ander, ja soo eenighe meynen somwijlen rugheling te loopen. Hier af sijn by ettelicke als *Thebit*, de *Alfonsinen*, *Purbachius*, *Copernicus*, *Ioannes Venerus*, verscheyden * spieghelinghen verdocht, elck *Theoriae.* na sijn goetduncken. Maer om van dit onbekent roersel mijn ghevoelen te segghen, het is te weten dat der sterren * wanschaeuwing grooter bevonden *Refractio.* wort in landen na den * aspunt dan na den evenaer, waer af wy seer mercke- *Polum.* lick voorbeelt hebben, deur de ervaring gheschiet op de vermaerde seylage van Willem Barentsen metten sijnen in Nova Zembla, wesende daer des aspunts verheffing van 76 tr. alwaer hemlien de Son eerst onder den * sichtein- *Horizonte.* der ginck den vierden November 1596, diese op den eersten behoorden verlooren te hebben, sulcx datse hoogher scheen danse eyghentlick was, of wan-

schaeu-

13th PROPOSITION.

An account of the stars' unknown motion; and the ecliptic's unknown deviation from the equator.

The fixed stars are said to have an unknown motion, at one time faster than at the other, for though they are always found at the same distances from one another, nevertheless the whole Heavenly Sphere has a motion from West to East, so that their ecliptical longitude, which starts at the vernal equinox, becomes continuously greater, and from *Ptolemy's* time to the present day has increased by more than 21° [1]). But according to the experiences that have been gained since, this motion is considered to be very irregular, at one time slower than at another, nay, as some think, it is sometimes a backward motion. About this many authors, such as *Thebit*, the *Alphonsines*, *Purbachius*, *Copernicus*, *Johannes Vernerus* [2]), have invented different theories, each according as he thought fit. But to give my opinion about this unknown motion, it is to be noted that the refraction of the stars is found greater in countries towards the pole than towards the equator, of which we have a very notable example through the experience gained during the famous voyage of Willem Barentsen and his men to Nova Zembla [3]) where the elevation of the pole was 76° and where the Sun did not go down beneath the horizon until the fourth of November 1596, though they should have lost it on the first, so that it seemed to be higher than it really was, or had

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[1]) For 21① in the Dutch text read 21 *tr.*

[2]) Abû-l-Ḥasan Thâbit ibn Qurra ibn Marwân al-Ḥarrânî (Harran 826—Bagdad 901), mathematician and astronomer at the court of the Caliph Almustadid. His theory of „trepidation" became known mainly through al-Zarkali and the tables of Toledo; a treatise of Thâbit was translated into Latin and printed at Leipzig (1503).

The Alphonsine Tables were constructed about 1250—1300 under the patronage of King Alphonso X of Castile. The original version is unknown, but modified tables bearing the same name existed in Paris about 1320; they came into general use and were repeatedly printed between 1483 and 1649: *Alphonsi, regis Castellae, caelestium motuum tabulae* (Venice 1483).

Georg von Peuerbach or Purbach (Peurbach in Austria 1423—Vienna 1461), professor in Vienna. His treatise, *Theoricae novae planetarum*, passed through many editions from 1507 onwards, during the whole of the 16th century.

Johannes Werner (Nuremberg 1468—*ibid.* 1528), astronomer, mathematician and cartographer, discussed the precession in his treatises: *De motu octavae sphaerae; Summaria enarratio Theoriae motus octavae Sphaerae* (Nuremberg 1522). – For *Venerus* in the Dutch text read *Vernerus*.

[3]) Novaya Zemlya. – Towards the end of the sixteenth century Dutch navigators tried to discover a passage towards the Indies through the arctic seas. These pioneers were obliged to winter on N.Z., where relics of their equipment have been found. See the account of the voyage by one of the participants, G. de Veer: *Reizen van Willem Barents, Jacob van Heemskerck, Jan Cornelisz. Rijp en anderen naar het Noorden (1594-1597)*. Edited by S. P. L'Honoré Naber (Linschoten Vereniging XIV, 1917); translated by the Hakluyt Society (1876).

The Novaya Zemlya phenomenon has been discussed by several modern authors, of whom we mention especially S. W. Visser (Proc. Acad. Amsterdam, Ser. B, vol. 59 (1956), p. 375. The exceptional refraction of 4°17′ may be explained by a pronounced ground inversion, extending over a large distance. A similar observation was made by Shackleton in 1915, when the refraction reached 2°.

schaeuwing had ghelijck elck berekenen mach, van ontrent 1 tr. 9 ①: Maer 81 daghen daer na, te weten den 24 Ianuarius 1597, soo heeft den randt der Son haer weerom begonnen te openbaren, welcke sy sooder geen wanschaeuwing gheweest en had, op den 9 Februarius eerst behoorden gesien te hebben, inder voughen datse hoogher scheen danse eyghentlick was, of wanschaeuwing had by de 5 tr. welcke in dese laetste ervaring veel meerder bevonden wiert als in d'eerste, waer af d'oirsaeck bekent gheworden is an *Philippus Lansbergius* vlietich * Gaslagher en vermaert Wisconstenaer, die daer te vooren in sijn ervaringhen deur oneven wanschaeuwinghen langhe in twijffel gheweest had, want hoewel de Son tot lijckstandighe plaetsen vant * Winters keerpunt was, als neem ick 50 of 56 tr. daer voor, en 56 tr. daer na, soo en heeft hyse nochtans in d'een en d'ander niet met een selve hooghde boven den sichteinder bevonden, maer meerder inde laetste dan in d'eerste plaets, achtende nochtans alsdoen dit feyl van weghen wanschaeuwing niet te connen commen, om dat, als gheseyt is, de Son in d'een en d'ander ervaring eveverre van het winters keerpunt was: Maer het boveschreven voorbeelt van Nova Zembla t'sijnder kennis ghecommen sijnde, heeft d'oirsaeck besloten te wesen, en mijns bedunckens met goede reden, dat de coude wintersche vochticheden in Februarius dicker en waterachtigher sijn, dan inde warmer maent October eynde des voorganghen Somers. Nu dan de wanschaeuwing in Nova Zembla soo uytnemende groot wesende, en tot ander plaetsen daer de ervaringhen ghebeurt sijn, als Pruyssen, Duytslandt, Spaeigne, Italie, Egipten, soo groot als yders Noorderlickheyt mebrengt, boven dien tot een selve plaets op d'een tijt des jaers grooter als op d'ander, sonder nochtans by de Beschrijvers der voornoemde spieghelinghen daer op acht ghenomen te wesen, soo valt daer uyt te besluyten, dat de onevenheden by hemlien bevonden, niet nootsakelick en sijn van weghen onevenheyt des loops der sterren, want al waer die gantsch even, soo souder om de verscheydenheyt der wanschaeuwing moeten schijnen onevenheyt te wesen: Oft anders gheseyt, soo by hemlien volcommen evenheyt bevonden waer, t'soude teycken sijn van onevenheyt int wesen. Tot breeder bevesting van het voorseyde valt noch te anmercken, ten eersten dat uyt de ervaringhen in Egipten gheschiet, even loop bevonden wiert, te hondert Iaren van 1 tr. totten tijt van 432 Iaren toe: Ten anderen dat de onevenheyt met rasscher loop, gheoirdeelt wort uyt ervaringhen die daer na in verscheyden Noorderlicker landen gagheslaghen sijn, t'welck met reden vermoeden gheeft de onberekende wanschaeuwinghen daer af oirsaeck te meughen wesen, of volcommelick, of ten deele, maer welck van beyden men neemt, de voorschreven spieghelinghen vallen onghegront, en schijnt dat soo de Schrijvers van dien dese verscheyden grootheyden der wanschaeuwinghen ghenouch bedocht hadden, datse de selve spieghelinghen soo niet en souden veroirdent hebben, en daerom laet ick die onbeschreven, achtende datter voor al behooren te sijn ervaringhen van ghenouchsaem sekerheyt, eermen tottet veroirdenen en besluyten van sulcke spieghelinghen comt. Hier toe soudet connen voorderlick sijn, datmen tot Alexandrie in Egipten, daer de voorschreven eerste ervaringhen ghebeurden, dede gaslaen de Sonnens inganck der lentsne en erftsne, met haer meeste afwijcking vanden evenaer, en datmen daer na saghe hoe die overcommen met derghelijcke ons Noorderlicker ervaringhen op den selven tijt gheschiet.

Observator.

Solstitio hiemali.

Tot hier toe is gheseyt vande onbekende roersels der vaste sterren: Angaende die der Dwaelders, van welcke in desen Anhang ghehandelt is, ick acht *Ptolemeus* a refraction, as anyone may calculate, of about 1°9′. But 81 days afterwards, to wit, on the 24th of January 1597, the rim of the Sun began to show again, though, if there had been no refraction, they ought not to have seen it until the 9th of February, so that it seemed higher than it really was or had a refraction of about 5°, which in this last experience was found much greater than in the first. The cause of this was revealed to *Philippus Lansbergius* [1]), industrious Observer and famous Mathematician, who previously had long been in doubt in his experiences owing to unequal refractions, for though the Sun was in homologous places in relation to the winter solstice — I assume 50° or 56° in advance of it and 56° behind it — yet he did not find it at the same height above the horizon in one place and the other, but at a greater height in the last place than in the first; yet he then thought that this error could not be due to refraction, because, as has been said, the Sun was at equal distances from the winter solstice in one experience and the other. But when the aforesaid example of Nova Zembla had come to his knowledge, he concluded the cause to be — in my opinion with good reason — that the cold, humid winter atmosphere in February is denser and contains more water than in the warmer month of October, the end of the preceding summer. Since therefore the refraction in Nova Zembla was so extraordinarily great, and in other places where the experiences were gained, such as Prussia, Germany, Spain, Italy, Egypt, as great as was conditioned by the Northerliness of each, while moreover in the same place it was greater at one time of the year than at another, but without the Describers of the aforesaid theories having paid heed to it, it may be concluded that the inequalities found by them need not necessarily be due to inequality of the motion of the stars, for even if this were altogether equal, there must seem to be inequality owing to the variation of the refraction. Or in other words: if they had found perfect equality, this would be a sign of real inequality. For a fuller confirmation of the above it is also to be noted that from the experiences gained in Egypt there was found equal motion, of 1° in a hundred years, up to 432 years. Secondly, that the inequality with faster motion is judged from experiences observed thereafter in different more Northerly countries, which justly raises the suspicion that the uncalculated refractions may be the cause of this, either altogether or in part. But no matter which of the two is taken, the above-mentioned theories are unfounded, and it seems that if the Authors thereof had taken sufficient account of these different amounts of the refraction, they would not have framed the said theories as they did; and for that reason I leave them undescribed, being of opinion that before all there should be experiences of sufficient certainty before one proceeds to frame and derive such theories. To this end it might be advantageous if at Alexandria in Egypt, where the aforesaid first experiences were gained, the Sun's passage through the vernal and the autumnal equinox were made to be observed, with its greatest deviation from the equator, and that it were thereafter seen how they correspond with similar more Northerly experiences gained at the same time.

Up to this point the unknown motions of the fixed stars have been discussed. As to those of the Planets, which have been dealt with in this Appendix, I also

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[1]) Philippus van Laensbergh or Lansbergen (Ghent 1561—Middelburg 1632), surgeon and clergyman, studied the motion of the planets and advocated the theory of Copernicus from 1619 onwards. His *Opera Omnia* were published at Middelburg, 1663. In his printed works, no considerations about abnormal refractions are found.

lemeus ervaringhen daerse op ghegront sijn oock voor onseker, want hoewel hy daer in groote looflicken aerbeyt ghedaen heeft, nochtans ghemerckt dat toeginck met een cleene cooper * Hemelloopstuych, van verscheyden ringhen ghemaeckt, elcke op haer as draeyende, sulcx datmen des tuychs duysteraer altijt evewijdich creech metten hemelschen: Voort dat hy de sterren deur sichtgaetkens der wijslijnen sach, met meerder plaets dan de grootheyt der sterre, onsekerder toegaende dan na het gebruyck van Tuycho Brahe, soo is hemlien die met sulcke reetschappen ommegaen, kennelick ghenouch de onsekerheden dieder in vallen, hoe sorchvuldich die oock ghemaeckt sijn, ghelijck *Ptolemeus* selfdaer af somwijlen vermaen doet. Maer der sterren hoochden of verheden van malcander met groote reetschappen ghemeten, en de rest deur rekeninghen der platte en clootsche driehoucken ghewrocht, daer machmen vastelicker op te wercke gaen.

Instrumento Astronomico.

Noch vielder in *Ptolemeus* ervaringhen eenige onsekerheyt van der Dwaelders schijnbaer duysteraerlangden, om dat de vaste sterren self daer de dadelicke meting op ghegront is, tot die tijt rouwelick beschreven waren met 10 ① voor cleenste maet, als in sijn tafelen blijckt. Voorts boven stelling van oneven onnatuerlicke loopen, soo betuyghen noch de dadelicke ghemeten grijphouccken der Dwaelders, voornamelick van Mars en de Maen, datse niet in sulcke verheyt en naerheyt des Eertcloots en commen als de spiegeling mebrengt, gelijck wy elders breeder verclaert hebben.

Benevens t'ghene tot hier toe gheseyt is noch vervought t'ghetuychnis van verscheyden Gaslaghers, als onder anderen *Regiomontanus*, *Bernhardus Waltheri*, en *Purbachius*, in druck uytgaende, soo en worden der Dwaelders plaetsen niet bevonden t'overcommen mette spieghelinghen: Al t'welck anghemerckt, ten schijnt niet seker ghenouch ofter der Dwaelders onbekende roersels sijn, of wesende, dat wy van haer ghedaente gheen ghenouchsaem bescheyt en hebben, daerom meyn ick dat soo ymant voor hem naem een nieuwe spiegheling van dien te beschrijven, dattèt oirboir waer sich voor al van soo veel ghewisse ervaringhen te voorsien, datse voor gront mochten verstreccken om op te bouwen. Doch hier mede mijn teghenwoordich ghevoelen verclaert sijnde, laet daerentusschen elck sijn goetduncken volghen.

T'BESLVYT. Wy hebben dan ghedaen een verhael op der sterren onbekende loop, en des duysteraers onbekende afwijcking vanden Evenaer, na den eysch.

DES DERDEN BOVCX
EYNDE.

consider *Ptolemy's* experiences, on which they are founded, to be uncertain, for though he has performed a great deal of praiseworthy work in this respect, nevertheless, considering that this took place with a small astronomical instrument of copper, made of different rings, each revolving about its axis, so that the ecliptic of the instrument was always obtained parallel to the celestial ecliptic, and since he saw the stars through small peepholes in the diopter rings, wider than the size of the star, which was a more uncertain procedure than by the practice of Tycho Brahe, it will be sufficiently obvious to those who handle such instruments to what uncertainties they give rise, however carefully they may have been made, as *Ptolemy* himself sometimes warns his readers. But when the altitudes of the stars or their distances from each other are measured with large instruments and the rest is achieved by calculations of plane and spherical triangles, this forms a firmer foundation on which to proceed.

There was also some uncertainty in *Ptolemy's* experiences concerning the Planets' apparent ecliptical longitudes, because the fixed stars themselves, on which practical measurement is based, up to that time had been roughly described with 10′ as the smallest measure, as appears from his tables. Further, apart from the supposition of unequal unnatural motions, the practically measured angular diameters of the Planets, chiefly of Mars and the Moon, also show that they do not come as far from and near to the Earth as the theory implies, as we have explained more fully elsewhere.

When to the statements made up to this point there is further added the testimony of different Observers, such as, among others, *Regiomontanus, Bernhardus Waltheri, and Purbachius* [1]), which has been printed, the Planet's positions are not found to agree with the theories. Taking all this into consideration, it does not seem to be certain enough whether there are unknown motions of the Planets or whether we do not have sufficient information about their character. I am therefore of opinion that if anyone intended to describe a new theory about this matter, it would be proper first of all to procure so many certain experiences that they might serve as a foundation on which to build such a theory. But since herewith my present view has been set forth, let everyone meanwhile use his own discretion.

CONCLUSION. We have thus given an account of the stars' unknown motions and the ecliptic's unknown deviation from the Equator; as required.

END OF THE THIRD BOOK.

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[1]) Johannes Müller (Unfind near Königsberg 1436—Rome 1476), celebrated astronomer, pupil and collaborator of Peurbach, worked in Vienna, in Italy, Hungaria, and Nuremberg.

Bernhard Walther (Nuremberg 1430—*ibid.* 1504), a wealthy citizen, gave financial aid to Regiomontanus and continued his observations. Only parts of his manuscripts have been preserved.

Georg von Peuerbach, see note p. 315.

CONTENTS OF THE HEAVENLY MOTIONS

(The Chapters, marked by an asterisk, have been selected for publication. The numbers between brackets refer to the pages of the original Dutch text).

VAN DE SPIEGHELING DER EBBENVLOET

OF THE THEORY OF EBB AND FLOW

From the *Wisconstighe Ghedachtenissen* (Work XI; i, 26)

INTRODUCTION

HISTORICAL REMARKS [1]

In Antiquity many authors described tidal phenomena and indicated their connection with the moon, though often also a comparison with the breathing of an animal was made. Pytheas of Massilia is said to have been the first to indicate the connection of the half-monthly variation of the tide with the phases of the moon. Strabo in his *Geography* [2] calls attention to the tidal currents in narrow straits, which are connected with the risings, settings, and meridian passages (upper and lower) of the moon. Pliny the Elder [3] is acquainted with the facts that the tide for any place has a constant time-interval with the moon, that the height of the tide varies with the phase of the moon, and also that near the equinoxes the tides are higher than at the solstices, and that on the coasts of the Atlantic they are higher than along the shores of the Mediterranean.

The revival of learning in the sixteenth century, which first had to resuscitate and assimilate the science of Antiquity, did not show much progress as to the knowledge of the Tides. The great navigations of those days indeed made known a wealth of new facts on the movements of the waters. But in the reports of the navigators the westward flow in the oceans was attributed to the westward diurnal motion of the heavens. In a work by Julius Caesar Scaliger [4] (the father of the renowned mathematician and chronologist Josephus Justus), published in 1557, the "desire of the moon" is mentioned as the cause of the tides; the alternation of ebb and flow is explained by the repulsion exerted by the long stretch of the Western Continent lying in the way of the westward stream.

Francis Bacon [5] also speaks of the general westward motion of the sea, and he believes that the short period of half a lunar day is due to obstruction by the shores. In his zeal to refute old superstitions he refuses to assume here an effect of the moon. "Yet it will not immediately follow... that things which correspond in the course and periods of time, or even in the manner of carriage, are in their nature subordinate, and the cause one of the other. For I do not go so

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[1]) An extensive historical survey of the investigations on the tides is found in: Rollin A. Harris, *Manual of Tides* (Appendix 8 to U.S. Coast and Geodetic Survey, 1897). – Pytheas wrote about 325 B.C.

[2]) Strabo (c. 54 B.C.—c. 24 A.D.), *Geography*, 1.3.11–12. *cap.* 54 & 55; 3.5.8. *cap.* 173–174.

[3]) Plinius (23—79 A.D.), *Natural History*, II. 212–222 (Loeb Classical Library, 1938, ed. Rackham).

[4]) J. C. Scaliger, scientist and physician (Padua 1484—Agen 1558), *Exotericarum Exercitationum ad Cardanum* (Exerc. LII).

[5]) Francis Bacon 1561—1626, *De fluxu et refluxu maris* (Ed. Spedding, Vol. V, p. 441).

far as to assert that the motions of the moon or sun are set down as the causes of the inferior motions which are analogous to them; or that the sun and moon (as is commonly said) have dominion over these motions of the sea; .... indeed in that very half-monthly motion (if rightly observed) it would be a very strange and novel kind of obedience, for the tides at the new and full moon to be affected in the same way, while the moon is affected in opposite ways . . ." [6])

Johannes Kepler in a letter to J. G. Herwartus (1598) says that the seas might adhere to the moon [7]). In his *Astronomia Nova* (1609) he writes: "If the earth should cease to attract its waters, all marine waters would be elevated and would flow into the body of the moon" [8]). Yet at other times he compares the tide to the breathing of an animal.

It is well known that Galileo Galilei in 1616 framed a different theory [9]). Ascribing the tides to a periodical irregularity in the velocity of the terrestrial surface, due to a combination of the diurnal rotation and the annual revolution, he makes them an important argument in favour of the Copernican world-system. This effect indeed must be present, but it is too small to be observed and is lost among the irregularities of the small tidal phenomena in the Mediterranean.

The treatise of Stevin on the Tides is part of his *Wisconstighe Ghedachtenissen (Mathematical Memoirs),* published at Leyden in 1608 (Work XI; i, 26); it was translated into Latin by Snellius (Leyden 1608, Work XI b) and into French by Girard (Leyden 1634, Work XIII; ii, 2). In these Memoirs, it is Book VI of the *Eertclootschrift (Geography).*

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[6]) Francis Bacon, *Works, l.c.*, p. 448.
[7]) Joh. Kepler, *Gesammelte Werke,* Vol. XIII, p. 193 (München, 1945).
[8]) Joh. Kepler, *Astronomia Nova* (*Gesammelte Werke*, Vol. III, p. 26. Ed. Max Caspar, München 1937).
[9]) Galileo Galilei, *Discorso sopra il flusso e reflusso del mare* (Ediz. Nazion. V, 371); also: *Dialogo* (4th dialogue) (Ediz. Nazion. VII, 442).

SUMMARY OF THE WORK.

The Tides are treated by Stevin as a chapter of geography. His knowledge of this phenomenon was determined by the conditions on the coasts of Western Europe, especially the North Sea. To him the tides were a phenomenon to be explained, as they were by Kepler, by an attraction of the terrestrial oceans by the moon. In this attraction there is nothing of the later Newtonian theory; it is simply an expression of direct experience. The same experience taught him a second fundamental fact: the waters were attracted not only towards the moon but also towards the opposite side. The third fundamental fact, also taken simply from experience, was the alternation of spring tide at the times of the full and the new moon, and neap tide at the times of the quarters. There is no trace of an attempt to account for this alternation by a combination of solar and lunar attraction; the tides to him were a purely lunar phenomenon.

Starting from this basis, Stevin derives the phenomena of the tides first by means of what in later years was called the static theory. The earth is supposed to be entirely covered with water, the surface of which takes the form of an elongated ellipsoid with the high tops exactly below the moon and at its opposite point, and the places of lowest level forming a circle on the globe at a distance of 90° between the tops. Projected upon the celestial sphere, they are the place of the moon in the sky and its opposite point, and a celestial circle at a distance of 90° from the moon. The observer's zenith, also projected upon the sphere, owing to the diurnal rotation describes a parallel circle with a declination equal to his terrestrial latitude. Thus it is a problem of spherical trigonometry: when the observer's zenith passes the moon (or its opposite) at the shortest distance — *i.e.* when it has the same right ascension — there is high tide; when it intersects the 90° circle of lowest level, the observer witnesses low tide.

These conditions are worked out for a number of special cases. When the moon is in the vernal or autumnal equinox, the low-level circle is the colure through the poles, at 90° right ascension; high tide and low tide alternate with a six-hour interval (neglecting the moon's interim motion in longitude). However, when the moon at 90° or 270° right ascension reaches its maximum declination (neglecting its latitude), the low-level circle passes through the equinoxes and the poles of the ecliptic; the parallel circle described by the observer's zenith intersects it at a point nearer to the pole of the ecliptic, at a right ascension that has to be computed by spherical trigonometry. Then the intervals between the moments of high and low tide are unequal; the rise of the water is slow, its fall is rapid. Since the height of the high tide depends on the distance from the zenith to the moon's position, it will — for medium latitudes — if the moon's declination is large, be different for the two high tides of one day. This difference increases with the observer's latitude, and beyond 61° (determined by the maximum declination 28°.6 of the moon) one of them disappears, so that instead of a 12-hour, a 24-hour period in the oceanic level appears. At the poles there is always an interval of 7 days between high tide and low tide.

Thus Stevin deduces the phenomena of the tides from simple static theory. But he is well aware that the real phenomena of the tides deviate from this theory and are far more intricate. In the next chapters he therefore gives an explanation, first of the fact that high tide occurs many hours (different for different coasts) after the moon's culmination; this is due to the resistance which the coasts of the continents offer to the progress of the tidal wave. The fact that in the seas of Western Europe the high tide proceeds in the wrong direction, from West to East, he explains by stating that small and inland seas are only imperceptibly attracted by the moon. An exact treatment of these phenomena was not yet possible at that time (a century before Newton). Stevin tries to give a demonstration in his third proposition; comparing a small and a large vessel with water, he shows that in the latter a greater weight of fluid can be sustained by a smaller sideways pressure. Thus the small attraction by the moon is more effective for the oceans than for minor waters, such as the North Sea; these receive their motion from the progress of the tide in the oceans. Hence, the tidal wave slowly runs up from the western sea eastward into the broad river estuaries.

Stevin emphasizes the fact that the tidal phenomena are complicated, and that their theory may be highly defective. Because the opinion had been expressed that the motion of the waters was due not to an attraction but to a repulsion exerted by the moon, he examines the consequences of such a force. They can be described by interchanging the expressions high tide and low tide in the preceding results, so that at medium latitudes the rise of the water is rapid and the fall is slow; they are at variance with the phenomena observed. And he concludes his treatise with the wish that all over the earth the tides should be carefully studied, in order that a satisfactory theory might be based on the observations.

SESTE BOVCK DES EERTCLOOTSCHRIFTS, VANDE SPIEGHELING DER EBBENVLOET.

Theoria.

CORTBEGRYP.

ANgesien ervaringen de sekerste gront sijn, als int voorgaende breeder gheseyt is, daermen ghemeene reghelen uyt treckt, om tot kennis aer saken te commen, en dat ons deur deser landen groote seylagen, bequamer middel ontmoet d'ander te vooren gheweest is, om te geraken tot veel ghewisse ervaringhen der eyghenschappen van ebbe en vloet: Soo heeft my tottet bevoorderen van sulcx, oirboir ghedocht van dese stof een * Spiegheling te beschrijven, ghegront ten deele op ervaringhen diemen nu heeft, ten deele op stelling die de natuerlicke redenlijckformich schijnt, dienende als begin, om by manier van beschreven const hier af te handelen, en deur breeder ervaringhen diemen namaels crijghen mocht, oirdentlick na grondelicker kennis te trachten.

Theoriam.

Angaende ymant dencken mocht, dattet van my voor t'uytgeven van desen vougheucker waer gheweest, sulcke dinghen eerst sekerlick ondersocht te hebben, of doen ondersoucken: Hier op segh ick dat sulcx niet een of weynich menschens werck wesende, soo heeft my dit de bequaemste wech ghedocht, om op corten tijt veel bescheyt en sekerheyt te crijgen, want veel menschen totte boveschreven gaslaginghen vermaent sijnde, t'can ghebeuren datter hun tot verscheyden plaetsen meer toe sullen begeven, dan deur mijn besonder voordering an besonder menschen meughelick soude wesen.

1 BE-

OF THE THEORY OF EBB AND FLOW

SUMMARY.

Since experience is the surest ground, as has been said more fully in the foregoing, from which to draw general rules in order to gain knowledge of things, and since thanks to the great navigations of these countries we have better means than before for obtaining many sure experiences of the properties of ebb and flow, it seemed suitable to me, in order to promote this, to describe a theory of this subject matter, based in part on experiences now available and in part on suppositions which seem to be in accordance with natural reason, which description may serve as a starting point, in order to deal with this in the manner of a textbook and properly strive to gain fuller knowledge by means of ampler experience that may be obtained later.

If anyone should be of opinion that it would have been more fitting if, before publishing this treatise, I had first examined these things with certainty or caused them to be so examined, I say to this that since that is not the work of one man or a small number of men, this seemed to me the best method for getting much information and certainty in a short time, for when many people have been admonished to make the above-mentioned observations, it may happen that in different places more people will proceed to do so than would be possible by my private admonition to some private persons.

BEGHEERTEN.

1 BEGHEERTE. *Postulatum.*

Wy begheeren toeghelaten te vvorden, dat de Maen en haer teghepunt het vvater des Eertcloots gheduerlick na hun suyghen.

VERCLARING.

MEN bevint deur daghelicksche gheduerighe ervaringhen, dat ebbe en vloet vande Maen gheregiert worden, oock den vloet ten hoochsten te commen in volle en nieu Maen, diemen dan sprinckvloet noemt, maer ten leeghsten in * vierdeschijn: Waer *Quadratura.* af sulcken ghemeenen kennis sijnde, datmen met ghenouchsaem sekerheyt van te vooren weet de uyr van toecommende ghetijden, totgroot voordeel der zeevaert, soo en ist niet noodich daer af toelating te begheeren. Maer want met elcken Maenkeer om den Eertcloot (die ontrent alle 25 uyren eens ghebeurt) twee vloeden en oock twee ebben commen, soo wort by ettelicke vermoet de Maen en haer teghepunt een eyghenschap te hebben, datse het water na hun suyghen inde hooghde: Doch ist onseker oft inde natuer soo toegaet, want deur haer persing (t'welck t'verkeerde van suyging is) souden oock dagelicx twee ebben en vloeden commen. Maer welck van beyden, of wat ander derde natuerlicke eyghenschap daer af d'oirsaeck is, meyn ick onbekent te sijn deur ghebreck van ervaring. Ick heb eenighe ons Indivaerders ondervraeght na de ghedaente van waterghetyen tot seker plaetsen, maer niet connen vinden t'ghene ick socht: Doch anghesien t'vermoen van ettelicke is, t'selve deur de voorschreven suyging te geschien, so ist dat wy begheeren sulcx toegelaten te worden, om also ons voorghenomen spiegheling een gront te gheven, welcke beschreven sijnde, sullen eyntlick int naest laetste voorstel segghen wat verandering uyt persing volghen soude.

2 BEGHEERTE.

Den Eertcloot heel met vvater bedeckt te sijn, sonder vvint of yet dat an ebbe en vloet verhindernis gheeft.

VERCLARING.

De natuerlicke oirden van ebbe en vloet, wort deur winden, oock deur landen boven t'water uytstekende, verhindert, sulcx dat tot alle plaetsen geen hoochste water en is, wesende de maen of haer teghenpunt int middachront, ghelijck de ghemeene reghel vereyscht volghende d'eerste begheerte, maer mach dan ten leegsten sijn, of min verschillen: Ten anderen dat de vloet niet ancommen en sal uyt den oosten na westen, maer uyt den westen of eenigen anderen oirt. Ten derden datmen tot sommighe plaetsen diens toppunt verre vande Maen is, den daghelickschen vloet hoogher bevint dan tot ander plaetsen over diens toppunt de Maen gheweest heeft, daerse volghende t'gestelde ten hoochsten soude sijn.

Maer

POSTULATES.

1st POSTULATE.

We postulate that the Moon and its opposite continually suck the water of the Earth towards them.

EXPLANATION.

It is found by daily continual experiences that ebb and flow are governed by the Moon, and also that the flood-tide is highest at full and new Moon, which is then called spring-tide, but lowest at quadrature. This being such common knowledge that the hour of future tides is known beforehand with sufficient certainty, to the great advantage of navigation, it is not necessary to postulate this. But because with every revolution of the Moon about the Earth (which takes place once in about every 25 hours) there are two flood-tides and also two ebb-tides, it is suspected by several people that the Moon and its opposite have the property of sucking the water up towards them. It is, however, uncertain whether it happens like this in nature, for in consequence of their pressure (which is the opposite of suction) there would also be two flood-tides and two ebb-tides every day. But I think that for lack of experience it is unknown which of the two or what other (third) natural property is the cause of this. I have inquired with some of our mariners sailing to the Indies about the nature of the tides in certain places, but have not been able to find what I sought. But since several people suppose that it is due to the aforesaid suction, we postulate this in order thus to give our intended theory a basis. And after this has been described, we will finally say in the last proposition but one what change would result from pressure.

2nd POSTULATE.

That the Earth is covered entirely with water, without the wind or anything else impeding ebb and flow.

EXPLANATION.

The natural order of ebb and flow is impeded by winds, also by lands sticking out above the water, so that not in all places is the tide highest when the moon or its opposite is in the meridian, as the general rule requires according to the first postulate, but it may be lowest or differ less. Secondly, the flood may come not from the east to the west, but from the west or some other direction. Thirdly, in some places, whose zenith is far from the Moon, the daily flood-tide is found to be higher than in other places, in whose zenith the Moon has been, where according to the supposition it ought to be highest.

Maer op dat alle dese ongheregheltheden, ons niet en verhinderen om te begrijpen de groote ghemeene eyghenschap van ebbe en vloet, die wy spieghelingsche wijse voornemen te beschrijven, soo begheeren wy hier boven toeghelaten te worden, den Eertcloot heel met water bedeckt te sijn, sonder wint of yet dat an ebbe en vloet hinder gheeft, om daer na vande ghedaente der beletselen onderscheydelicker te meughen spreken: Want gelijck * spieghelingh der Meetconst voorderlick is * totte meetdaet, hoe wel daer in nochtans platten en rechte linien der meetbaer velden en lichamen, niet de volcommenheyt en hebben die de bepalinghen der Spieghelinghen mebrenghen, alsoo can dese spiegheling oock voorderlijck sijn totte daet, voornamelick int stuck der zeevaert, hoe wel nochtans de formen der zeen, niet de volcommenheyt en hebben die dese begheerte en de volghende bepalinghen inhouden.

Theoria Geometriæ.
Praxin Geometriæ.

BEPALINGHEN.

BEPALING.

Wesende ghetrocken een rechte lini van des Eertcloots middelpunt totte Maen: Het punt daerse des vvaters oppervlack gheraeckt noemen vvy Manens vloettop: En t'punt daer teghenover Teghepunts vloettop.

VERCLARING.

Latet rondt A B C D den Eertcloot beteyckenen, diens middelpunt E, welcke teenemael bedeckt sy met water, totten omtreck F G H I toe, en dat sonder storm, wint, of eenich beletsel, na luyt der boveschreven 2 begheerte: Voort sy het rondt K L de Maenwech, waer in K de Maen bediet, en L haer teghepunt: Welcke twee deur haer trecking niet toe en laten, het water om den Eertcloot sijnde, hem totte rontheyt te begheven, maer een eysche form doen hebben: Ick treck daer na de rechte lini K E L, snyende des waters oppervlack in F en H: Voort treck ick I E G rechthouckich op K L. Dit so sijnde, F is des vloets hoochste punt, of top na de Maen toe, t'welck ick daerom noem Manens vloettop. En om derghelicke redenen H haer teghepunts vloettop.

2 BEPALING.

Het rondt op den Eertcloot diens plat de rechte lini tusschen beyde de vloettoppen int middel deursnijt, en daer op rechthouckich is, noemen vvy Ebront.

VERCLARING.

Als het rondt I F G overcant ghesien, heet Ebront, om dat daer in altijt ebbe of leeghste water gheschiet. En valt daer in ten alderleeghsten om dat E I de cortste lini is, diemen van des Eertcloots middelpunt E, tot des waters oppervlack trecken can.

But in order that all these irregularities may not prevent us from understanding the great general property of ebb and flow, which we intend to describe in theory, we postulated above that the Earth is covered entirely with water, without the wind or anything else impeding ebb and flow, after which we may discuss more particularly the character of the impediments. For just as Theoretical Geometry forms a suitable preliminary to practical geometry, although in the latter the planes and straight lines of the fields and bodies to be measured do not have the perfection which the definitions of the Theories imply, the present theory may also form a suitable preliminary to practice, especially in the matter of navigation, although nevertheless the forms of the seas do not have the perfection which the present postulate and the following definitions imply.

DEFINITIONS

1st DEFINITION.

When a straight line is drawn from the centre of the Earth to the Moon, we call the point where it touches the surface of the water the Moon's Flood Top, and the point opposite it the Opposite's Flood Top.

EXPLANATION.

Let the circle *ABCD* denote the Earth, whose centre is *E,* and let it be covered entirely with water, up to the circumference *FGHI,* such without storm, wind or any impediment, according to the above-mentioned 2nd postulate. Further let the circle *KL* be the Moon's orbit, in which *K* denotes the Moon and *L* its opposite. Owing to their attraction these two do not permit the water surrounding the Earth to take a spherical shape, but cause it to have the shape of an egg. I then draw the straight line *KEL,* intersecting the water's surface in *F* and *H.* Further I draw *IEG* at right angles to *KL.* This being so, *F* is the highest point of flood, or the top directed towards the Moon, which I therefore call the Moon's flood top. And for the same reasons I call *H* its opposite's flood top.

2nd DEFINITION.

We call the circle on the Earth, whose plane intersects the straight line between the two flood tops in the middle and is at right angles thereto, Ebb Circle.

EXPLANATION.

Thus the circle *IEG* [1]), seen transversely, is called Ebb Circle, because ebb or lowest tide always occurs in this. And it is lowest of all in this, because *EI* is the shortest line that can be drawn from the centre of the Earth *E* to the water's surface.

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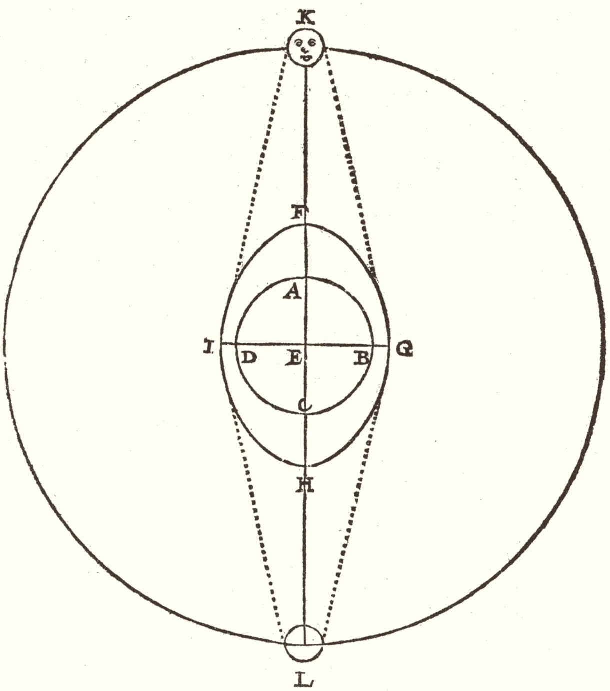
[1]) IFG is a printer's or clerical error in the Dutch text.



NV DE

VOORSTELLEN.

1 VOORSTEL.

T'onderſoucken de ghemeene ghedaente van ebbe en vloet.

Om lijckformicheyt te hebben der gheſtalt van vloettoppen en ebront tot alle plaetſen des Eertcloots, ſoo iſt voorderlick te doen maken een ebbenvloettuych welcke op een Eertcloot gheleyt, en daer op verſchoven ſijnde na ons wille, altijt de ſelve twee vloettoppen en ebront anwijſe: Hier toe heeft ſijn VORSTELICKE GHENADE hem in dees ſpiegheling octfenende, doen bereyden ſeker ronden van ſtijf papier, daer me ſulcx te weghe ghebrocht wiert, t'welck yder tot dergheliјcke luſt hebbende, oock ſo ſou meughen doen. Doch want op de hemelclooten der vaſte ſterren, ſeker twee ronden en vier punten gheteyckent worden, die ons verſtrecken meugen voor ebronden en vloettoppen van eenighe beſonder plaetſen, ſoo ſullen wy die voorbeeltſche wijſe daer toe ghebruycken, want den ſin daer me verſtaen ſijnde, ſoo ſalſe metten boveſchreven ebbenvloettuych openbaer weſen int ghemeen overal.

NOW THE PROPOSITIONS

1st PROPOSITION.

To examine the general figure of ebb and flow.

In order to have uniformity in the form of flood tops and ebb circle in all places of the Earth, it is efficacious to have a tidal instrument constructed, which, when laid on a Globe and displaced thereon as desired, shall always show the two flood tops and ebb circle. To this end his PRINCELY GRACE, when he practised the present theory, had certain circles of stiff paper prepared, with which this was brought about, a thing which anyone having a mind to it might also do in this way. But because on the celestial globe of the fixed stars are drawn two circles and four points which may serve as ebb circles and flood tops of some particular places, we will use them for this purpose by way of example, for if the meaning has been understood in this way, it will be clear in general with the above-mentioned tidal instrument.

1 *Voorbeelt gaende de Maen onder de lentsne, dats boven t'middelront.*

Sectionem vernalem & autumnalem.

In handen hebbende een Hemelcloot, ick stel my voor * lentsne en herbsne de twee vloettoppen te sijn, wesende de Maen onder t'begin des duysteraers, en sal alsdan ebront sijn het rondt deur den 90 tr. en de aspunten des Eertcloots. Dit aldus wesende, soo lang de Maen onder de lentsne loopt, salt tot elcke plaets opt middelrondt daer de Maen boven is hoochwater sijn: En 6 uyren daer na (meer van weghen haer eyghen loop ontrent $\frac{1}{4}$ dats 6 $\frac{1}{4}$ uyren) sal het ebront tot die plaets ghecommen sijn, en daerom alsdan t'water ten leeghsten: En binnen ander 6 $\frac{1}{4}$ uyr daer nae weerom hooghe vloet, en soo overhant gheduerlick mette volghende ebbe en vloet.

2 *Voorbeelt gaende de Maen onder des duysteraers 90 tr.*

Ten tweeden stel ick my op den Hemelcloot voor, des duysteraers 90 tr. en haer teghenoverpunt den 270 tr. de twee vloettoppen te sijn, wesende de Maen onder den 90 tr. en sal alsdan t'ront deur de lentsne, en des duysteraers aspunten ebront sijn. Hier me en sullen de tijden tusschen ebbe en vloet niet evegroot vallen als int eerste voorbeelt, maer verschillen, en dat tot d'een plaets meer als d'ander na t'verschil haerder breeden. Om nu t'selve te vinden tot een ghegeven breede, als van 50 tr. ick verhef den aspunt op sulcken hooghde boven den sichteinder, en breng den 90 tr. des duysteraers onder t'middachront, keer daer na den cloot tot dattet ebront deursnijt het middachront inden voorschreven 50 tr. der breede, dats int toppunt: En sie hoe veel trappen des evenaers daerentusschen verloopen sijn, bevinde neem ick 121 tr. 13 ①, die, 15 tr. op de uyr gherekent, bedraghen 8 uyr 5 ①, voor den tijt van t'hoochste water tottet leeghste: En van daer tot d'ander vloet sal sijn 3 uyr 55 ①, te weten t'verschil tusschen 8 uyr 5 ① en 12 uyren; welverstaende dat hier tot elck noch soude moeten vergaert worden t'ghene de Manens eyghenloop veroirsaeckt, t'welck ick cortheyts halven achterlaet.

MERCKT.

Sijn VORSTELICKE GHENADE dit boveschreven 2 voorbeelt metten Hemelcloot ondersouckende, en voorder lettende opt vervolgh van verscheyden ebben en vloeden achter malcander, heeft daer in de volghende oirden bemerckt.

Den tijt van ebbe na de boveschreven vloet, sal sijn van 3 uyr 55 ①: En de volghende vloet weerom 8 uyr 5 ①. Sulcx dat by aldien de Maen gheduerlick liep in des duysteraers 90 tr. (t'welck om de leerings wil soo mach ghestelt worden) d'oirden van ebbe en vloet soude dusdanich sijn.

Ebbe	8 uyr 5 ①
Vloet	3 uyr 55 ①
Ebbe	3 uyr 55 ①
Vloet	8 uyr 5 ①
Ebbe	8 uyr 5 ①
Vloet	3 uyr 55 ①
Ebbe	3 uyr 55 ①
Vloet	8 uyr 5 ①
Ebbe	8 uyr 5 ①

En

1st Example, the Moon moving under the vernal equinox, i.e. *above the Equator.*

Having in my hands a celestial globe, I imagine the vernal and the autumnal equinox to be the two flood tops when the Moon is at the beginning of the ecliptic; then the circle through 90° and the poles of the Earth will be the ebb circle. This being so, as long as the Moon is moving under the vernal equinox, it will be high tide in every place on the equator where the Moon is in the zenith. And 6 hours later (about ¼ hour more, on account of its own motion, *i.e.* 6¼ hours) the ebb circle will have reached that place, and therefore the water will then be lowest. And another 6¼ hours later it will be high flood-tide again, and so on continually with the next ebb and flow.

2nd Example, the Moon moving under the point at 90° of the Ecliptic.

Secondly I imagine on the Celestial Globe that the point at 90° and its opposite at 270° [of ecliptical longitude] are the two flood tops, the Moon being under 90°, then the circle through the vernal equinox and the poles of the ecliptic will be the ebb circle. Herewith the intervals between ebb and flow will not be equally long, as they were in the first example, but they will be different, such in one place more than in another, according to the difference of their latitudes. In order to find it at a given latitude, *e.g.* of 50°, I raise the pole to that height above the horizon and bring the 90° of the ecliptic under the meridian, then turn the globe until the ebb circle intersects the meridian at the aforesaid 50° of latitude, *i.e.* in the zenith, and I ascertain how many degrees of the equator have passed between the two positions. I find *e.g.* 121°13′, which — taking 15° to the hour — is equivalent to 8 hours 5 minutes for the interval from the highest to the lowest tide. And from there to the next high tide will be 3 hours 55 minutes, to wit, the difference between 8 hours 5 minutes and 12 hours, it being understood that to each there would still have to be added what the Moon's own motion causes, which I omit for brevity's sake.

NOTE.

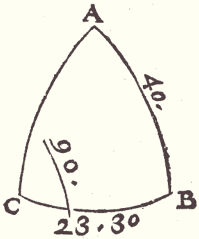
When his PRINCELY GRACE examined the above-mentioned 2nd example with the Celestial Globe and further observed the sequence of many successive tides, he noted in this the following regularity.

The time from the above flood-tide to the next ebb-tide will be 3 hours 55 minutes, and then to the next flood-tide again 8 hours 5 minutes, so that if the Moon were continually at 90° of the ecliptic (which may thus be assumed for didactic purposes), the order of the tides would be as follows:

Ebb-tide	after	8	hours	5	minutes
Flood-tide	„	3	hours	55	minutes
Ebb-tide	„	3	hours	55	minutes
Flood-tide	„	8	hours	5	minutes
Ebb-tide	„	8	hours	5	minutes
Flood-tide	„	3	hours	55	minutes
Ebb-tide	„	3	hours	55	minutes
Flood-tide	„	8	hours	5	minutes
Ebb-tide	„	8	hours	5	minutes

En ſoo gheduerlick voort. Doch valt hier daghelicx noch ſulcke veranderring by,als de Maenloop veroirſaeckt.

Maer om de voorſchreven vinding des tijts deur rekening der clootſche driehoucken af te veerdigen, ick aenſie int eynde der voorgaende wercking op den Hemelcloot, de geſtalt des driehoucx begrepen tuſſchen de drie punten als Eertcloots aſpunt, Duyſteraers aſpunt, mettet toppunt,en merck den houck an des duyſteraers aſpunt recht te weſen: De ſijde tuſſchen beyde de aſpunten te doen 23 tr.30 ①: En de ſijde van des Eertcloots aſpunt tottet toppunt 40 tr. te weten ſchilbooch vande gegevē breede 50 tr. Sulckē driehouc teycken ic hier als A B C, alwaer A het toppunt bediet, B des Eertcloots aſpunt, C des duyſteraers aſpunt,weſende den houck ande ſelve C recht, de ſijde A B van 40 tr. en B C 23 tr.30 ①. Sulcx dat den driehouck drie bekende palen heeft, waer me ghesocht den houck B, wort deur het 32 voorſtel der clootſche driehoucken bevonden van 58 tr. 47 ①: Die ghetrocken van 180 tr. blijft 121 tr.13 ①,en ſoo groot is den verloopen evenaerbooch die doet voor t'begheerde 8 uyr 5 ①.



TBEWYS.

B A, B C, voortghetrocken totten evenaer,begrijpen daer des evenaers booch als grootheyt des houcx B, dats 58 tr.47 ①: Maer de voortghetrocken B C valt in des duyſteraers 270 tr. tuſſchen welcke en des evenaers 90 tr. ſijn 180 tr. die ten tijde des hooghen waters int middachront was, daerom vanden tijt des hooghſten waters totten tijt des leeghſten dattet ebront deurſneet het middachrondt inden 50 tr. der breede, ſijn verloopen 180 tr. min den houck B 58 tr.47 ①, dats 121 tr. 13 ①, als int werck.

1 VERVOLGH.

T'water is op de Eertcloots aſpunten ten leeghſten, loopende de Maen onder den evenaer als gheſeyt is int 1 voorbeelt; En ten hoochſten weſende in haer uyterſte noortſche en zuytſche breede, waer uyt volght datmen daer met elck Maenſchijn maer tweemael hooch water en heeft,en tweemael leeghwater: En dattet ten leeghſten ſijnde daer na ontrent 7 daghen lanck waſt,en weerom ontrent 7 daghen daelt,en ſo overhant voort. Doch crijcht deſe oirdentlicke ebbe en vloet ſoo daer als overal eenighe verandering deur dien de vloettoppen hoogher rijſen, in ſaming en teghestant van Son en Maen,dan alſſe een vierendeelronts ſchijnbaerlick van malcander ſijn.

2 VERVOLGH.

Tis kennelick dat ebbe en vloet opt middelront elck altijt ghedueren ontrent $6\frac{1}{4}$ uyren, tot wat plaets de Maen oock is. Laetſe by voorbeelt ſijn in des Duyſteraers 90 tr. Dit ſoo ghenomen, het ront deur le lentſne en weerelts aſpunten ſtreckende is dan ebront, dat openbaerlick 6 uyren in evenaerloop verſchilt vanden ſelven 90 tr. des duyſteraers: En dergheliјcke bevint ſich oock alſoo tot alle plaetſen daer de Maen is, ghelijckmen lichtelicker ſiet deur t'behulp eens papieren ebbenvloettuychs daer int begin des 1 voorſtels afgheſeyt is.

And thus continually on. But to this, daily changes have to be added as caused by the Moon.

However, in order to effect the aforesaid finding of the time by means of spherical triangles, at the end of the foregoing operation on the Celestial Globe I note the form of the triangle contained between the following three points: the pole of the Earth, the pole of the Ecliptic, and the zenith, and I note that the angle at the pole of the ecliptic is right, that the side between the two poles is 23°30′, and that the side from the pole of the Earth to the zenith is 40°, to wit, the complement of the given latitude of 50°. I here draw this triangle, namely *ABC,* where *A* denotes the zenith, *B* the pole of the Earth, *C* the pole of the ecliptic, the angle at the said *C* being right, the side *AB* 40°, and *BC* 23°30′, so that the triangle has three known terms. When by these means the angle *B* is sought, by the 32nd proposition of spherical triangles [1]) it is found to be 58°47′. When this is subtracted from 180°, there remains 121°13′, and this is the arc of the equator passed through, which is equivalent to the required 8 hours 5 minutes.

PROOF.

When *BA, BC* are produced to the equator, they there contain the equator's arc which is the magnitude of the angle *B, i.e.* 58°47′. But *BC* produced passes through 270° of the ecliptic, and between this and 90° of the equator there are 180°, which were in the meridian at the time of high tide; therefore from the time of the highest tide to the time of the lowest, when the ebb circle intersected the meridian at 50° of latitude, 180° have passed *minus* the angle *B* of 58°47′, *i.e.* 121°13′, as in the trigonometrical operation.

1st SEQUEL.

The water is lowest at the poles of the Earth when the Moon moves under the equator, as has been said in the 1st example, and highest when it is at its extreme northerly and southerly latitudes, from which it follows that there they have twice high tide and twice low tide in every lunation, and that when it is lowest, thereafter it rises for about 7 days and ebbs again for about 7 days, and thus continually. But this regular ebb and flow will undergo some change there as well as everywhere because the flood tops rise higher when Sun and Moon are in conjunction and opposition than when they are apparently a quarter circle apart.

2nd SEQUEL.

It is evident that on the equator low and high tide each always lasts about 6¼ hours, no matter in what place the Moon is. Let it be, for example, at 90° of the Ecliptic. This being assumed, the circle passing through the vernal equinox and the poles of the ecliptic [2]) is then the ebb circle, which evidently differs 6 hours in equatorial motion from the said 90° of the ecliptic. And the same is also found in all places where the Moon is, as is seen more easily by means of a tidal instrument of paper as referred to at the beginning of the 1st proposition.

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[1]) Stevin's *Spherical Trigonometry* (Work XI; i, 13), p. 234.

[2]) For *weerelts aspunten* in the Dutch text read *duysteraers aspunten.*

3 VERVOLGH.

Het blijckt op den Eertcloot, dat plaetsen naerder den aspunt, dan de Maen den evenaer, alsdan vant ebront niet gherocht en worden: Sulcx datse op dien tijt soo leeghen ebbe niet en crijghen, als wanneer het ebront daer over comt.

MERCKT.

Alsoo sijn VORSTELICKE GHENADE het inhout des voorgaende 3 vervolghs deur een Hemelcloot ondersocht, heeft met een willen weten, wat tijt datter tot sulcke plaets verloopt, tusschen het leeghste der ebbe en hoochste des vloets; Maer bevant (soo yghelick sal die dergelijcke doet) altijt 12 uyren, besloot daer uyt dat sulck geval somwijlen soude moeten overcommen alle plaetsen diens breede over de 61 tr. 30 ① streckt, want soo veel blijfter alsmen de Manens grootste evenaerbreede 28 tr. 30 ①, treckt van 90 tr.

Nu ghedenckt my eenich zeevarent volck te hebben hooren bevestighen, datse tot plaetsen gheweest hadden daert 12 uyren ebde, en 12 uyren vloeyde, doch dat sulcx niet lang en duerde, dan dat daer na weerom een ongheregheldе ebbe en vloet volghde. Maer of dit om dese oirsaeck gheschiet, daer soudemen bescheyt af connen weten, alsmen de ervaringhen te werck stelde daer int 9 voorstel af gheseyt sal worden.

4 VERVOLGH.

Tis kennelick dattet een der twee vloettoppen t'welck een gestelde plaets naest comt, aldaer hoogher vloet veroirsaeckt als t'ander. Waer uyt volght dat wesende de Maen over de noortsijde, soo sullen des Eertcloots plaetsen over de noortsijde gheleghen hoogher vloet crijgen vande Manens vloettop, als van haer teghepunts vloettop. Maer wesende het teghepunt over de noortsijde, dat alsdan t'verkeerde ghebeuren sal.

MERCKT.

Uyt het voorgaende valt te besluyten, datter op den Eertcloot vier merckelicke verscheydenheden van ebben en vloeden sijn angaende de gheduericheyt.

Ten eersten opt middelront altijt van ontrent $6\frac{1}{4}$ uyren, als int 1 voorbeelt en 2 vervolgh.

Ten tweeden buyten t'middelront, tot op ontrent den 61 tr. 30 ① der breede, alwaerse verschillen connen na t'inhout des 2 voorbeelts.

Ten derden vanden 61 tr. 30 ① tot byden aspunt, daerse tot sommige tijden elck van 12 uyren sijn, als int Merck des 3 vervolghs.

Ten vierden onder den aspunt, daer ebbe en vloet altijt elck ontrent 7 daghen duyren als int 1 vervolgh.

5 VERVOLGH.

Tis kennelick dat deur de boveschreven twee voorbeelden en vervolghen, in welcke vloettoppen en ebronden op den Hemelcloot gheteyckent waren, lichtelick can verstaen worden de ghemeene regel van ander voorbeelden, diens vloettoppen en ebront daer op niet en sijn, doch daer op beteyckent connen worden metten ebbenvloettuych daer int begin deses voorstels af

gheseyt

3rd SEQUEL.

It appears on the Earth that places nearer to the pole than the Moon is to the equator are not reached by the ebb circle, so that at that time they do not get as low an ebb-tide as when the ebb circle passes through them.

NOTE.

When his PRINCELY GRACE examined the contents of the foregoing 3rd sequel by means of a Celestial Globe, he wanted to know at the same time how much time elapses in that place between the lowest point of the ebb-tide and the highest point of the flood-tide. But he found (as will anyone who does so) this was always 12 hours. From this he concluded that this case would sometimes have to occur for all those places whose latitude is more than 61°30′, for this is what remains when the Moon's greatest equatorial latitude of 28°30′ is subtracted from 90°.

Now I remember that I have heard certain mariners assert that they had been in places where the water ebbed during 12 hours and flowed during 12 hours, but that this did not last long and was followed again by an irregular ebb and flow. But whether this is due to this cause might become known if the experiences to be dealt with in the 9th proposition were put to use.

4th SEQUEL.

It is evident that the one of the two flood tops which comes nearest to a given place causes a higher flood-tide there than the other. From this it follows that when the Moon is on the north side, the places on the Earth lying on the north side will get a higher flood-tide from the Moon's flood top than from the flood top of its opposite. But when the opposite is on the north side, the contrary will happen.

NOTE.

From the foregoing it is to be concluded that on the Earth there are four notable diversities between the tides as to their alternating duration.

Firstly, on the equator always about 6¼ hours, as in the 1st example and the 2nd sequel.

Secondly, outside the equator, up to about 61°30′ of latitude, where they may differ, according to the 2nd example.

Thirdly, from 61°30′ to near the pole, where at some times they both last 12 hours, as in the Note to the 3rd sequel.

Fourthly, at the pole, where ebb- and flood-tide each always lasts about 7 days, as in the 1st sequel.

5th SEQUEL.

It is evident that from the above two examples and sequels, in which flood tops and ebb circles were drawn on the Celestial Globe, the general rule for other examples can easily be understood, for which the flood tops and ebb circle are not present thereon, but can be drawn thereon with the tidal instrument geseyt is. TBESLVYT. Wy hebben dan ondersocht de gemeene gedaente van ebbe en vloet, na den eysch.

2 VOORSTEL.

Te verclaren d'oirsaeck vvaerom na de hooghe sprinckvloeden leegher ebben volghen dan na leeghe vloeden.

T'wort dadelick bevonden dat de ebbe die nae sprinckvloet volght, leegher comt dan de ebbe na vloet ontrent vierdeschijn: Sulcx datmen in zeewercken die op diepe drooghe gront moeten ghemaeckt worden, als onder anderen nu ter tijt inde beleghe rde Stadt Ostende, na sprinckvloet wacht, om het strant leeghe drooch te hebben. D'oirsaeck hier af is openbaer om dese reden: Anghesien inde form van d'eerste bepaling t'water met sprinckvloet, die in saming en teghestant ghebeurt, in meerder menichte hoogher ghetrocken wort an F en H, dan ten tijde van vierdeschijn, soo moetet alsdan opt ebront I G meer ghebreken, en daer minder en leegher wesen dan ten tijde van vierdeschijn.

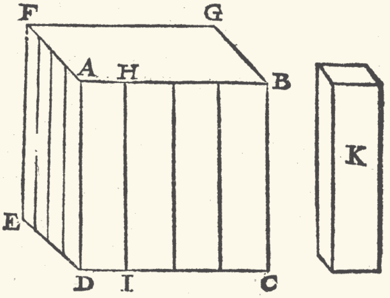
TBESLVYT. Wy hebben dan verclaert d'oirsaeck waerom na de hooghe sprinckvloeden leegher ebben volghen dan na leeghe vloeden, na den eysch.

Tot hier toe sijn de voorstellen gheweest vande ghemeene eyghenschappen van ebbe en vloet: De volghende sullen van besonder wesen, te weten van de oirsaken waerom op besonder plaetsen des Eertcloots, de voorgaende ghemeene reghels gheen plaets en houden.

3 VOORSTEL.

Te verclaren de reden, vvaerom cleene vvateren vande Maen en haer teghepunt, soo hooch niet ghetrocken en vvorden als groote.

Men merckt niet dat cleene waterkens, als binnelantsche meerkens, grachten, water in een glas of ander vat, vande Maen opghetrocken worden: Nochtans mocht ymant dencken, so de eyghenschap der Maen waer t'water na heur inde hooghde te trecken, dat soude so wel over sulcke cleene behooren te geschien, als over ander der groote zeen: Ia datse de cleyne noch hoogher behoort te trecken, deur dietse lichter sijn dan de groote. Om hier af de reden te verclaren, laet A B C D E F G een vat vol waters sijn, elcke sijde als A B C D een rechthouck wesende 4 voet lanck, en 4 voet breet, en sal tegen de selve sijde A B C D perssen een ghewicht even an t'ghewicht van 32 voeten waters deur het 15 voorstel vande beginselen des waterwichts, en t'geheel water sal begrijpen 64 voeten. Ghenomen voort dat elcke sijde die wy gestelt hebben op 4 voet lanck en breet, ghemaeckt sy van 4 plancken elck lanck 4 voet, en breet 1 voet, als de planck A H I D met dierghelijcke, en sal teghen elcke planck ancommen een ghewicht even an t'ghe-



mentioned at the beginning of this proposition. CONCLUSION. We have thus examined the general figure of ebb and flow; as required.

2nd PROPOSITION.

To explain the reason why the high spring-tides are followed by lower ebb-tides than are low flood-tides.

It is found in practice that the ebb-tide following spring-tide is lower than the ebb-tide after flood-tide at quadrature, so that builders of coastal defences, which have to be made on deep, dry ground, such as *e.g.* at the present time in the besieged City of Ostend, wait until after spring-tide, to have the beach low and dry [1]). The cause of this is evident for the following reason: Since in the figure of the first definition the water at spring-tide, which occurs at conjunction and opposition, is raised higher in larger quantities at *F* and *H* than at quadrature, on the ebb circle *IG* there must be a greater shortage and there it must be less and lower than at the time of quadrature.

CONCLUSION. We have thus explained the reason why the high spring-tides are followed by lower ebb-tides than are low flood-tides; as required.

Up to this point the propositions related to the general properties of ebb and flow. The following are to relate to particular properties, to wit, to the reasons why in particular places of the Earth the foregoing general rules do not apply.

3rd PROPOSITION.

To explain the reason why small waters are not raised as high as big waters by the Moon and its opposite.

Small waters, such as inland lakes, canals, water in a glass or some other vessel are not perceived to be raised by the Moon. And yet a man might think that if the property of the Moon were to raise the water up to it, this would have to happen with such small waters just as well as with others, of the big seas; nay, that it even ought to raise the smaller ones higher still, because they are lighter than the big ones. In order to explain the reason of this, let *ABCDEFG* be a vessel full of water, each side (*e.g. ABCD*) being a rectangle 4 feet long and 4 feet broad; against this side *ABCD* there will press a weight equal to the weight of 32 [cubic] feet of water, by the 15th proposition of the elements of hydrostatics, and the whole of the water will be 64 [cubic] feet. Assuming further that each side, which we have said to be 4 feet long and broad, be made of 4 boards, each 4 feet long and 1 foot broad, such as the board *AHID* and the like, then the weight pressing against each board will be equal to the weight

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[1]) This passage gives rather precise information on the year when Stevin's *Ebb and Flow* was written. Ostend was besieged from 1601 to 1604 by the Archduke Albert and the Spanish general Spinola; finally it was taken and entirely destroyed.

wicht van 8 voeten waters, als vierendeel vande 32 voeten, en sullender in als sijn 16 sulcke plancken. Ghenomen voort dat elcke planck om in die standt ghehouden te worden, een teghendrucking moet hebben van buyten, even so stijf als t'water van binnen, dat is t'ghewicht ghelijck voor gheseyt is van 8 voet waters: Hier toe neem ick te wesen 64 verscheyden crachten, of om by claerder voorbeelt te spreken 64 mannen, te weten soo veel alsser voeten waters int vat sijn, en sal commen tegen elcke planck 4 mannen. Laet nu K een ander cleender vat vol waters sijn, ghemaeckt alleenelick van 4 plancken, even en ghelijck mette voorgaende, en sal dat vat begrijpen 4 voet waters, staende 4 voet hooch. Dit soo sijnde, dat ymant nu aldus seyde: Anghesien 64 mannen ant groot vat connen houden 64 voeten waters 4 voet hooch, soo connen 4 mannen ant cleen vat houden 4 voet waters 4 voet hooch, en wilde over sulcx teghen elcke der vier plancken des cleen vats K, stellen alleenelick een man, daer tegen yder planck des groot vats ghestelt sijn 4 mannen, dat waer ghemist, om dat teghen elcke planck des cleen vats, even soo veel ghewicht perst als teghen elcke planck des groot vats deur het 11 voorstel vande beginselen des waterwichts. Sulcx dat teghen de 4 plancken des cleen vats, niet en souden moeten ghestelt sijn 4 mannen, om t'water in die standt te houden, maer 16 mannen: En vervolghens 4 mannen en sullen 4 voeten waters int cleen vat soo hooch niet connen opperssen, als 64 mannen sullen connen opperssen 64 voeten waters int groot vat. Waer uyt wijder volght, dat soo een bevelhebber over sulck volck, tot een cleen water een menichte van mannen veroirdende, in sulcken reden totte mannen teghen een groot water, als het cleen water tottet groot, dat hy daer me de cleene wateren soo hooch niet perssen en sal als de groote: En alsoo is dergheliicke te verstaen mette Maen en haer teghepunt, welcke deur haer trecking over eveveel waters eveveel ghewelts doende, en trecken de cleene wateren soo hooch niet als de groote: En vervolghens hoe wel sy heur treckende werck soo sterck doen op cleene waterkens als op groote, soo en cant nochtans op de cleene om de boveschreven oirsaken niet bemerckt worden.

TBESLVYT. Wy hebben dan verclaert de reden waerom cleene wateren vande Maen en haer teghepunt, so hooch niet ghetrocken en worden als groote, na den eysch.

4 VOORSTEL.

Te verclaren de reden vvaerom de vloet tot veel plaetsen niet an en comt van oosten na vvesten, ghelijck de gemeene reghel der spiegheling mebrengt.

Het blijckt deur het 2 voorstel, dat cleene waters soo hooch niet ghetrocken en worden als groote: Hier uyt volght dat rivieren cleene waters sijnde, in haer selven gheen vloet en hebben, en dat de vloet diemender in siet, niet uyt de rivier, maer uyt de zee comt, waer uyt wijder volght, dat alst vande mont der rivier opwaert, of na de lantsijde, van westen na oosten streckt, dat dan de vloet daer in moet loopen, teghen de ghemeene reghel van westen na oosten, en die strecking der rivier anders sijnde, so comt den vloet oock anders in. Als de vloet commende uyt zee in Schelde loopt van Berghen op Zoom na Antwerpen zuytwaert van daer na Baestroo ghelijck haer de form der rivier leyt. T'gene wy hier gheseyt hebben vande zee en een rivier, verstaet hem oock alsoo met een

groote

of 8 [cubic] feet of water (being one fourth of the 32 [cubic] feet), and there will be 16 such boards in all. It is further assumed that each board, to be kept in that position, must be subjected to a counter-pressure from the outside as great as that of the water on the inside, *i.e.* the weight of 8 [cubic] feet of water, referred to before. For this I assume there are 64 distinct forces or, to give a clearer example, 64 men, to wit, as many as there are [cubic] feet of water in the vessel; then against each board will press 4 men. Now let *K* be another — smaller — vessel full of water, made of only 4 boards, equal and similar to the foregoing; then that vessel will contain 4 [cubic] feet of water, standing 4 feet high. This being so, if a man now said as follows: Since at the big vessel 64 men can retain 64 feet of water, 4 feet high, at the small vessel 4 men can retain 4 feet of water, 4 feet high, and because of this he wanted to put only one man against each of the four boards of the small vessel *K,* whereas against each board of the big vessel are placed 4 men, this would be wrong, because the same weight presses against each board of the small vessel as against each board of the big vessel, by the 11th proposition of the elements of hydrostatics [1]); so that against [each of] the 4 boards of the small vessel there would not have to be placed 4 men to keep the water in that position, but 16 men. And consequently 4 men will not be able to raise 4 feet of water in the small vessel to the same height as 64 men will be able to raise 64 feet of water in the big vessel. From this it follows further that if a commander of such men were to order to a small water a number of men having the same ratio to the men he ordered to a big water as the small water itself has to the big water, he will not thus raise the small as high as the big waters. And the same is to be understood for the Moon and its opposite, which, by their attraction exercising the same force on the same quantity of water, do not raise the small as high as the big waters. And consequently, though they exercise their attraction as much on small as on big waters, nevertheless it cannot be perceived on the small ones, for the above-mentioned reasons.

CONCLUSION. We have thus explained the reason why small waters are not raised as high as big waters by the Moon and its opposite; as required.

4th PROPOSITION.

To explain the reason why in many places the flood does not come from east to west, as would be in accordance with the general rule of the theory.

It appears from the 2nd proposition that small waters are not raised as high as big waters. From this it follows that rivers, being small waters, do not in themselves have a flood, and that the flood seen in them does not come from the river, but from the sea; from which it follows further that if the direction from the mouth of the river upwards, or to the side of the land, is from west to east, the flood must enter, against the general rule, from west to east, and if the direction of the river is different, the flood also enters differently. Thus the flood coming from the sea to the Scheldt passes from Berghen op Zoom to Antwerp southwards, from there to Baestroo according as the form of the river

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[1]) This has been demonstrated in *The Elements of Hydrostatics*, Proposition XI. See Vol. I, p. 421.

groote zee en een cleene: Als by voorbeelt, men siet langs de Fransche, Vlaemsche en Hollantsche stranden, den vloet incommen uyt westen na oosten, en dat deur dien de groote wijde Spaensche zee streckende tot America toe, ghescheyden wort met Engelant en Schotlant, vande cleene Duytsche zee, in welcke sulcke verheffing niet wesende als in d'ander, soo comt de groote vloet uyt de groote zee tusschen Enghelant en Vranckrijck daer invallen van westen na oosten, ghelijck wy vooren van t'vallen des vloets in een rivier gheseyt hebben; T'welck in veel zeen schijnende teghen de ghemeene reghel der spiegheling te wesen, nochtans om bekende oirsaken soo wesen moet.

TBESLVYT. Wy hebben dan verclaert de reden waerom de vloet tot veel plaetsen niet an en comt van oosten na westen, ghelijck de ghemeene regel der spiegheling mebrengt, na den eysch.

5 VOORSTEL.

Te verclaren d'oirsaeck, vvaerom dattet tot veel plaetsen gheen hoochste vvater en is, vvesende de Maen of haer teghenoverpunt int middachront, ghelijck de ghemeene reghel der spiegheling mebrengt.

Angesien de vloet der cleene zeen, veroirsaeckt wort deur de vloet der groote ghelijck int 4 voorstel gheseyt is, soo volght daer uyt dat de plaetsen der cleene zee naest de groote ghelegen, eer hooch water moeten hebben als ander verder plaetsen der cleene zee: Daerom alwaert inde groote zee altijt hooch water wesende de Maen of haer teghepunt int middachront, soo en cant op den selven tijt tot ander plaetsen verre ghenouch inde cleene zee gheen hoochste water sijn: Ghelijck men siet in ons cleene Duytsche zee, alwaer de oostlicker plaetsen later vloet hebben dan de westelicker. TBESLVYT. Wy hebben dan verclaert d'oirsaeck waerom dattet tot veel plaetsen gheen hoochste water en is wesende de Maen of haer teghenoverpunt int middachront, ghelijck de ghemeene reghel der spiegheling mebrengt, na den eysch.

6 VOORSTEL.

Te verclaren d'oirsaeck vvaerom sprinckvloet tot sommighe plaetsen deurgaens eenighe dagen later comt dan met volle of nieu Maen.

Int 5 voorstel is gheseyt, dattet hoochste water tot ettelicke plaetsen eenighe uyren gheschiet na de comst der Maen of haer teghenoverpunts int middachront, waer uyt niet vreemt en is dat de sprinckvloeden die altijt metten hoogen vloet commen, aldaer oock soo veel uyren later vallen dan na de ghemeene reghel: Maer ettelicke daghen van volle en nieu Maen te verschillen ghelijck dadelick bevonden wort (want voor Hollandt, soo ick van zeevolck verstae, verschillet over de twee daghen, westwaert min, oostwaert meer) dat mocht ymant bedencking gheven. Om hier af d'oirsaeck te verclaren, ick segh voor al kennelick te wesen deur t'ghene gheseyt is int 4 voorstel, dat sprinckvloet der rivieren en cleene zeen, niet en commen uyt haer selven, maer veroirsaeckt sijn deur de sprinckvloet der groote zee, waer uyt volght dattet verschil des tijts tusschen de directs it [1]). What we have here said about the sea and a river applies in the same way to a big and a small sea. For example, along the French, Flemish, and Dutch shores the flood is seen to come from west to east, such because the big, wide Spanish sea, extending to America, is separated by England and Scotland from the small Dutch sea, and since in the latter there is not the same elevation as in the other, the big flood enters from the big sea between England and France from west to east, as we have said before about the entrance of the flood into a river; and though this seems to be against the general rule of the theory in many seas, this must be so for known reasons.

CONCLUSION. We have thus explained the reason why in many places the flood does not come from east to west, in accordance with the general rule of the theory; as required.

5th PROPOSITION.

To explain the reason why in many places the tide is not highest when the Moon or its opposite is in the meridian, as would be in accordance with the general rule of the theory.

Since the flood of the small seas is caused by the flood of the big seas, as has been said in the 4th proposition, it follows that those places of the small sea which are situated nearest to the big sea must have high tide sooner than other remoter places of the small sea. Therefore, even if it were always high tide in the big sea when the Moon or its opposite is in the meridian, at the same time the tide may not be highest in other places far enough away in the small sea, as is seen in our small Dutch sea, where the more eastward places have the flood-tide later than the more westward places. CONCLUSION. We have thus explained the reason why in many places the tide is not highest when the Moon or its opposite is in the meridian, as would be in accordance with the general rule of the theory; as required.

6th PROPOSITION.

To explain the reason why in some places spring-tide usually occurs a few days later than full or new Moon.

In the 5th proposition it has been said that in several places the highest tide occurs a few hours after the Moon or its opposite is in the meridian, so that it is not strange that the spring-tides always occurring at the high flood-tide also occur there so many hours later than according to the general rule. But that they should differ several days from full and new Moon, as is found in practice (for in Holland, as I understand from mariners, the difference is more than two days, to the west less and to the east more), this might give rise to objections. To explain the cause of this, I say first of all that it is evident from what has been said in the 4th proposition that spring-tides of rivers and small seas are not due to themselves, but are caused by the spring-tide of the big sea, from which it follows that the difference in time between the spring-tides of a big and a small

[1]) See the map on pag. 358.

ſprinckvloeden van een groote en cleene zee, grooter can ſijn dan t'verſchil des tijts tuſſchen de ſprinckvloet der groote zee en een rivier, uyt oirſaeck dat de rivierens ſprinckvloet om het afcommende water dat achter veel hoogher is, niet ſoo verre loopen en can als de ſprinckvloet der cleene zeé, die van achter ſoo veel niet hoogher en is. Om hier af deur een form breeder verclaring te doen, laet A B C D E F G het oppervlack der cleene zee beteyckenen, waer af de verheventheyt A, ſy ande mont der groote zee beduydende den hooghen ſprinckvloet, welcke binnen ſes uyren daer na ten leeghſten ſijnde, en ſes uyren daer na weerom ten hoochſten, ſoo wort die cleene zee in gheduerlicke beweging ghehouden, met verſcheyden hooghe ſprinckvloeden, en ebben tuſſchen beyden, d'een achter d'ander, al veroirſaeckt uyt den eerſten hooghen ſprinckvloet A, wordende allencx cleender en cleender, ſulcx datmen ten laetſten geen

ebbe en vloet meer en merckt. Maer wanttet eenighe daghen can anloopen eer den eerſten hoogen ſprinckvloet A, comt totten laetſten hooghen ſprincvloet G, en dat de hooghe vloeden in alle nieu en volle Maen oock den ſprinck me brenghen als gheſeyt is, ſoo en can den grootſten vloet diemen ſprinck noemt an A ten grootſten weſende, haer aldergrootſten vloet te weten ſprinck an G, niet veroirſaken dan ſoo veel tijts na nieu of volle Maen, als den eerſten ſprinck an A, behouft om over al de ſprinckvloeden B, C, D, E, F, tot G te commen, t'welck na dattet verre is ettelicke daghen an can loopen: Als by voorbeelt: Angeſien het na luyt der Almenacken, met volle en nieu Maen hooch water is tot Calis ten 10 uyren: Tot Nieupoort ten 11 uyren: Tot Oſtende ten 11½ uyren. Tot Blanckeberge te 1 uyr: Tot Vliſſinghen ten 2 uyren: Tot Bergen op Zoom ten 4 uyren: Tot Antwerpen ten 6 uyren: Tot Baeſtroo ten 8 uyren: Soo is de vloet 1 uyr doende met te commen van Calis tot Nieupoort: ½ uyr van Nieupoort tot Oſtende: 2 uyren van Oſtende tot Blanckeberghe: ½ uyr van Blanckeberghe tot Vliſſing: 2 uyren van Vliſſing tot Berghen op Zoom: 2 uyren van Berghen op Zoom tot Antwerpen: 2 uyren van Antwerpe tot Baeſtroode: Maeckt t'ſamen om den vloet te commen van Calis tot Baeſtroo 10 uyren. Waer uyt blijckt dat wanneert te Calis ſprinckvloet is, ſo moetet te Baeſtroo 10 uyren daer na eerſt ſprinckvloet weſen. En die dergheliјcke berekende op een plaets van Baeſtroo veel verder weſtwaert dan Calis, ſoude alſo in plaets van 10 uyren meughen vinden verſchil van eenighe daghen: Daerom al iſt dat ſprinckvloet ter plaets daer geen hinder en is, met volle Maen comt, volghende de ghemeene reghel der ſpiegheling, ſoo moet nochtans om t'belet der uytſtekende landen, tot ſommighe plaetſen de ſprinckvloet eenighe daghen nae volle en nieu Maen commen. TBESLVYT. Wy hebben dan verclaert d'oirſaeck, waerom ſprinckvloet tot ſommighe plaetſen deurgaens eenighe daghen later comt dan met volle of nieu Maen, na den eyſch.

7 VOORSTEL.

Te verclaren d'oirſaeck vvaerom tot ſommige plaetſen verder vanden vloettop gheleghen als ander, nochtans hoogher vloet comt.

Na

sea may be greater than the difference in time between the spring-tide of the big sea and a river, because the spring-tide of a river, on account of the water flowing downward, which is much higher behind, cannot proceed as far as the spring-tide of the small sea, which is not so much higher behind. In order to explain this more fully by means of a figure, let *ABCDEFG* denote the surface of the small sea, of which let the elevation *A* be at the mouth of the big sea, denoting the high spring-tide, and since this is lowest six hours afterwards and six hours later highest again, this small sea is kept in continual movement, with several high spring-tides and ebb-tides between them, one after the other, all caused by the first high spring-tide *A,* which gradually become smaller and smaller, so that finally no ebb and flow is perceived any more. But because it may be some days before the first high spring-tide *A* reaches the last high spring-tide G and the high floods at every new and full Moon also bring on the spring-tide, as has been said, the greatest flood-tide, which is called spring-tide at *A,* when at its greatest, can only cause its greatest flood of all, to wit, the spring-tide at *G,* as much time after new or full Moon as the first spring-tide at *A* requires te reach *G via* all the spring-tides *B, C, D, E, F,* which may be several days, according to the distance. For example [1]: according to the Almanacs at full and new Moon it is high tide at Calais at 10 o'clock, at Nieuwpoort at 11, at Ostend at 11.30, at Blankenberge at 1, at Flushing at 2, at Bergen op Zoom at 4, at Antwerp at 6, at Baestroo at 8. The flood thus takes 1 hour to come from Calais to Nieuwpoort, ½ hour from Nieuwpoort to Ostend, 2 hours from Ostend to Blankenberge, ½ hour [2]) from Blankenberge to Flushing, 2 hours from Flushing to Berghen op Zoom, 2 hours from Berghen op Zoom to Antwerp, 2 hours from Antwerp to Baestroode. This makes together 10 hours for the flood to come from Calais to Baestroo. From this it appears that when it is spring-tide at Calais, it must be spring-tide at Baestroo as much as 10 hours later. And if anyone were to calculate this for a place much further to the west of Baestroo than Calais, he might find instead of 10 hours a difference of some days. Therefore, though spring-tide comes at full Moon in a place where there is no impediment, according to the general rule of the theory, nevertheless in some places, on account of the impediment formed by lands sticking out, the spring-tide must come some days after full and new Moon. CONCLUSION. We have thus explained the reason why in some places spring-tide usually occurs a few days later than full or new Moon; as required.

7th PROPOSITION.

To explain the reason why in some places situated further away from the flood top than others the flood-tide may yet be higher.

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[1]) See the map on p. 358.

[2]) These numbers fit only if it were high tide at Blankenberge at 1.30 p.m.

Na de ghemeene reghel der ſpiegheling,ſo behoort den vloet ten hoochſten te weſen anden vloettop,en de plaetſen die den vloettop naerder ſijn,behooren den vloet oock hoogher te hebben als ander verder daer af ghelegen; nochtans ſietmen dadelick tot veel plaetſen t'verkeerde ghebeuren. Om hier af d'oirſaeck te verclaren, ſoo ſegh ick eerſt by voorbeelt aldus: Men ſiet dat een bare waters commende teghen een hooft in zee uytſtekende, datſe daer vooren verhoocht,ja daer overloopt veel hoogher als ander baren in zee: De reden daer af is deſe: Soo t'voorſte der baer teghen t'hooft ſteutende alleen waer ſonder vervolgh van water, het ſoude als kennelick is ſtracx te rugghe keeren ſonder ſulcke hooge verheffing te doen: Maer de reſt der ſware groote baer op haer ganck weſende, en can op ſoo corten tijt niet keeren,dan het voorſte water wort opghehouden van het volghende,en dat volghende oock van ander volghende, ſulcx dat t'een t'ander onderhoudende en voortdringhende,het gheraeckt tot die voorſchreven verheffing, hoogher dan t'ghemeen water der zee. Dit verſtaen ſijnde, laet ons het heel verheven water eens vloets nemen voor een baer,welcke ettelicke mijlen lanck ſijnde, ten is gheen wonder dat daer af tot ſommighe plaetſen daerſe recht tegen an comt, hoogher verheffing gheſchiet dan in zee,als by voorbeelt de vloet der groote Spaenſche zee,treckende na de leeghe cleyne Duytſche zee als boven geſeyt is,en vallende teghen de landen des inhams van Bretaigne, crijcht een verheffing boven de ebbe van 10 of 11 vamen,dats hoogher dan de vloet tot ander plaetſen die den vloettop naerder ſijn,ja hooger dan onder den vloettop ſelf, en dat om bekende reden. TBESLVYT. Wy hebben dan verclaert d'oirſaeck,waerom tot ſommighe plaetſen verder vanden vloettop ghelegen als ander,nochtans hoogher vloet comt,na den eyſch.

8 VOORSTEL.

Te verclaren d'eygenſchappen dieder van ebbe en vloet ſijn ſouden, ſooſe gheſchiede deur perſing der Maen en haer teghepunt.

Wy hebben tot hier toe de ſaeck ghenomen, al of de Maen en haer teghepunt het zeewater na hun trock of ſoghen, volghende t'inhoudt van d'eerſte begheerte: Maer ſooder een verkeerde eyghenſchap van perſing in waer, ſoo ſouden eenighe reghelen verkeert vallen. Om van t'welck by voorbeelt te ſpreken, laet inde volghende form de letteren van beteyckening weſen als inde form der 1 bepaling,uytghenomen dat de Maen K,en haer teghepunt L, nu t'water niet en trecken als daer,maer perſſen als hier, ſulcx dat de lini F H nu corter ſy dan I G. T'welck ſoo ghenomen, F en H ſullen ebtoppen ſijn, I G vloetront,en veel gedaenten van ebbe en vloet ſullen op verkeerde wijſe commen vande voorgaende des 1 en 2 voorbeelts,oock des 1, 2, 3, en 4 vervolghs vant 1 voorſtel.

Ten eerſten,ſoo ſalt ter plaetſen daer de Maen boven is leegh water ſijn,teghen d'eerſte ſtelling.

Ten tweeden, gaende de Maen onder des duyſteraers 90 tr. ſoo ſal t'ghene int 2 voorbeelt des 1 voorſtels berekent wort op 8 uyren 5 ①,ſijn van 3 uyr 55 ①; En weerom verkeert dat daer berekent wiert op 3 uyr 55 ①, ſal ſijn van 8 uyr 5 ①.

According to the general rule of the theory the flood-tide ought to be highest at the flood top, and the places which are nearer to the flood top also ought to have the flood-tide higher than others which are further away from it; nevertheless, in practice the contrary is seen to occur in many places. In order to explain the cause of this, I first say as follows, by way of example: It is seen that a billow of water striking on a mole sticking out into the sea is raised in front of it, nay, overflows it, much higher than other waves in the sea. The reason of this is the following: If the foremost part of the billow striking on the mole were by itself, without any water following it, it would — as is evident — at once turn back without rising to such an elevation. But the rest of the heavy billow, being already on its way, cannot turn back in so short a time; then the foremost water is retained by the next, and this next also by more that follows, so that, one retaining and urging on the other, the aforesaid elevation, higher than out at sea, is reached. This being understood, let us take the whole of the elevated water of a flood-tide for a wave; then since this is many miles long, it is no wonder that in some places against which it strikes directly it gives rise to a higher elevation than occurs out at sea; thus, for example, the flood-tide of the big Spanish sea, flowing to the low, small Dutch sea, as said above, and striking against the lands of the bay of Brittany, gets an elevation of 10 to 11 fathoms above ebb-tide, *i.e.* higher than the flood-tide in other places which are nearer to the flood top, nay, higher than at the flood top itself, such for known reasons. CONCLUSION. We have thus explained the reason why in some places situated further away from the flood top than others the flood-tide may yet be higher; as required.

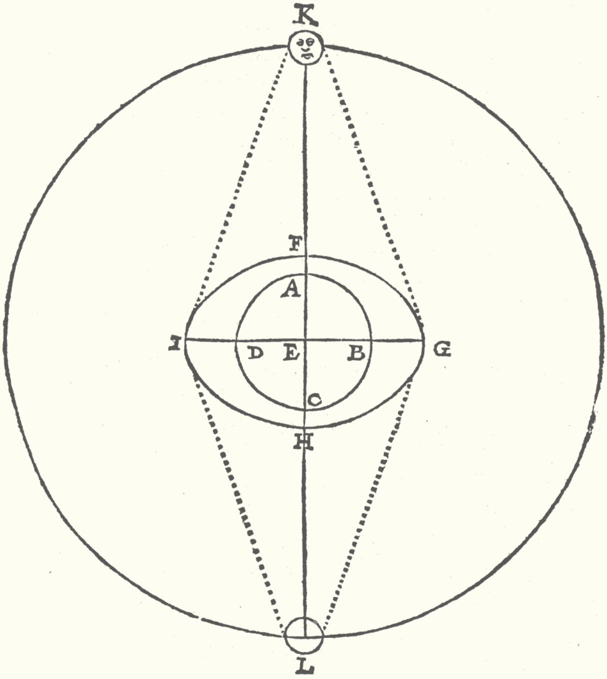
8th PROPOSITION.

To set forth the properties which ebb and flow would have if they were occasioned by pressure from the Moon and its opposite.

Up to this point we assumed that the Moon and its opposite attracted or sucked the sea-water towards them, according to the contents of the first postulate. But if it were due to the opposite property, namely pressure, some rules would become reversed. In order to speak of this by way of example, in the following figure let the reference letters be the same as in the figure of the 1st definition, except that the Moon K and its opposite L do not now attract the water as there, but press it as here, so that the line FH be now shorter than IG. When this is so taken, F and H will be ebb tops, IG the flood circle, and many qualities of ebb and flow will be the converse of the foregoing of the 1st and 2nd examples, as also of the 1st, 2nd, 3rd, and 4th sequels to the 1st proposition.

Firstly, in those places where the Moon is in the zenith it will be low tide, contrary to the first theory.

Secondly, when the Moon is under 90° of the ecliptic, the interval which in the 2nd example of the 1st proposition is calculated to be 8 hours 5 minutes will be 3 hours 55 minutes; and conversely, what was calculated there to be 3 hours 55 minutes, will now be 8 hours 5 minutes.



Ten derden, ſal t'water op des Eertcloots aſpunten ten hoochſten ſijn, loopende de Maen onder den evenaer, en ten leeghſten weſende in haer uyterſte noortſche en zuytſche breede, teghen de reghel des 1 vervolghs vant 1 voorſtel.

Ten vierden, een der twee ebtoppen dat een gheſtelde plaets naeſt comt, veroirſaeckt aldaer leegher ebbe als t'ander: T'welck verſchil heeft vant 4 vervolgh des 1 voorſtels.

Ten vijfden, blijcket dat plaetſen naerder den aſpunt, dan de Maen den Evenaer, alſdan vant vloetront niet gherocht worden, ſulcx datſe op dien tijt ſo hooghen vloet niet en crijghen, als plaetſen daer het vloetront over comt, tegen de reghel vant 3 vervolgh des 1 voorſtels.

Nu welcke ſtelling van beyden recht is, of wat ander derde int natuerlick weſen mach ſijn, daer af acht ick ons dadelicke ervaringhen te ghebreken; maer hoemen daer toe ſoude meughen commen, van dies ſal ick mijn ghevoelen int volghende voorſtel ſegghen. TBESLVYT. Wy hebben dan verclaert d'eygenſchappen dieder van ebbe en vloet ſijn ſouden, ſooſe deur perſing der Maen en haer teghepunt gheſchiede.

9 VOORSTEL.

Te verclaren hoet ſchijnt datmen de ſaeck an ſoude meugen legghen, om te gheraken tot grondelicker kennis van ebbenvloet dander nu af is.

Als

Thirdly, the water will be highest at the poles of the Earth when the Moon moves under the equator, and lowest when it is at its extreme northerly and southerly latitudes, contrary to the rule of the 1st sequel to the 1st proposition.

Fourthly, the one of the two ebb tops which comes nearest to a given place causes there a lower ebb-tide than the other; which differs from the 4th sequel to the 1st proposition.

Fifthly, it appears that places nearer to the pole than the Moon is to the Equator are not then reached by the flood circle, so that at that time they do not get as high a flood-tide as places through which the flood circle does pass, contrary to the rule of the 3rd sequel to the 1st proposition.

Now I consider that we lack practical experience to decide which of the two theories is the right one, or what other (third) theory may be in accordance with nature; but in the following proposition I will give my opinion about the way in which such experience might be obtained. CONCLUSION. We have thus explained the properties which ebb and flow would have if they were occasioned by pressure from the Moon and its opposite.

9th PROPOSITION.

To explain how it seems we might proceed to gain fuller knowledge of ebb and flow than we now have.

When ebb-tide, flood-tide, and spring-tide are not in accordance as to time and magnitude with the general calculation of the tides to which mariners

Als ebbe, vloet en ſprinck, niet en overcommen in tijt en grootheyt, mette gemeene rekening der ghetijden daer de Schippers hun na ghevougen, als wat vrougher of later ghebeurde, of datter leeghe vloet comt, met wint die nochtans hooghe vloet veroirſaeckt, of verkeert hooghe vloet met wint die cleene vloet mebrengt, men ſegt dan ghemeenelick daer moet ander wint in zee ſijn, of gheweeſt hebben, daert nochtans miſſchien ſal meugen ghebeurt ſijn om redenen der ghemeene reghel die wy boven beſchreven hebben, of by anderen daer af beſchrijvelick ſijn. Doch want hier uyt ſomwijlen volcht verlies van ſchip lijf en goet, ſoo en ſchijnet niet buyten reden datmen tracht na ſekerheyt en kennis der oirſaken deſer ſtof. Maer want uyt veel ghewiſſe ervaringhen de ſekerſte reghels gemaeckt worden, ſo ſoudet ſeer voorderlick ſijn dat veel menſchen, tot allen plaetſen des Eertcloots daert te pas can commen, daghelicx gaſloughen en opſchreven al t'ghene ſy dadelick daer af ſien gebeuren, als tot wat uyr ebbe, tot wat uyr vloet comt, metende oock de hooghde van yder vloet, en de leeghde van yder ebbe, daer by opteyckenende de winden of ſtilten dieder dan ſijn, ſulcke ſchriften daer na int ghemeen, en (ſoo den Gaſlagher ſelf gheen ſpieghelaer en is) ter hant der ſpieghelaers commende, ſy ſouden ſien hoe alles mette ghemeene ſpiegheling overquaem, acht nemende op des ghemeene reghels belet van landen en winden. Tis oock te weten dat d'onderſoucking der ghemeene ghedaente van ebbe en vloet daer int 2 voorſtel afgheſeyt is, bequamelicxt ſoude meughen gheſchien op cleene Eylandekens in een groote zee geleghen, en (om geen hinder der ebbe en vloet te hebben) ſeer verre van lant, als Sint Helena en dierghelijcke. Hier uyt ſoudemen meughen mercken, ten eerſten ofter recht onder de Maen en haer tegepunt twee vloettoppen loopen met een ebront, volghende t'eerſte gheſtelde: Of twee ebtoppen met een vloetront, volghende het tweede gheſtelde des 8 voorſtels.

Ten tweeden, als de Maen onder den evenaer loopt, of dan ebbe en vloet $6\frac{1}{4}$ uyren achter malcander volghen na t'inhout der ſpiegheling verclaert in des 1 voorſtels 1 voorbeelt.

Ten derden, als de Maen met groote afwijcking vanden evenaer loopt, of dan ſulcke tijden tuſſchen ebbe en vloet, verſchillen na t'inhout der ſpiegeling verclaert in des 1 voorſtels 2 voorbeelt; of nae t'inhoudt des 2 lidts vant 8 voorſtel, want ons dat oock verſekeren ſoude van ſuyging of perſing.

Ten vierden, of t'water ontrent des eertcloots aſpunten ten leeghſten is loopende de Maen onder den evenaer, volghende t'eerſte gheſtelde, als int 1 vervolgh des 1 voorſtels, of ten hoochſten volghende het tweede gheſtelde, als int 8 voorſtels derde lidt, t'welck ons van ſuyging of perſing oock ſoude wetenſchap gheven.

Ten vijfden, of alſdan dien vloet hoogſt comt, wiens treckende cracht te weten der Maen of haer teghepunts, naeſt des * Doenders toppunt is, na t'inhout der ſpiegeling verclaert in des 1 voorſtels 2 voorbeelt; Of anders dat de vloeden even hooch commen, maer datter ſulck verſchil inde ebben valt, t'welck uyt perſing volghen ſoude. Alle welcke dinghen ſoo ſe bevonden wierden te overcommen met een der twee voorgaende ſpieghelinghen, t'ſoude verſtrecken tot ſekerheyt des handels: Maer verſchillende men ſoude deur dat ghevonden meughen trachten na verbetering. TBESLVYT. Wy hebben dan verclaert hoet ſchijnt datmen de ſaeck an ſoude meughen legghen, om te geraken tot grondelicker kennis van ebbe en vloet d'ander nu af is, na den eyſch.

Efficientis punctum verticale.

EBBENVLOETS

EYNDE.

conform, namely their setting in a little earlier or later, or that the flood-tide is low with a wind which nevertheless occasions a high flood-tide, or conversely the flood-tide is high with a wind which causes a low flood-tide, it is usually said that there must be or must have been a different wind at sea, thought it may perhaps have happened on account of the general rule we have described above, or according to others still to be described. But because loss of ship, life, and property sometimes results from this, it does not seem inopportune to try to get certainty and knowledge of the causes of this matter. But because the surest rules are made from many certain experiences, it would be very propitious if in all places of the Earth where this may be appropriate many people were to observe and write down daily what they see happening in actual fact, *e.g.* at what time there is ebb-tide, at what hour flood-tide, measuring also the height of every flood-tide and the depth of every ebb-tide, and also noting the wind or calm at that moment; if such writings thereafter become public and (if the Observer himself is no theoretician) fall into the hands of the theoreticians, these will have to find out how far everything was in accordance with the general theory, taking into account the impediment to the general rule caused by lands and winds. It is also to be noted that the examination of the general qualities of ebb and flow, as referred to in the 2nd proposition, might be effected most efficaciously in small islands situated in a big sea and (in order not to have ebb and flow impeded) very far from the land, *e.g.* St. Helena and the like. From this, one might note: firstly, whether directly under the Moon and its opposite there are two flood tops with one ebb circle, according to the first theory, or two ebb tops with one flood circle, according to the second theory of the 8th proposition.

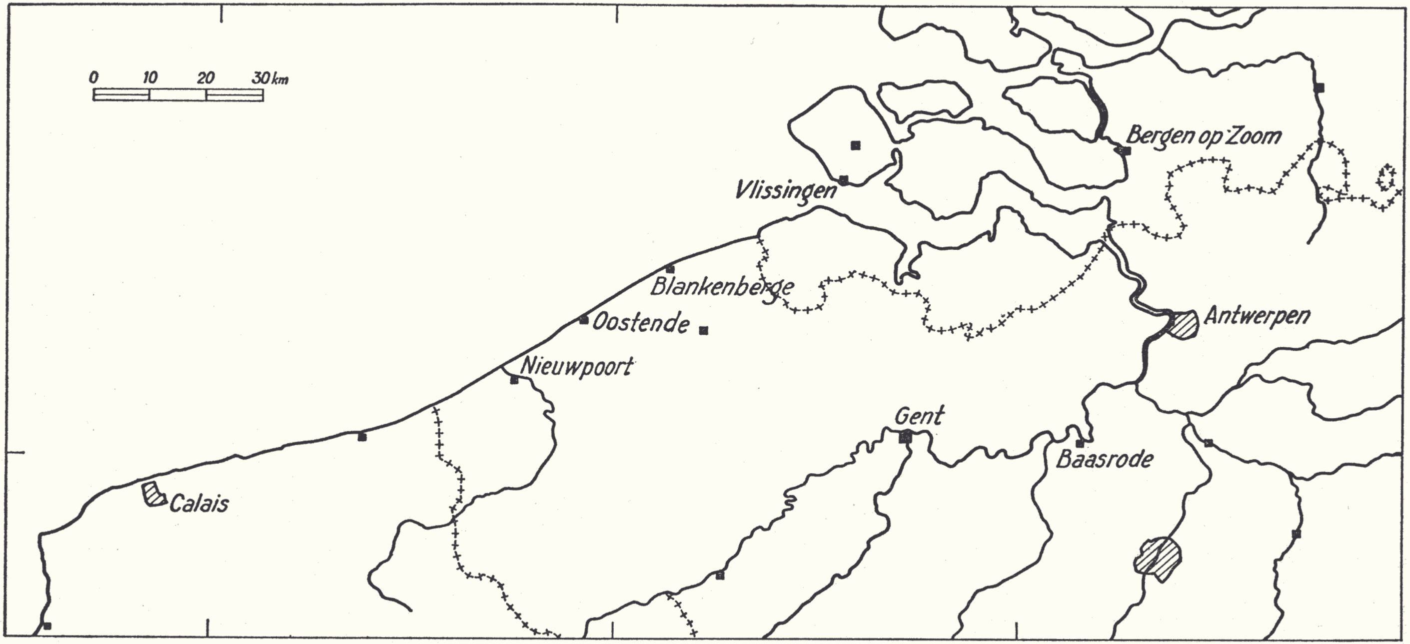
Secondly, whether when the Moon moves under the equator, the tides succeed one another after 6¼ hours, according to the contents of the theory set forth in the 1st example of the 1st proposition.

Thirdly, whether when the Moon has great deviation from the equator, these intervals between ebb-tide and flood-tide differ according to the contents of the theory set forth in the 2nd example of the 1st proposition, or according to the second section of the 8th proposition, for this would also give us certainty whether it is a matter of suction or of pressure.

Fourthly, whether the water is lowest about the poles of the earth when the Moon moves under the equator, according to the 1st theory, *e.g.* in the 1st sequel to the 1st proposition, or highest according to the second theory, *e.g.* in the third section of the 8th proposition, which would also furnish us with knowledge as to suction or pressure.

Fifthly, whether then that flood-tide will be highest for which the attraction, to wit, that of the Moon or its opposite, is nearest to the zenith of the Observer, according to the wording of the theory set forth in the 2nd example of the 1st proposition; or otherwise whether the flood-tides will be the same height, but there is such a difference between the ebb-tides, which would result from pressure. And if all these things were found to be in agreement with one of the two foregoing theories, this would provide certainty in the practical work. But if no agreement were found, one might strive for correction by means of the data found. CONCLUSION. We have thus explained how it seems we might proceed to gain fuller knowledge of ebb and flow than we now have; as required.

END OF EBB AND FLOW.



The estuary of the Scheldt and the adjoining seacost.

THE NAUTICAL WORKS

OF

SIMON STEVIN

DE HAVENVINDING

THE HAVEN-FINDING ART

INTRODUCTION

§ 1

INTRODUCTION AND GENERAL REMARKS

The main rules followed in Stevin's day — and even for a long time after — in conducting a ship safely across the ocean to her destination were:

a. conduct the ship to the latitude of the parallel through the place she is bound for;

b. sail east or west along this parallel.

A place situated on the seaboard of the continent was certain to be reached in this way, but the time of arrival could not be predicted. The rules equally applied if the land to be made was an island, but this involved considerable risk, as was proved all too frequently in actual fact. Indeed, a navigator would sometimes guess he was west of an island, whereas actually he was east of it. If in such a case he headed east, he went in the wrong direction, an error which was not detected until much later and after much doubt. Stevin refers to such a case (p. 427), where a ship sought for St. Helena for several weeks and the navigator "sailed several times around it before he got there" [1]).

The resultant prolongation of the voyage and the complete uncertainty as to the time at which a landfall would be made formed two out of the numerous and very great dangers attendant on navigation in those days. In fact, prolongation of the voyage entailed longer exposition to all sorts of wind and weather, a greater risk of water shortage, illness, and loss of lives. Such uncertainty involved the possibility that in the hours of the night one might suddenly approach land, land which was usually a lee shore and not lit by warning lighthouses. Under such circumstances good seamanship demanded that navigators should keep a very sharp look-out and exercise the greatest caution, while the ship must make no headway during the night, when dangers could not be sighted. Further, the lead had to be kept going. But in spite of all this, in many cases when land suddenly loomed ahead, there was neither time nor opportunity to turn the ship round and avoid peril of death. Indeed, how often a sailing-vessel is driven on by the wind at haphazard, thus meeting its doom, generally with disastrous consequences!

If the determination of longitude by land and by sea had been possible, the seaman would have found coasts and islands in the right place on the chart. He would have been able to mark the position of his ship on the chart and thus

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[1]) When, more than sixty years later, in his *Kort Onderwijs aengaende het gebruyck der Horologien tot het vinden der lenghten van Oost en West* (see: *Oeuvres Complètes de Chr. Huygens*, Vol. XVII, La Haye 1932, pp. 191-237) Christiaan Huygens explains the use at sea of his pendulum clock, he cites a perfectly similar instance of a considerable misconjecture and great uncertainty. This concerned a fleet which in 1664 wanted to make land at Fogo, one of the Cape Verde Islands, on account of water shortage.

would have known its position relative to the coast, in consequence of which the uncertainty would have been removed. But this possibility did not exist. From the early sixteenth century the basic idea of the method of determining longitude with the aid of the chronometer and by means of lunar distances indeed had been known. But the practical application of both these methods was not arrived at until the second half of the eighteenth century. Aboard they were not commonly used until the nineteenth century.

During the sixteenth, seventeenth, and eighteenth centuries a solution of the great problem was diligently and passionately sought in countries like Portugal, Spain, France, England, and the Netherlands, where the seafaring trade flourished and the art of navigation was practised and developed scientifically. The problem was studied in those countries both by sailors, who daily felt the lack of knowledge, and by scholars. The latter made proposals on a great many points. But many more ideas — fantastic ideas in our eyes — were advanced by seafaring and non-seafaring men, who thought they could make useful suggestions and thus hoped to receive the high pecuniary rewards offered for the solution of the problem. The thought and endeavours of all these people form the subject-matter of an important and fascinating chapter of the history of navigation, a chapter which bears the heading "The Problem of Longitude".

A self-contained and completed section of this chapter must be considered to be formed by the efforts which sought the solution in the deviation of the magnetic needle from the astronomical meridian [2]). The title of this part should

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[2]) The vertical plane through the vector representing the geomagnetic force bears the name of the plane of the magnetic meridian, and the intersection of this plane with the horizontal plane is called the magnetic meridian, direction of the magnetic north, or magnetic north and south line. It is the direction in which a magnetic needle pivoting on a vertical pin aligns itself under the influence of terrestrial magnetism. In general this direction does not coincide with the astronomical meridian, which is also called the true north and south line. The magnetic meridian may deviate to the east or to the west of the astronomical meridian. The angle between these two meridians is called declination. The merchant navy speaks of *variation* of the compass, a name which will be used henceforth. In the navigable parts of the world the variation is an acute angle. (see the figure on page 368)

For a given place on the earth the amount of the variation is not constant. It is liable to a very slow increase or decrease (secular variation), further to a small diurnal fluctuation, and sometimes to sudden irregularities.

At the present day maps exist on which lines have been drawn joining the places where the variation has the same value, expressed in degrees; these lines are called isogonics. The lines joining the places at which the variation is nil are called agonics. These maps mention the secular variation. They make it possible to find the amount of the variation for a given point at sea and for a given year. It is clear that the diurnal fluctuation and the above-mentioned irregularities cannot be taken into account.

When a magnetic needle is not directed exclusively under the influence of earth magnetism, but mounted on board a ship where disturbing influences due to surrounding iron occur, the magnetic needle will deviate from the magnetic north and south line. This is called *deviation.*

In those days ships were made of wood. Deviation thus could not be due to the ship's iron, but it *could* be caused by the nearness of guns, objects of iron in the neighbourhood of the compass, knives in the pockets of the helmsman, etc. It is known that a warning was sounded against such influences as far back as the seventeenth century. It may therefore be assumed that navigators were on their guard with regard to this point and that the compasses referred to in the present introduction showed no deviation, so that this deviation will not be taken into account and no more reference will be made to its existence.

be: "the determination of longitude at sea by means of the variation of the compass" [3]). The matter played a part around 1500 to the second half of the eighteenth century.

A good many people in Holland as well as elsewhere thought and wrote about this subject. In the sixteenth century opinions about the possibility of using the variation for the determination of longitude differed widely, between absolute rejection and hopes of complete success. Stevin was among the convinced champions. His *Haven-Finding Art,* published in 1599 [4]), is a lucid scientific treatise, in which the author, making use of existing observational data, states his view of the magnetism of the earth. In an intelligible idiom he sets forth a method which is to enable the seaman, on the strength of the variation of the compass needle observed by him, to conduct his ship unerringly to her destination, without being kept in uncertainty and without having to know either the geographical longitude of the place for which the ship is bound or the longitude of the ship at sea. In Stevin's time, however, the data about the magnetism of the earth were extremely scanty and quite insufficient to base thereon such a method of determining longitude. It was not until the eighteenth century, when more information about the variation of the magnetic needle on the oceans had been collected, that the method became of practical value and, for want of anything better, was used and appreciated by mariners. The basic idea of Stevin's system then proved to be correct. At the present day, when the variation has been measured everywhere and charts show its value for any given place in any given year, a rough determination of longitude by this method would be possible. But the need of this solution is no longer felt. Nowadays ships are steered across the ocean by astronomical navigation, while the chronometer, checked by means of radio time-signals, forms the backbone of this system. It is hardly necessary to refer to up-to-date electronic aids to navigation in this place.

However, all this does not alter the fact that Stevin has supplied the material for a particularly interesting page out of the history of navigation. His treatise may be called a very remarkable book, remarkable for its place in this history as well as for its contents.

§ 2

THE PLACE OF *THE HAVEN-FINDING ART* AMONG SIXTEENTH-CENTURY TEXTBOOKS ON NAVIGATION

The question may be asked whether *The Haven-Finding Art* is one among the early publications on navigation in Holland.

The navigator of the second half of the sixteenth century, sailing along the seaboard of western Europe and to the Baltic was in a position to consult short elementary treatises on navigation, which were included in some "rutters" and atlases destined for this trade. Such books gave a description of the route as well as some information about the compass, about the calculation of the hours of high and low water, about plane charts, etc., the latter with a view to the

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[3]) In § 5 this subject is to be briefly discussed under this title.
[4]) *Works*, X.

compilation of maps [5]). But in consequence of the rapid development of deep sea navigation in Holland there was a growing need of wider nautical knowledge, required for carrying on this trade. It is natural that people looked to the country which had long been in possession of knowledge and experience of deep sea navigation to supply this want. It was Spain which provided the required instruction, both directly and also indirectly *via* England. This development may be briefly outlined here.

In 1580 a Dutch translation of Medina's *Arte de navegar* [6]) appeared at Antwerp [7]). Although the book had been written by one who was no sailor, it was looked upon as a standard work, and it had become very famous and widely diffused. Of the original work, which had appeared 35 years earlier, Italian, French, and English translations had long been in existence. To the Dutch edition the Antwerp mathematician and nautical expert Michiel Coignet added an excellent commentary, *Nieuwe Onderwijsinghe* [8]). This latter work, which was better adapted to practice than was Medina's *Arte,* argues the great nautical knowledge of the expert author. The said translation of Medina, combined with Coignet's appendix, was reprinted several times at Amsterdam, invariably by Cornelis Claeszoon, publisher of a great many books on nautical subjects. In 1598 he already published the fourth edition, which points to a rapid diffusion even in Holland. It appears that the book, in its combination with Coignet's treatise, played a useful part in the advancement of nautical knowledge.

The textbook of Zamorano [9]), which was greatly valued in Spain and passed through four reprints there, appeared in a Dutch translation at Amsterdam in 1598 [10]). The book of William Bourne [11]), based on Spanish sources, which was highly popular in England and was repeatedly reprinted there, appeared in a Dutch translation at Amsterdam in 1594 and once again in 1599 [12]).

The fact that the enumeration of the textbooks available in Holland in 1599 is thus exhausted shows that Coignet in the southern and Stevin in the northern Netherlands were foremost among those who devoted treatises to scientific navigation, although mention must also be made of Adriaan Veen, who in the same period and during the early years of the seventeenth century tried to serve ocean navigation with his charts in the form of a spherical cap [13]). The sailors, who traditionally learned to sail and find their way at sea in practice, through instruction by old salts and their own watchfulness, and who regarded this as

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[5]) See: Cornelis Anthoniszoon, *Onderwijsinge vander zee, om stuermanschap te leeren, derdewerf nu ghedruckt, neerstelijck ghecorregeerdt ende verbetert.* Anno 1558. Described by J. Keuning, Tijdschrift Kon. Ned. Aardrijkskundig Genootschap, 1950, p. 687. See further the works of Lucas Jansz. Waghenaer and others.

[6]) Pedro de Medina, *Arte de navegar.* Valladolid 1545.

[7]) Pedro de Medina, *De zeevaert oft conste van ter zee te varen. In onse nederduytsche tale overgeset ende met annotatiën verciert bij M. Merten Everaert Brug.* Antwerp 1580.

[8]) Michiel Coignet, *Nieuwe onderwijsinghe op de principaelste puncten der zee-vaert.* Antwerp 1580.

[9]) Rodrigo Zamorano, *Compendio de la arte de navegar.* Seville 1581.

[10]) Rodrigo Zamorano, *Cort onderwijs van de Conste der zeevaert.* Amsterdam 1598.

[11]) William Bourne, *A Regiment for the Sea.* London 1574.

[12]) William Bourne, *De const der zee-vaerdt.* Amsterdam 1594.

[13]) Contained in his book: *Tractaet vant Zee-bouck houden op de ronde gebulte pas-kaert,* Amsterdam 1597, which forms part of his *Napasser,* Amsterdam 1597. Adriaan Veen tried to overcome the defects inherent in the sea charts of those days by teaching his readers to work out the sailing problems on segments of the globe.

sufficient, were as a rule reluctant to learn from arm-chair theorists, unacquainted with day-to-day practice. Stevin was of course aware of this reluctance, but it did not deter him, sound scientist as he was, from putting his work and his discoveries at the disposal of seamen. Stevin, being convinced that he had something valuable to offer, made an attempt to improve safety at sea and, if possible, to solve a burning question, a problem which the navigational world was yearning to solve. His work roused the interest of Prince Maurice, for he gratefully relates (p. 431) how the Stadtholder had acquainted himself with the subject and had become convinced of the possibility thus opened for greater safety in navigation. The Prince gave orders for navigators henceforth to determine carefully, with the aid of suitable instruments, the variation of the magnetic needle in the places at which they touched, and to submit the results of their observations to the Admiralty. The latter was to publish the data thus collected. In this way the Stadtholder backed up Stevin. His measures advanced the research on terrestrial magnetism, the result of which was expected to benefit navigation.

Finally it may be observed that the "Privilege" with which *The Haven-Finding Art* opens shows that the States General of the United Netherlands by letters patent of 18th March 1599 granted to Christoffel van Raphelingen, printer at Leiden and a grandson of the famous Christoffel Plantijn (Christophe Plantin) at Antwerp, for a period of six years, the sole right of printing, publishing, and selling this book. We also read there that Van Raphelingen intended to publish the treatise not only in Dutch but in Latin, French, and other versions as well.

Van Raphelingen brought out Latin and French editions almost simultaneously, but he did not after all publish the treatise in other translations. The English translation, which we owe to Edward Wright, was printed and published in London, also in 1599 [14]).

The name of the writer does not occur in the original Dutch book. Stevin's authorship appears from the Latin translation, which is due to Grotius. In the "dedication", dated 1st April 1599, Stevin is mentioned as such (p. 4), and the same is the case in Wright's preface (page B-2 *verso*) to his translation, the title of which is: *The Haven-finding Art*. Reference may further be made to Snellius, who says in his *Tiphys Batavus* [15]): "But this subject has been treated by others and it induced our Stevin *(Stevino nostro)* to write his *Haven-Finding Art (suae Limeneuretices)*. You will find it in the work *Wisconstighe Ghedachtenissen* of H.R.H. Prince Maurice, which was translated into Latin and published by us previously."

§ 3

THE CONTENTS OF *THE HAVEN-FINDING ART* OF 1599

a. *STEVIN'S "CONJECTURE" ABOUT TERRESTRIAL MAGNETISM*

It is known, says Stevin, that for a long time past, especially after the great voyages of exploration to the Indies and America, people have sought for means to determine longitude at sea, so as to make it possible to reach one's destination. However, all these attempts failed. Some investigators had hoped that the vari-

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[14]) *Works*, X.

[15]) Willebrord Snellius, *Tiphys Batavus sive histiodromice de navium cursibus et re navali.* Leiden 1624, p. 67.

ation of the compass might furnish the basis of a method for the determination of longitude, and they had assumed a magnetic pole, by which they understood a point on the earth towards which any magnetic needle, no matter where it was mounted, would point. But experience had taught that the variation had nothing to do with such a pole. This idea therefore had been found to be unsound. By emphatically rejecting such a magnetic pole, Stevin reveals one of the foundations of his standpoint, which was diametrically opposed to that of his great contemporary Petrus Plancius, the famous cartographer and scholar who prepared the ground scientifically for the important first nautical expeditions of the Dutch and taught the navigators who were sent out on these voyages.

Stevin goes on to say that investigations had indeed furnished a method which made it possible to head for the desired harbour unerringly, even if neither the longitude of the place of the ship nor that of the harbour in question was known. The only requirement was that one had to know the amount of the variation for the place of destination and had to be able to determine this for the place of the ship by observation. One then had to follow the parallel of the port of destination to a point where the variation was identical with that holding for that place. By observing whether the variation measured was increasing or decreasing the navigator learned whether he was sailing in the right direction. This method appeared so sound to Stevin that in his opinion the position found by dead reckoning had to be considered less reliable than the result obtained in this way. He regards the variation of a given place as constant.

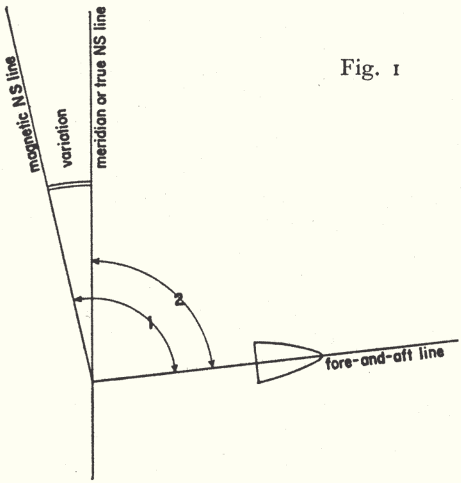

Fig. 1

Stevin dismisses the objection that several places with the same variation might be situated on the same parallel. This need not cause errors, since such places were very far apart.

There was yet another reason why knowledge of the variation was necessary. The navigator of an ocean-going ship had to know it if he was to be able to deduce the true course from the course steered by the compass, in order to learn how the ship was moving on the surface of the earth in relation to the meridians [16]). As a rule the seaman assumed that his compass —

[16]) The course which is steered and kept while sailing by the compass is the angle between the compass needle and the fore-and-aft line. It is read into degrees or points on the division at the circumference of the compass card. Since the present discussion does not take account of deviation due to iron in the vicinity, the direction pointed by the compass needle implies the direction of the magnetic north and south line. In this case therefore the course steered is the angle between the magnetic north and south line and the fore-and-aft line. Because this course is measured by reference to the magnetic north and south line it is called the magnetic course (1).

The direction of the ship may also be measured by reference to the meridian. This

(Continuation on next page).

constructed in such a way as to allow for the variation found at his port of departure — continued to point to the true north throughout the voyage. Most sailors had no idea that the variation changed as they moved across the earth. In consequence errors arose in the data about directions, or in the positions of places, which data they submitted upon coming home, with the object of marking them on charts and on globes. Large-scale inquiries and collection of data in the matter of measurement of the variation were desirable to avoid such inaccuracies. As has been said, the Stadtholder urged home this desirability by his measures and orders.

It is in this connection that Stevin recommends the use of a compass with an adjustable needle, a type which was in use side by side with other compasses in Holland as well as elsewhere. With such instruments it was possible to set the needle for the value of the variation at the place in question, so that the north and south line invariably indicated the true north or the direction of the astronomical meridian and the courses steered by such a compass were true courses. Compasses of this type continued to be made and used by the Dutch until far into the nineteenth century.

For the benefit of those who wanted to study this matter the book contains a table of the values of the variations in a number of places, in so far as they had already been observed, which values Plancius had "collected by protracted labour and not without great expense from different corners of the earth, both far and near". It was to Plancius, writes Stevin, that credit would be chiefly due if by this system it were to be possible for the seaman to reach different lands and harbours. Stevin feels obliged to express his gratitude, for the numerical data were taken from Plancius, and it must have been quite a job indeed to collect, elaborate, and arrange them. But in his theoretical discussions Stevin does not follow the train of thought of Plancius. He adopts a course of his own, which — as has been said — was of importance for the pratice of navigation during a period long after his day, whereas the theories of Plancius were soon exploded.

Before publishing the table and proceeding to discuss it in detail, Stevin, being a scientist who hopes to make a contribution to the solution of the problem of determining longitude at sea, addresses the seaman who will have to supply the further data. He provides him with a golden rule aiming at the advancement of both theory and practice. Even if further observations of variation, latitude, and longitude should furnish values different from those listed in the table and if consequently different explanations should have to be framed, this was not to deter the seaman from the inquiry; on the contrary, he was to do everything that might help to collect more information about the method, with the ultimate object of jointly finding its true character. In his concluding sentence (p. 459) Stevin says once more that if the results of later inquiries should be different from the outlook on the

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(Continuation of note 16)

course is called the true course; it is the angle between the meridian (or true north and south line) and the fore-and-aft line (2).

The course steered, which is read on the compass—in this case the magnetic course—is reduced to the true course with the aid of the variation. The amount of the variation in degrees therefore has to be added or subtracted. Once the true course is known, the direction of the path followed by the ship can be drawn on the chart in relation to the meridian.

variation based on the table of Plancius, it will be necessary to modify the conclusions.

It was actually in this way that the theory was developed. The orders of the Stadtholder were obeyed and Stevin's wish was fulfilled, though much later than he may have hoped. Indeed, we know that the gaining of an understanding of terrestrial magnetism was a process which required several centuries. The seaman may consider himself fortunate in having advanced the art of navigation by making ample contributions to the knowledge concerning the indications of the magnetic needle on the earth (see § 5). Even in the present century he made some valuable contributions. The scientific investigation of terrestrial magnetism is still far from being complete even to-day, but it is no longer a subject of study for the seamen.

For 24 places on the northern and for 19 places on the southern hemisphere the table gives the variation, the latitude, and the longitude, the latter being reckoned by reference to the meridian through Corvo, one of the westernmost islands of the Azores. The easternmost places given are situated in longitude 160° E. They are the mouth of Canton River in China in latitude 23° N. and "Bunam, 46 German miles to the east from the eastern end of Java" in latitude south [17]). No data are supplied for the part of the world to the west of Corvo and to the east of Canton, *i.e.* a part comprising 200 degrees of longitude. The observations which Plancius had received about that part of the world from Spanish, English, and Dutch sailors were not in conformity with one another, "because they had been taken without suitable instruments and sufficient knowledge". Plancius was looking forward to receiving new and more accurate data.

The variation is nil in the island of Corvo. Going eastward on the northern hemisphere, we find easterly variation which increases to 13° 24′ at Plymouth, upon which it decreases to zero at the island of Hjelmsöy, at about 30 miles west of the North Cape, and according to the table in longitude 60° E. On the southern hemisphere, east of the meridian through Corvo, the variation is also easterly, increasing from zero to the maximum of 19° in the vicinity of Tristan da Cunha and then decreasing again to zero in longitude 60° E. It appears that the two maximum values lie in longitude 30° E., *i.e.* on the mid-meridian of the lune contained between longitude 0° and 60° E.; this lune is called a "perck" (segment) by Stevin.

Stevin draws special attention to the regularity he seems to detect in these figures, and on this ground he arrives at a bold conclusion. In fact, he "concludes" that the value zero of the variation holds for the whole meridian through Corvo — from one pole to the other — and equally for the meridian through Hjelmsöy. The maximum values fall on the mid-meridian of longitude 30° E. He assumes that the increase of the easterly variation holds for the whole lune

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[17]) 46 German miles stand for a distance of 184 nautical miles, because 15 German miles are reckoned for one degree of latitude of 60 nautical miles. Bunam or Bima, on the north coast of "Java Menor", is to be found on the map of China, etc. by Van Langeren 1595 (reproduced in Vol. XLIII of the Linschoten Ver., Map V) and on the map of Java (in the *Caert-thresoor*, 1598, p. 61). It is referred to as Buma on the page of the 2nd edition of the map of the world of Plancius, reproduced in Wieder, *Monumenta Cartografica*, Vol. II, 1926, p. 40 III. The place Bima and Bima Bay are situated on the north coast of the island of Sumbawa.

between longitudes 0° and 30° E., and the decrease for that between longitudes 30° and 60° E.

The phenomena in the lune between longitudes 60° and 160° E. confirm this picture. Here we have westerly variation, increasing from zero to the maximum value of 33° near Novaya Zemlya in the northern, and to 22° at the island of St. Brendan's in the southern hemisphere, upon which it decreases to zero in the meridian of longitude 160° E. again in both hemispheres. The places in which the two maxima occur again lie on the mid-meridian of the lune, *viz.* in longitude 110° E.

The conclusion now is obvious. The meridian of longitude 160° E. is a line joining points where the variation is nil, a line which is now called an agonic. The meridians of longitude 0° and 60° E. too are agonics. The increasing variation holds for the whole lune between longitudes 60° and 110° E., the decreasing variation for the lune between longitudes 110° and 160° E.

When in his attempt to give a picture of terrestrial magnetism Stevin here assigns a particular property to all places situated on the same meridian because in one or two points of that meridian either a value zero or a maximum value is found for the variation, he takes a view held by others before him. This point is to be discussed more fully later on. Stevin leaves it an open question how one is to conceive the difference of the variation on the mid-meridian, considering that unequal values are found in the northern and the southern hemisphere. To him the main thing is whether during a movement either to the east or to the west an increasing or a decreasing variation is observed.

For the part of the world comprising 200 degrees of longitude for which Plancius possessed no observational material, but for which he was expecting fuller and more reliable data, Stevin merely states how he "somewhat suspects" the magnetic needle will point there. On the same lines he now continues as follows.

The property of the magnetic needle of pointing to the north on the meridian through Corvo and through those of longitude 60° and 160° E. is also assumed by him for those of longitude 180°, 240°, and 340° E., *i.e.* the meridians in the longitudes 180° away from longitudes 0°, 60°, and 160°. All these lines are agonics. In this way the surface of the earth is divided into six lunes, with alternate easterly and westerly variation, each subdivided into one half with increasing and one with decreasing variation. In each of these lunes the maximum falls on the mid-meridian, *i.e.* the 30th, the 110th, the 170th, the 210th, the 290th, and the 350th meridian.

In a remark at the end of his theoretical discussion Stevin cautiously calls his picture a "conjecture", which may not be confirmed by observation. He regards the whole picture as descriptive of the division of the earth's surface in connection with the behaviour of the magnetic needle. However, he has great confidence in the core of his idea, *viz.* the possibility of reaching a place on the earth with the aid of the latitude and the value of the variation. This confidence appears from the instance given on pp. 456 - 458 of a voyage from Amsterdam to Brazil. Here again he considers the results more reliable than the longitude deduced from the course and the distance sailed. Thus he is able to conclude his discussion with the hopeful expectation: "If therefore the needle-pointing and the latitude are duly observed in all corners of the world and made known to everybody, it will be possible to sail the world in another way than hitherto".

Stevin's method also enables ships — which have been scattered by a storm, for instance — to assemble at an appointed place at sea.

We have thus seen how the learned author ventures to draw conclusions from a small number of observations, holding only for one sixth of the earth's surface. The regularity he has found is met with again in the lune between longitudes 60° and 160° E., upon which he utters the "surmise" that it will also hold for the rest of the earth's surface. The picture here outlined was to help the seaman to find his destination as long as the great problem of the determination of longitude at sea had not yet been solved.

Stevin makes no attempt to account for all these things in his treatise. He explicitly rejected a magnetic pole, a point on which he was right as long as this pole was considered the centre at which the magnetic needle had to point. In his refusal to assume a magnetic pole Stevin took up a standpoint diametrically opposed to that of Mercator as well as Plancius.

In our view his "conjectures" and "surmises" appear extremely speculative. Many contemporaries, hankering as they were after the possibility of determining longitude at sea, held a different view of the matter from ours.

b. *THE MEASUREMENT OF THE VARIATION OF THE COMPASS ACCORDING TO STEVIN*

As was to be expected, Stevin's treatise expounds in what way the variation of the compass can be determined on board a moving ship, even though — according to the author — many people were acquainted with this subject.

The instructions found on pp. 461 - 465 do not call for much comment. They state that if a mariner's compass is used, this should be one in which the magnetic needle coincides with the north and south line of the compass card. In taking a bearing one has to hold a vertically suspended cord near the compass in such a way that the shadow cast by it passes through the centre of the graduation. The place of the shadow on the graduation is read before and after noon at equal altitudes of the sun. The point situated midway between the two observations indicates the direction of the meridian. Since the needle coincides with the north and south line, the amount and the direction of the variation are thus found.

The observation could also be taken by means of a compass-needle pivotally mounted in a box, against the inner wall of which the graduation had been marked. During the observation the box had to be held in such a way that the needle coincided with the zero line of the graduation. Otherwise the method remained unchanged.

There were some navigators who used a compass fitted with an adjustable azimuth circle, which is described on p. 467. On p. 468 it is illustrated, unfortunately not completely, for though the drawing does show the bowl, the needle, and the azimuth circle, it does not include the system of suspension. The latter it said to have been an invention due to Reynier Pietersz, who had suspended the instrument "on two different pins, in the manner of the mariner's compass". For steering compasses this suspension system was quite familiar at the time. It is usually called an invention of Cardanus (1501-1576), an Italian mathematician and physicist, astronomer, and professor of medicine at Pavia. But with regard to an azimuth compass it was indeed justifiable to speak of an invention, as will appear in § 4 b.

Along the graduated circumference of the quadrant an alidade, equipped with sights, was adapted to pivot. This made possible a rough measurement of the altitude of the sun or another heavenly body. The azimuth quadrant turned on a pivot mounted in the centre of the glass cover. To ensure its remaining perpendicular to the glass cover of the bowl as it turned, it was provided with two lateral supports, as mentioned in the text. The bowl was weighted on the underside so as to ensure the horizontal position of the cover. Along the circumference of the bowl as well as on its inner wall, graduations had been provided, in such a way that the zero points of the two coincided. On the former the direction of the quadrant could be read. The needle was set to the zero point of the latter.

The observation took place as follows. The adjustment of the compass needle was brought about by the rotation of the compass bowl within its gimbal ring. Owing to the deviations from the course to left and right due to the movement of the ship as she sailed, this adjustment was undone. Accordingly, for the neutralization of this so-called yawing the bowl had to be repeatedly turned to the right and to the left by hand. At the same time the quadrant had to be directed towards the sun. Once the quadrant had been given the right direction, the turning and readjustment of the bowl alone sufficed to maintain the proper adjustment of the needle as well as that of the quadrant. When this had been achieved, the angle between the quadrant — or, which comes to the same thing, the azimuthal direction of the sun — and the compass needle was read on the graduated circumference.

Later in the day a second bearing was taken at an altitude of the sun equal to that of the morning observation, after which the amount of the deviation of the needle from the meridian appeared from the readings of the morning and the afternoon, i.e. the deviation was equal to half the difference between these readings.

In the same way it was possible to determine the deviation by night at equal altitudes of the same star. With the aid of the moon this was impossible — as Stevin writes — in view of its rapid proper motion and its large parallax, due to its short distance from the earth [18]).

The allegation that the rapidity of the moon's motion gives rise to errors is already to be found in William Bourne, *A Regiment for the Sea,* in the chapter on "How to find the true meridian" (in the edition of 1577 on p. 28). About the taking of bearings it is stated: "you may do the like by night by any of the starres that you perfectly do know, doing as you do by the sun in all points, but you cannot do it so wel and truly by the moone, by the meanes of the swiftnesse of the moones motion in the Zodiack". There exist Dutch translations of Bourne's book, published at Amsterdam in 1594 and 1599. It seems fairly probable that this work was available to Stevin.

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[18]) These words will have to be interpreted as a not untimely warning: do not take bearings of the moon. The reason why this should not be done is not expressed very felicitously. It is especially the rapid change in the declination between the instants of observation which may give rise to errors, for an increase or decrease of 2° is quite conceivable. Other inaccuracies fall within the limits of the errors of observation. Perhaps Stevin considered it superfluous to give an exact explanation.

At the end of the chapter there is yet another warning. When the ship is in a region where the variation changes rapidly as one sails eastward and westward, so that the variation is not the same for the morning and the afternoon position, this appears from the fact that the deviation found by combining the first forenoon and the last afternoon observation does not agree with that obtained by the combination of the last forenoon and the first afternoon observation. According to Stevin this need not point to inaccuracy on the part of the navigator. The phenomenon might even serve to gain some idea of the speed with which the variation changed during a known number of sailing hours [19].

Let us devote some attention to this remark of Stevin's.

It is only if the change of the variation due to the movement of the ship is considerable and moreover is not equal during equal movements — *i.e.* if the change is irregular — that the phenomenon referred to can be revealed. In that case high demands as regards accuracy of measurement are made on the compass. It is beyond doubt that the compasses of those days could not yet meet these demands.

But Stevin, who knew no more about the amount of the variation in various points on the earth than he mentions in *The Haven-Finding Art,* can have had no idea of the speed with which the variation changed in consequence of the said movement of the ship, nor can he have known whether this change was regular or irregular. He is not likely to have possessed observational data about this. If this had been the case, he would no doubt have mentioned it in this context.

In answer to the question what may have induced Stevin to make the above remark two suppositions may be suggested. Either we have to regard it as an entirely theoretical one, which arose in the mind of a mathematician who had reflected profoundly about his problem, or — what is more likely — the source must be sought in England again. In fact, in this connection attention may be drawn to an interesting booklet, *viz.* Robert Norman's *The Newe Attractive,* London 1581 [20].

In Chapter 9 Norman [21] disputes the view of those who assume that the change of the variation from one place to another is regular; he explains quite clearly that he feels justified in speaking of regularity if with an equal move-

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[19]) Stevin has omitted to draw attention to an influence due to the change in the latitude through the movement of the ship and one in the declination of the sun between the forenoon and the afternoon observation. Either he forgot to mention it or he considered these influences to be of minor importance for practical purposes. The change in the declination, which can be no more than a few minutes of arc, indeed does not have to be taken into account. The change in the latitude will seldom have amounted to more than one degree.

[20]) Robert Norman, *The Newe Attractive: containing a short discourse of the magnes or loadstone, and amongst other his vertues of a new discovered secret and subtill propertie, concerning the declining of the needle, touched therewith under the plaine of the Horizon.* London 1581. Reprinted in 1585, in 1596, and again in 1614.

[21]) Norman († 1596), who in the 18 or 20 years of his seafaring life had diligently collected data about the behaviour of the compass-needle and who, after having settled on shore, became known as "Norman, the compass-maker", was the discoverer of the inclination of the magnetic needle, which phenomenon he describes in his booklet. "A matter never before found or written by any", thus William Borough calls this phenomenon in the preface to his book *A Discourse of the Variation of the Cumpas or Magneticall Needle*, London 1581, which was intended as an "annexe" to Norman's treatise.

ment of the ship is associated an equal change of the variation. Martin Cortes, too, had assumed this regularity. But Norman calls the assumption an error. According to him no such regularity exists. In some regions the change is rapid and sudden, in others it is slow. For the same place on the earth, however, the deviation remains constant.

Norman expresses the wish that sailors on their voyages may observe the variation with accurate instruments, a practice which will benefit them at a later date, in particular in those regions where the variation changes rapidly over short distances. Norman does not state how the phenomenon is revealed by observation, as Stevin does.

It may be considered probable that Stevin was acquainted with Norman's view. Further it is quite conceivable that the wish of Norman just mentioned was the origin of Stevin's similar wish, as well as of Prince Maurice's order of 1599 about the collecting of observational data.

c. *THE LATIN TRANSLATION*

The jurist Hugo Grotius (1583—1645), a figure famed in world history, at the age of 16 years had translated *The Haven-Finding Art* faithfully into elegant Latin. The title of the translation is:

> *Limenheuretica, sive portuum investigandorum ratio.*
> *Metaphraste Hug. Grotio Batavo.*
> *Ex officina Plantiniana apud Christophorum Raphelengium.*
> *Academiae Lugduno-Batavae Typographum.* 1599. [22]

Grotius wrote for the booklet a dedicatory epistle, addressed to the Doge, the Senate, and the people of Venice and dated Delft, 1st April 1599. This date shows that the translation appeared almost simultaneously with the original Dutch version. It is not merely on account of the courtesy of the wording that this dedication is worth reading. It also contains some personal impressions of the author. It drives Stevin's meaning home to the reader more clearly than he himself had done and it throws full light on the importance attached to Stevin's work by the leader of the country, Lieutenant-Admiral Prince Maurice.

Grotius relates that he had met the Venetian ambassador while accompanying the Dutch embassy sent to Paris. After making a polite comparison between Venice and the Republic he states he had resolved to dedicate a work to the Venetians. The favourable occasion which was worthy of them and which enabled him to add a contribution of his own — a reference to his dedicatory epistle — had now arisen. He was able to offer and recommend a booklet containing instructions given by the Prince to the commanders of the navy and to their boards, to be followed by them. The Lieutenant-Admiral himself had previously studied the subject.

After a circumstantial discussion of the development of ancient navigation and the knowledge of the compass, Grotius recalls how on voyages from east to west the compass-needle had been found to deviate gradually and not inconsiderably

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[22]) *Works*, X.

from the true north, which had caused great doubt and uncertainty among seamen. Thanks to prolonged observation of the magnetic declination at different times and places it had been found by the most learned mathematicians — as one of whom he considers Prince Maurice — that this was no mere accident, but that in nature a certain regularity *(ratio et norma)* existed according to which the pointings of the needle varied. The Prince had now presented these instructions, written about the matter by his mathematician Stevin, to those in authority in maritime affairs, in order that, if there should be found to exist disagreement between theory and personal observation, every effort might be made to deduce a rule from different experiments.

In order that as many data as possible might be collected, the Prince had decided to present the booklet to the Doge, so that the Venetian navigators might take similar observations, which would make for greater certainty in the finding of any destination.

Grotius concludes his dedicatory epistle with a general recommendation of the method and with the wish that "this small present" might be sympathetically received, "which will be of benefit to both parties and to the whole of the human race".

The high expectations that were entertained — by the Prince in particular — of the fruits of Stevin's work could hardly be expressed more eloquently.

d. *THE ENGLISH TRANSLATION* [23])

Not long after the appearance of the Dutch edition of *The Haven-Finding Art* an English translation was published, prepared by the mathematician and nautical expert Edward Wright (1558—1615), famed in the history of navigation. He added a solemnly worded dedicatory epistle to Charles, Earl of Nottingham, Lord High Admiral of England, dated 23rd August 1599. Wright very urgently solicits the admiral's interest in the subject, which was considered by Prince Maurice to be of exceptional value for deep-sea navigation.

Wright had undertaken the translation in order to spread the knowledge of this method among all English seafarers, although he was aware that there were English navigators who had found the position of their ship by means of latitude and variation "more then ten yeres since". However, his countrymen were not to be behind the Dutch, and that was why he recommended this useful booklet — he calls it "this Dutch Pilot" — to the admiral, requesting him to induce English seamen to test the method and to take observations all over the world. "Proofe already made by some of our skilfullest English navigators" had already raised good hopes of success, a success which would benefit not only seamen, but also "the whole body of the Christian commonwealth".

After this, in an introduction Wright wishes "Richard Poulter, the maister and brotherhood of Trinitie House and all English mariners and sea-men in generall that love the perfection of their owne profession, health and happiness". Wright here uses the words in which Grotius had addressed the Doge and the Venetians, and he expatiates on the wide importance of the knowledge of the variation for the improvement of navigation.

The two introductions are followed by the translation of the body of *The*

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[23]) *Works*, X.

Haven-Finding Art, with the exception of the Appendix at the end. The figures are reproduced extremely faithfully.

The name of Edward Wright lives on in the history of navigation on account of his famous book *Certaine Errors in Navigation Detected and Corrected,* of which three editions appeared, *viz.* in 1599, in 1610, and the last in 1657, published posthumously and edited by Joseph Moxon. When Wright revised the work for the second edition and addressed Henry, Prince of Wales, in a dedicatory epistle, in which he described in glowing colours the perfect state of shipping and navigation at that time, he evinced such great confidence in the system of haven-finding that this nautical writer, who was one of the greatest experts of his country and his age, wrote: "this variation performeth almost so much in effect as the invention of the longitude".

Since the translation of *The Haven-Finding Art* already existed, Wright did not consider it necessary in 1610 to include this treatise in his book, but he did add *An Addition Touching the Variation of the Compasse,* an essay of only three pages. There was a definite reason for his writing this addition. The opening paragraph at once gives the explanation of it. In fact, it is stated that "some have been of opinion that there be two magneticall poles". The author mentions no names, but the identity of his adversaries is revealed by Moxon, in his preface to the reader in the third edition of *Certaine Errors* referred to above. It was Anthony Linton and his followers who propagated this theory, which resembles that of Plancius. Linton published a small book [24]) containing an enumeration of all sorts of improvements to be made in navigation. One of them concerned the possibility of determining the longitude from the variation. The starting-point was that although "a great learned man and his followers absolutely denie that there is any fixed pole magneticall", there must be two fixed magnetic poles, which furnished the basis for the possibility of determining a ship's position. Both magnetic poles were situated on the surface of the earth, within the arctic circles, one in the Arctic and the other in the Antarctic regions. They were altogether different from the geographical poles and were unlike anything written up to that time about magnetic poles. Linton imagined these two poles to be joined by a magnetic axis passing through the centre of the earth. A magnetic equator with parallels and meridians could be imagined at right angles to the line joining these poles. The meridian through the two magnetic and the two geographical poles had to be regarded as the prime meridian. If the variation and the latitude of a place were known, it was assumed to be possible to determine the longitude of this place by "arithmeticall calculation". Conversely, if the latitude and the longitude of a given place were known, the variation for this place could of course also be calculated.

In his *Addition* Wright refutes this opinion. In accordance with his views, always assuming that the magnetic pole must be the central point towards which the magnetic needle points, wherever it is placed on the earth, logically there would have to be as many pairs of magnetic poles as agonics were known. All those poles cannot be reduced to two. Wright draws a brief conclusion: Linton's

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[24]) *Newes of the complement of the art of navigation and of the mightie Empire of Cataia together with the Straits of Anian.* by A. L., London 1609.
The author mentions his name, Anthonie Lynton, in the dedicatory epistle on page 1.

supposition "is absurd and therefore no such magneticall poles". In other words: away with them!

Wright finds the impossibility of such magnetic poles confirmed in a series of values of the variation of the compass which follows the *Addition.* The observational data had partly been supplied by the author himself, but he had obtained the majority from English and foreign mariners. The table runs to twelve pages and contains 283 values for places widely dispersed over the earth, a very considerable extension indeed of the original material of Plancius of 1599, published by Stevin at the time.

It thus appears that Wright made no alterations in Stevin's train of thought. There *can* be no magnetic pole, at least none of the kind imagined at that time. The variations of the compass are enumerated and listed such as they had been observed. No attempt is made to give an explanation of terrestrial magnetism.

And there the matter rested. As already said, Moxon published the third edition of Wright's book, because he considered it undesirable for the English nation "that this so usefull a book should sleep it self to death". The *Addition* of 1610 was printed unchanged at the end, along with the list of the observed variations. As an appendix he included *The Haven-Finding Art,* the text being that of the translation of 1599. It is curious that the same book thus contains both the list of the variations of 1599 and the enlarged list of 1610, now published again after almost fifty years. Although this seems to indicate stagnation, progress did exist in the matter of the knowledge of the variation; this will be discussed in § 5.

e. *THE FRENCH TRANSLATION*

The French translation [25]) bears the following title:

Le Trouve-Port, traduict d'Alleman en Françoys à Leyde en l'Imprimerie de Plantin, par Christoffle de Ravelengien, Imprimeur juré de l'Université de Leyde, 1599, avec privilège. [26])

The vignette on the title-page is the same in the Dutch and the French edition. The unknown translator has closely followed the original text. The booklet contains no dedicatory epistle, like the two other translations, nor any preface to the reader or the like, so that no data are present giving special information about this version. On the last page the *Privilege* is reproduced. It is given in Dutch and is perfectly identical with that in the original edition. It is probably the same type.

f. *STEVIN'S VIEW OF THE SYSTEM OF "THE HAVEN-FINDING ART" IN 1608*

Nine years after the publication of *The Haven-Finding Art* the great collective work *Wisconstighe Ghedachtenissen, beschreven door Simon Stevin, Leiden 1608* [27]) appeared, in two volumes.

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[25]) The French translation of *The Haven-Finding Art* is not present in any Dutch library. Two copies of it are to be found in the Bibliothèque Nationale de Paris.

[26]) *Works*, X.

[27]) *Works*, XI.

Volume I includes, as the "Fifth Book of Geography", the *Haven-Finding Art* (pp. 163—175) [28]. A comparison of the text with that of 1599 reveals points of resemblance as well as differences.

Long passages were taken over literally, such as the description of the finding of the variation of the compass at sea and that of the azimuth compass of Reynier Pietersz. The table with the values of the variations, originating from Plancius, has been reprinted and once more forms the starting-point of the discourse. The expression of gratitude to Plancius too is present. Just as he did in 1599, Stevin also emphatically rejects the magnetic pole in the text of 1608. In both cases he utters this central idea quite at the beginning.

A sentence has been added in which the author says emphatically that he is going to speak of "haven-finding", not of "longitude-finding, which would be a wider subject and would be of greater worth". The work has been abridged by the omission of several passages. The order of other passages has been changed.

From the omissions a changed view may be inferred.

Whereas in 1599 the statement that the meridians through Corvo and Hjelmsöy were agonics and that a maximum value for the variation occurred on the mid-meridian was for Stevin a "conclusion" based on the data, in 1608 he writes: "from this it is suspected that these properties can be assumed". Stevin therefore has grown more cautious in his wording.

The text concerning the agonic in longitude 160° E. and the meridian of longitude 110° E., with the maximum value of the variation, is taken over. But the parts east of longitude 160° E. and west of Corvo are no longer spoken of. The figure illustrating the six agonics and the text concerning the six lunes, each with an increasing and a decreasing variation, have been dropped. Apparently he now considered his earlier "conjecture" too speculative to maintain it.

In 1599 Stevin wrote that Plancius was expecting "any day" to receive new observations, taken on board ships which had been away for more than fourteen months. In 1608 they do not seem to have reached him yet, for he now writes of Plancius: "but he expects to receive more accurate information about it". Although therefore the lack of observations from remote regions is the reason why the list of the variations is neither enlarged nor corrected after nine years, one strongly suspects that acquaintance with newer data induced Stevin to make the above-mentioned alterations in the next and to be more cautious in his wording.

Like the starting-point of his theory, his confidence in his system of finding a destination through knowledge of the latitude and the variation of the compass is unaltered. The text about the deviation of the compass in the vicinity of Amsterdam, the result obtained during the voyage to Brazil, the preference of the position obtained to that by dead reckoning, the story of the ship that missed St. Helena, to be found on pages 4 to 6 in the edition of 1599, all have been taken over literally in the edition of 1608. Stevin concludes with the following words: "Thus far the appearance of the variations following from the data of the table has been described. If other, more exact observations should prove different in the future, other conclusions will have to be drawn from them, and in navigation the best must always be followed".

The *Wisconstighe Ghedachtenissen* were translated into Latin and published in two volumes under the title:

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[28]) *Works*, XI, i, 25.

Hypomnemata mathematica. Leiden 1608. [29])

Volume I contains:

Liber quintus geographiae de Limenheuretica, metaphraste Hugo Grotio Batavo,

with in the margin the words: *Portuum investigandorum ratione.*

From this title it may be seen that Grotius again prepared the translation, for which naturally he now followed the text of 1608. No further remarks therefore have to be made about this, with the exception of one.

When in 1599 Stevin had written: "From this it is concluded", Grotius had translated: *ex hisce concludere volumus.* But when in the edition of 1608 Stevin used a weaker expression and wrote: "from this it is suspected", Grotius still wrote: *ex his concludere volumus.* Since the translator thus continued to speak of "concluding", he failed in this respect to render Stevin's meaning accurately.

Finally it has to be mentioned in what form *The Haven-Finding Art* was included in

Les Oeuvres mathématiques de Simon Stevin ... Le tout reveu ... par Albert Girard, 1634. [30]).

In Volume II, p. 171, we find:

Cinquiesme livre de la Géographie. Du Trouve-Port ou la manière de trouver les Havres.

This again is the text of 1608. Here the change in Stevin's words has been taken into account. Whereas the French translation of 1599 reads: *"de ceci on veut conclure"*, in that of 1634 we find: *"de cecy on a opinion"*.

There are no indications of any other editions or versions of *The Haven-Finding Art* having appeared since then.

§ 4

THE MEASUREMENT OF THE VARIATION OF THE COMPASS

a. *THE MEASUREMENT OF THE VARIATION OF THE COMPASS BEFORE STEVIN'S DAY.*

It has been said before that Stevin, when proceeding to discuss the determination of the variation of the compass, begins with the remark that this subject was "known to many people". This was no doubt true. In fact, sixteenth-century navigators had examined special phenomena observed by them at sea and they had the good sense to publish their measurements. Writers of nautical textbooks dealt with the subject. In the latter part of the sixteenth century Dutch seamen too on their ocean voyages determined the amount of the variation and recorded it in their log-books. But in practice most sailors did not concern themselves about the variation. They knew by heart by what courses they could reach their destination and sailed "by sight". The art of finding their way they had "mostly learned by their own experience and by the instruction of old and experienced pilots", as Coignet wrote in his *Nieuwe Onderwijsinghe* of 1580 and in its later editions. Since for the application of the system of "haven-finding" it was necessary to know the variation of the magnetic needle, it is obvious that for the sake

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[29]) *Works*, XI, b.
[30]) *Works*, XIII.

of completeness Stevin devoted some space to the subject. What he writes about it is based on foreign works available at the moment, and further — as far as Holland was concerned — on the work of Reynier Pietersz. Stevin was not original on this point, as will appear from the following discussion.

It is known that Columbus on his first voyage across the Atlantic took bearings of Polaris [31]) in the early and the latter part of the nights of 13th, 17th, and 30th September 1492. We do not know whether he did or did not take these observations with the aid of a special instrument. A bearing instrument is first described in a Portuguese manuscript of 1514 [32]), but unfortunately this description is not very clear, so that no reliable idea can be formed of it.

A perfectly clear description is given by Francisco Faleiro, a Portuguese in the service of Spain, in his textbook *Arte del Marear* of 1535 [33]). In Chapter VIII, in which the northeasting and the northwesting of the compass-needle are discussed, the author describes the construction of an instrument for determining the variation of the needle. It is a round, flat disc, in the centre of which a magnetic needle has been mounted. About the compass pivot as centre a graduation, consisting of four times 90°, has been marked on the disc. Coinciding with the line 0°-180° and perpendicular to the plane of the disc is a small rectangular plate, which serves as a shadow pin. In the figure illustrating the text the latter is not shown.

The observation is taken at noon. The instrument is held in the hand in such a way that it is horizontal and the plate casts no shadow. The line 0°-180° then lies in the meridian and the needle indicates the variation on the graduation. It is also possible to take the observation in the forenoon and the afternoon at equal altitudes of the sun and to read the two indications of the needle. During the second observation the mean of the readings indicates the direction of the meridian, while the angle which the needle now makes with this direction is the variation sought. Faleiro prefers the latter method of taking the observation because it can be performed many times a day. The observation at the sun's greatest altitude on the other hand might be inaccurate because the moment of noon could not be determined exactly. Finally observations at sunrise and sunset, of which in the same way the mean is taken, afford a third method for obtaining the desired result. Although nowadays this instrument appears to us to be little suited for use on board, because, while holding it horizontal and at the same time directing it, one has to compensate the rolling, pitching, and yawing movements and changes of the heeling of the ship by movements of the hand, from Faleiro's text it is quite evident that it was destined for use at sea [34]).

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[31]) "Taking a bearing of Polaris" or of the sun is the process of measuring the angle between the compass needle on board and the azimuthal direction of Polaris or the sun, as the case may be.

[32]) This manuscript is discussed in § 5, p. 393

[33]) Francisco Faleiro, *Tratado del esphera y del arte del marear*. Seville 1535. Facsimile reprint edited by J. Bensaude, *Histoire de la science nautique portugaise*. Vol. 4, Berne/Munich 1915. Chapter VIII, Del nordestear de las agujas, p. 79.

[34]) Alonso de Santa Cruz—an inhabitant of Seville, who died there in 1572—in 1536 was appointed *cosmografo real* and was a renowned cartographer. He also made contributions to the development of the art of navigation, in particular to the problem of the determination of longitude. Following his attendance of a *junta* of cosmographers, astronomers, and scholars he wrote a treatise entitled:

(Continuation on next page).

In discussing the compass and its properties, Medina [35]) draws attention to the taking of bearings during navigation, saying: "the Pilot should adjust his needle to Polaris" [36]), waiting for the moment at which the "guardians" or "wheels" (β and γ *Ursae Minoris*), are northeast and southwest of Polaris, because the latter is then in the meridian. Chapter VI describes how the direction of the needle can be tested by taking two bearings of the sun at equal altitudes, the shadow cast on the card by a style being read before and after noon, when the shadow has the same length. However, this is a method for use on land, to find whether the instrument has been properly made, not for determining the variation of the compass at sea. Medina did not believe in the existence of variation, and thus he made no contribution to the development of this nautical subject.

Zamorano [37]) (Chapter XVIII) also teaches the observation of the Pole Star, but he calls the method "uncertain" *(ocasionada a error,* says the Spanish text). In addition his book contains the above-mentioned testing method. But he presents a new method for the measurement of the variation of the compass at sea and on land, *viz.* taking a bearing of the sun at sunrise and at sunset. At the end of his book he depicts a simple instrument, by means of which the point of sunrise or sunset in the horizon is found graphically, the declination and the latitude being known. This instrument therefore furnished the solution of the problem, which is essentially one of spherical trigonometry. With one instead of two observations the desired objective was now reached. It was the Englishman Thomas Harriot (1560-1621) [38]) who afterwards compiled for this purpose a table of ampli-

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(Continuation of note 34)

Libro de las longitudines y manera que hasta agora se ha tenido en el arte de navegar. This treatise, which he dedicated to Philip II, has been preserved in manuscript at Madrid. The year of its appearance is not recorded, but this cannot be prior to 1536, since he calls himself *cosmografo mayor* of the king. In 1921 it was published.

He devotes a good deal of space to one of the numerous methods for the determination of longitude discussed by him, *viz.* to that by means of the northeasting and northwesting of the compass needle. In this context the same instrument as that of Faleiro is described, with the only difference that here a central shadow pin is mentioned. The meridian and the magnetic declination are determined by observation of the sun in the forenoon and the afternoon, at equal altitudes. The inventor of this instrument is reported to be Felipe Guillen, dispenser and inhabitant of Seville, an intelligent and inventive man, who enlisted in the Portuguese navy in 1525. In the treatise the instrument is called "*muy comun en Portugal*". Although Guillen is looked upon as the inventor, by Al. de Santa Cruz, it is much more probable that it is of Portuguese origin.

A description of the construction and the use of the instrument, equipped with a vertical central shadow pin, is also to be found in Nunes, *Tratado da Esphera,* 1537 (p. 141) (See the facsimile edition of this book by J. Bensaude, Berne/Munich 1915). Also in: Hellmann, *Rara magnetica,* and in: *Terrestrial Magnetism and Atmospheric Electricity,* 1943-1945. Harradon, *Some early contributions to the history of geomagnetism.*

[35]) Pedro de Medina, *Arte de navegar.* Valladolid 1545. Book 6, Chapter III, p. 83 and Chapter VI, p. 86.

[36]) "*El piloto para marcar sus agujas mira el estrella del norte para las marcar por ella....*" (*marcar* = to adjust).

[37]) Rodrigo Zamorano, *Compendio de la arte de navegar.* Seville 1581. Reprints in 1582, 1586, 1588, 1591, 1596. Dutch translation in 1598. English translation by Edward Wright, included in his work *Certaine Errors in Navigation,* 2nd edition of 1610, and again in the 3rd edition of 1657. The numerous editions and translations indicate the wide diffusion of this book.

[38]) E. G. R. Taylor, *The Mathematical Practitioners of Tudor and Stuart England.* Cambridge 1954, pp. 182 and 333.

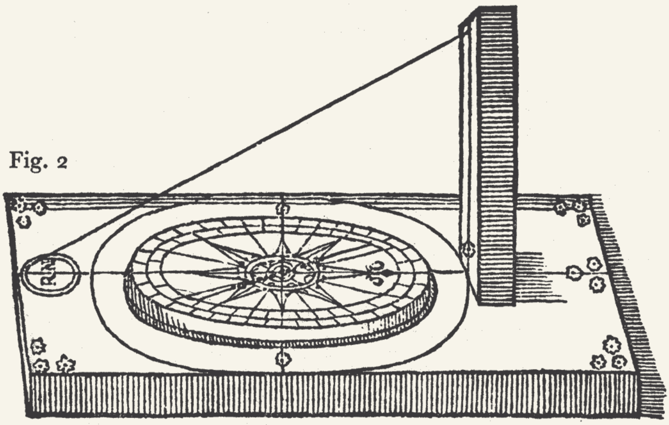
tudes for every degree of declination of the sun and for every degree of latitude up to 54° [39]).

A book widely used and valued in England in the last decades of the sixteenth century was the textbook of William Bourne entitled *A Regiment for the Sea,* which first appeared in 1574 [40]). A Dutch translation, under the title of *De Const der zee-vaerdt,* was published at Amsterdam in 1594, and again in 1599. For the determination of the variation of the compass at sea the author gives five methods, *viz.* observation in the horizon at sunrise and sunset and taking the mean of the readings, taking a bearing of the sun before and after noon at equal altitudes, ditto at noon, when it is in the meridian, two observations of a star, and finally taking a bearing of Polaris. Observation of the moon is stated to be inadvisable.

With the exception of Faleiro, the authors hitherto cited do not say anything about the use of an instrument specially fitted for taking observations on board. It thus has to be assumed that the observation was taken with an ordinary mariner's compass of the type that was being used to steer by, and that the shadow cast on the card by a vertically suspended string or a vertical pin in the middle of the glass cover indicated the azimuthal direction of the sun.

An instrument specially constructed for the purpose, *i.e.* a true azimuth compass, is first found in William Borough (1537-1598), who during his seafaring years studied the variation of the compass-needle in order to make possible accurate navigation. About this subject he wrote an important treatise [41]), which he

Fig. 2



Instrument for the measurement of the variation after William Borough
(See note 41)

[39]) Such tables were widely used in the eighteenth century.
Tables of the amplitude of the sun are also to be found in: Joao Baptista Lavanha, *Regimento Nautico.* Lisbon 1595, 2nd ed. 1606.

[40]) Reprints in 1576, 1577, and 1580. After the author's death in 1588 the book was republished by Thomas Hood in 1592, 1596, and 1601. The numerous reprints and the translation indicate that the book was widely diffused and well known in nautical circles.

[41]) William Borough, *A Discourse of the Variation of the Cumpas or Magneticall Needle, wherein is mathematicallie shewed the manner of the observation, effects and application thereof, made by W. B.* London 1581. Reprints in 1585, 1596, and 1614. The title-page has the statement: "*and is to be annexed to The New Attractive of R.N.*"

regarded as a supplement to the work of Robert Norman. In the present context attention is drawn only to two very simple instruments with the aid of which it was possible to take a bearing and which are illustrated and described in the treatise.

As shown by Fig. 2, the first consists of a piece of board, on which has been fixed a round box, on the bottom of which a wind-rose is marked. Mounted in the centre of the latter is the pivot on which the magnetic needle turns. On this piece of board has been mounted a vertical style on which a line has been drawn. A second line has been made on the piece of board, from the foot of the first line and through the centre of the card. This is the centre line of the instrument. A string has been stretched from the top of the style to the end of the line on the piece of board. In order to enable the observation to be taken, the piece of board was put on a horizontal table and turned until the shadow of the string fell on the line through the centre of the card and at the same time covered the line on the vertical style. On the card the position of the needle is read, the angle between the needle and the azimuthal direction of the sun thus having been measured.

Because in Borough's opinion "imperfections" still attached to this instrument, he designed another of improved construction (Fig. 3). The needle is contained

A new Inſtrument for the Variation.

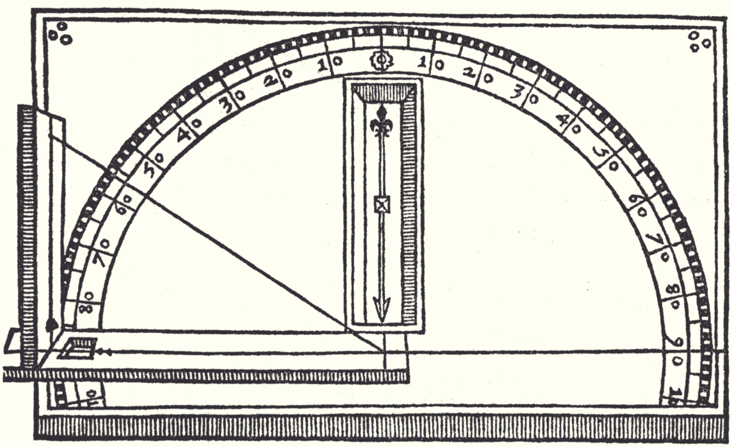


Fig. 3

Improved version for Borough's Instrument (see note 41)

in a narrow box, in consequence of which only small deflections are possible. The instrument always has to be turned until the needle coincides with the zero line of the graduation. In the illustration the instrument has been folded out into the plane of the drawing. It can be seen that it consisted of a horizontal lath (alidade), mounted to turn on a pivot in the centre of the graduation, and of a vertical lath fastened on the first. On both a mark had been made and the ends were joined by a string. In the horizontal lath, above the marks of the graduation, there was an opening, through which the position of the lath could be read.

This adjustable azimuth compass may no doubt be called a considerable improvement and an invention.

When the observation was to be taken, the instrument was put on a horizontal surface or held horizontal in the hand and was turned in the right direction by the setting of the needle to the zero line of the graduation. The alidade was then turned until the shadow of the string fell on the lines of the two laths, upon which the position of the alidade was read on the graduation, and thus the angle between the needle and the azimuthal direction of the sun had been measured. It is to be noted that when the instrument was used on board a ship moving in all sorts of directions, by a movement of the hand the piece of board had to be given the position in which the prescribed adjustment of the needle was maintained. The observer thus was faced with the far from easy task of seeking two adjustments at the same time. In this respect the new azimuth compass of Borough was inferior to the first-mentioned construction, for in that case only the base of the instrument had to be given the right direction.

It is a fortunate circumstance that a fine print has been preserved, which illustrates a navigator who is taking a bearing of the sun. It occurs in a book of prints entitled *Nova reperta,* published at Antwerp about 1600 [42]). The latest inventions known at the time are illustrated. This print, reproduced on the opposite page concerns the system of determining longitude from the declination of the magnetic needle. The useful application is ascribed to Plancius. A particularly heavy ship is shown. Four relatively small, bulging sails are spread. They seem to be drawing well, but curiously enough the sheets are not taut and have the same elegant curve as the sails. Considering the appearance of the sails the ship must be running, whereas the direction in which the flags are fluttering shows that the wind is approximately ahead instead of aft. The anchor, the flukes and the stock of which lie in one and the same plane instead of being at right angles to one another, is unfit for use in this way. The artist of the time therefore appears to have proceeded very freely indeed. We have to interpret his print with the greatest caution.

On the raised quarterdeck there is a small table, covered with a cloth, at which the navigator is sitting on a seat. The compass is in front of him on the table. The bowl is a round box. Fixed on its edge is the azimuth quadrant, which consists of a small vertical style and a quarter circle. It appears not to be adjustable, nor has it got an alidade, but perhaps a sight could move along the circumference of the quarter circle, by which means it may have been possible to determine roughly the altitude of the sun. The quadrant is directed towards the sun, a position which the observer must have obtained by turning the box. If we rely upon the print in this respect, we may therefore conclude that this instrument was directed and manipulated like the first azimuth compass of Borough, but that the non-adjustable quarter circle, probably with a sight, is a new addition.

An extremely important stride forward in the system of "taking a bearing of a heavenly body" is due to the Rev. William Barlow (1544-1625), a son of the bishop of Chichester, who was greatly interested in navigation and engaged in investigations on magnetism. His book *The Navigator's Supply* (1597) contains an illustration of the "compasse of variation" designed by him and constructed by the instrument-maker Charles Whitwell. It may be described as follows.

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[42]) Joan Stradanus, *Nova reperta.* Engraved by Joan Galle. Antwerp, n.d. (about 1600).

In a square wooden box the round metal compass bowl is suspended in gimbals. The card is divided into full points and provided with a beautiful fleur-de-lys for the north point, as was the custom in those days. An azimuth instrument similar to the present-day cross bar sight is adapted to turn on a pivot in the centre of the glass cover. Hinges are shown at the foot of both sights. Both could thus be placed in the horizontal position when the box had to be closed with the sliding lid. Against the inside of the bowl there was a graduation. The position of the azimuth instrument as well as that of the north point of the compass card had to be read on this graduation if the angle between the needle and the azimuthal direction of the heavenly body observed was to be found. Although the use of gimbals and of a "thread and sights" may be looked upon as two important strides forward in the development of the azimuth compass, the fact that two readings had to be made simultaneously on a moving ship — with the card pivoting relative to the graduation and the azimuth instrument too, because this was moved by hand — formed a great inconvenience still attaching to this instrument.

In France Jean de Séville, dit le Soucy, a medical practitioner and mathematician at Rouen, in his *Compost manuel* 43), a book written in particular with a view to the determination of longitude, gave an unfortunately obscure description of a compass with which the variation could be measured. There is no indication that this book may have been known to the Dutch at that time, nor that any other French book may have affected their knowledge concerning the observation of the variation of the compass.

Let us now see what information was available in Holland at that time.

Since 1580 Coignet's *Nieuwe Onderwijsinghe,* appended to the Dutch translation of Medina, was no doubt the best source a seaman could use if he wanted to learn more about deep-sea navigation. It was a useful textbook for him, to be preferred to Medina in view of its simplicity, the clear language in which it was written, and above all things because it was adapted entirely to the practice of navigation. Coignet avoids long digressions about the variation of the compass, "for this would appear useless to mariners", and he is brief and to the point in giving his opinion, which agrees with that of experienced seamen. This opinion is the following. It is necessary to use on board a compass the needle of which was set under the same angle with the north and south line of the card as was the case in the compass used by the maker of the chart which the navigator consulted at sea. "Thus you will not err". Experience had proved the necessity of this. It was of greater value than the enunciations of science, "since the matter at sea cannot so far be easily stated in a general rule".

We thus find that the Dutch textbook does mention the deviation of the compass-needle as a fact, but that deliberately little is said about this matter and the determination of the amount of the variation is not discussed. Of course the observations of the variation taken during the first voyage to the Indies, the data collected on the northern voyages of Heemskerck and Barents — to mention a few important sources — bear witness to the fact that there were pioneers who had full command of the subject. With regard to the rank and file of mariners this could not be said, since most of them had gained practical experience at sea

43) Jean de Séville, dit le Soucy, *Le compost manuel calendier et almanach perpetuel, principalement pour la longitude de l'est et ouest.* Rouen 1586. Reprint 1595.

and had not learned the art of navigation from a book. The views held by many of them are revealed by Jan Huychen van Linschoten [44]), when he writes "that some masters think it is not necessary to know how much the northeasterly or northwesterly variation of the compass amounts to, giving as their reason that our ancestors were unacquainted with the compass and nevertheless drew charts of the coasts". Upon which the author gives as his opinion "that this may indeed be excused in some navigations, but in most long voyages it is highly necessary to know the northeasting and northwesting".

Considering the available information and the prevailing views in Holland, Stevin in compiling his *Haven-Finding Art* must have been convinced of the necessity of including the method of determining the variation in addition to his own theory. Just as in each case, when entering a special field of knowledge, he acquainted himself with the progress of the science concerned, he must here — not finding the source in his own country — have turned his eyes towards other countries. It is beyond doubt that he knew the works of Zamorano and Bourne, either in the original or in the Dutch translation. And although no translation exists of Borough's *Discourse of the Variation of the Cumpas,* this treatise, which was printed three times before 1599, must have been so widely known to those who studied the magnetism of the earth that neither its existence nor its contents can have escaped Stevin's notice. Nor does he mention in his text anything in this field beyond what was known from the said sixteenth-century authors.

But when he describes the azimuth compass of Reynier Pietersz to his readers, he gives something else than Borough and Barlow had found and described. This is an invention due to a compatriot of his, and we see how Stevin makes use of an instrument which at that time possessed the best properties with a view to its use in practice [45]).

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[44]) Jan Huychen van Linschoten, *Reijs-Gheschrift*, 1595, p. 19. *Beschrijvinghe van de Coursen nae Oost-Indien.* Printed in Vol. XLIII of *Werken der Linschoten Vereeniging*, pp. 34, 35.

[45]) In the Spanish language there exists a booklet which, as the title reveals, is entirely devoted to such an instrument and which in connection with our subject is a very important treatise because it deals with the measurement of the variation, the importance of the variation for the determination of longitude, etc. It is:

Andres del Rio Riaño, *Tratado de un instrumento por el qual se conocera la nordesteacion o noroesteacion de la aguja de marear, navegando por la mayor altura del sol o de otra estrella o por dos alturas yguales y de la utilidad que del se á de seguir.* Seville, n.d.

One copy of this treatise is present in the library of the Museo Naval at Madrid. From an exhaustive search by the director of this museum it appears that it is not to be found in any other library. The copy in question is thus unique.

The year of its publication is mentioned neither on the titlepage nor anywhere else in the book. It was known at the time to Navarrete, who mentions it in his *Biblioteca marítima Española*, 1851, Vol. I, p. 97. He relates that it appeared subsequent to another work of Riaño, which was published in 1585. Further it is cited in the *Ensayo de bibliografía marítima Española,* Barcelona 1943, under No 220. In a note it is said that Navarrete puts it after 1585 and that Mendez Bejarano assumes it appeared in 1589. In the Museum it is put at 1585.

If this were true, it might just be possible that one specially interested in the subject, like Stevin, might have been acquainted with its contents. Further inquiry, however, proves it to be impossible.

The reader will encounter many statements implying an indication of time, *e.g.* "the

(Continuation on next page).

b. *REYNIER PIETERSZ AND HIS "GOLDEN COMPASS".*

Reynier Pietersz, as Stevin calls him, or Reynier Pieter van Twisch [46]), as his name appears in various resolutions, is known to have followed the sea for many years, in the service of shipowners at Hoorn, among others. In 1595 he is referred to as "pilot, living at Hoorn", in 1598 as "engineer". At this town he died in 1613. The Chronicle of Hoorn [47]) speaks, more than a century after his death, in the highest terms of this citizen, "a man greatly skilled in navigation". In the entry for 1598 the reason of his fame is given. In fact, he had "practised an exceptional skill in measuring the longitude, as simply as the southern and the northern latitude are commonly done". According to these words therefore he made the long-sought determination of longitude as simple as the determination of latitude, which had been known for several centuries. The solution was effected with the aid of an instrument which the inventor called the "Golden Compass". The brief description of it states "that it floated [48]) and thus always placed itself horizontal, in order to be able to use it in ships which list badly at sea or are tossed in other ways". The chronicler describes the process of taking the observation as follows: "the reading took place by means of the light of the sun, which fell through a certain sight on to the graduation marked around, and this indicated exactly, by means of the deviation of the needle from the true meridian, how far one was to the east or to the west".

From the fact that his inventions are mentioned in various resolutions it appears that the determination of the variation of the compass occupied his mind very intensively. We will now give some details.

On 12th July 1595 the States of Holland and West Frisia granted him letters patent on an invention of nautical instruments, which, "being useful to general navigation", he wanted "to benefit others as well" [49]). The reference

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(Continuation of note 45)

fact that the whole fleet was lost in this year". Only three such statements will be cited here.

On page 5 *verso* a problem involving the determination of longitude by means of the moon is discussed, which is put at May 1, 1607. Who was likely to do this in 1585?

On page 3 *verso* of the "address to the reader" reference is made to the 8th chapter of the book which "*el padre Clavio*" wrote about sundials. Chr. Clavius, a well-known Jesuit father, who died at Rome in 1612, wrote:

Gnomonices libri octo, in quibus non solum horologiorum solarium. . . .",
which appeared at Rome in 1602.

There are several references to a book by Joan Garcia de Cespedes, *Regimiento de marear*. This name is not known among those of nautical writers. A name that *is* known is that of Andres Garcia de Cespedes, whose book *Regimiento de navegacion* appeared in 1606. No doubt it is the latter book which is meant. It can safely be said that 1585 is not the year in which Riaño's booklet appeared. It will probably have to be dated in 1607.

It is quite certain that the booklet has to be dated after the invention of the azimuth compass by Reynier Pietersz. It need not therefore be cited in connection with the development of the azimuth compass before Stevin's day.

[46]) The name is also spelled Van 't Wisch, Twisck, and Twisk. Twisk is the name of a village in the neighbourhood of Medemblik, in the province of North Holland.

[47]) Th. Velius, *Chronijk van Hoorn*, 4th edition, annotated by S. Centen, 1740, p. 501.

[48]) *I.e.* it was suspended horizontally.

[49]) G. Doorman, *Octrooien* 1940, p. 281. *Resolutiën Staten van Holland en West-Friesland.* 12th July 1595, fol. 278.

is to the determination of latitude, of longitude, and of the variation of the compass in remote regions, "hitherto never found or revealed so fully by anyone".

The same inventions are spoken of in somewhat clearer language in the letters patent granted to the inventor by the States General on 8th March 1597 for a period of twelve years [50]). This document speaks of "two new instruments, invented" and already made by Reynier Pietersz, "being highly useful aids and very essential to navigation, the one to be used in order to learn, by measurement of the sun, the deviation of the needle and also to measure the longitude, east and west, *i.e.* how far one is removed from every meridian". The other must have been an instrument for measuring the altitudes of the sun or a star, but not with the aid of the visible horizon, as takes place in a fog or in observations on land.

In the document it is stated that a drawing accompanied it. Unfortunately this drawing has not been preserved, so that we lack further data about the appearance and the construction of these instruments, which are explicitly called "new inventions".

A year later he addressed to the States of Holland and West Frisia a request for a subvention in connection with the expense incurred in the construction of the two instruments just mentioned, upon which on 13th March 1598 [51]) the States requested Jos. Scaliger, Snellius, Ludolf van Ceulen, and Stevin, together with the deputies of Amsterdam, Rotterdam, Hoorn, and Enkhuizen, to examine them and to report on them. The fact that the judgment of these learned and still famous men was sought shows very clearly that the inventions of Reynier Pietersz were considered important and that great expectations were entertained of them. Perhaps it was due to this that Stevin became acquainted with Reynier Pietersz and his invention.

In 1611 the latter applied once more to the States, asking them for a subvention [52]).

It is not known on what lines the learned committee made their report. The Chronicle of Hoorn does contain a judgment based on a practical examination of the "Golden Compass", which reads as follows: it was "thus put to the test by several experienced pilots in many places, both to the east and to the west, even on the coasts of Guinea and the East and West Indies, and was everywhere found correct and without error, to the surprise of those who undertook the tests".

Although the authentic drawing, which might have provided a conclusive confirmation, is lacking, it is beyond doubt that the "Golden Compass" of Reynier Pietersz as described in the Chronicle of Hoorn and the cited resolutions is the same as that which Stevin illustrates, describes, and recommends in *The Haven-Finding Art.* On a great many points this invention of his compatriot brought improvements of the foreign instruments with which he may have been acquainted.

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[50]) G. Doorman, *Octrooien* 1940, p. 97. Also printed in De Jonge, *Opkomst,* Vol. I, p. 176. Original: Public Record Office, Records of the States General, item 3328 of the inventory, fol. 243.

[51]) De Jonge, *Opkomst,* Vol. I, p. 176. *Resolutie Staten van Holland,* 13th March 1598. De Jonge here adds a note reading: This instrument is illustrated in the *Haven-Finding Art* of Simon Stevin.

[52]) G. Doorman, *Octrooien* 1940, p. 288. *Staten van Holland en West-Friesland,* 1st September 1611, p. 174.

Indeed, Borough's azimuth compass consisted of a small piece of board which had to be put horizontally on a table. It was thus suitable for use on land, but on a moving and rolling sailing-vessel no instrument can be mounted so as to be permanently horizontal, nor can such an adjustment be ensured by holding it in the hand. For the "Golden Compass" cardanic suspension was used. For a steering compass this feature was not novel. What *was* novel was its application in an azimuth compass, so that the horizontal position was ensured. Stevin was thus justified in speaking of an invention.

A second feature, which appears neither from the cited letters patent nor from *The Haven-Finding Art,* was the following. The graduation was marked on the inside of the bowl. It was to the zero point of this graduation that the needle had to be set. In consequence it was necessary during the taking of the observation to move the bowl in the direction contrary to that of the ship's movement. At the same time the quadrant had to be directed towards the sun. But as soon as the right direction had been given to the quadrant, the position of the needle as well as of the quadrant was maintained by the mere movement and readjustment of the box. It can readily be understood that this method of making the cardanically suspended bowl rotatable produced more accurate results than those that could be obtained with Borough's instrument.

The revolving quadrant, furnished with an alidade, may also be called an improvement, but it is especially the method of adjustment to which this remark applies. This method appears from the description found in the Chronicle of Hoorn, from which it may be understood that the sunlight falling through the sights cast a spot of light on the graduation along the circumference of the bowl. One observer could thus manipulate the instrument and bring about the two desired adjustments simultaneously. For this he merely had to look from above at the circumference of the bowl and at the needle.

Briefly, with the „Golden Compass" of Reynier Pietersz Stevin no doubt presented the most efficient instrument which was to be found in those days and which moreover satisfied the demands made by his system of „haven-finding". And yet in practice its success seems to have been scanty, at least if we are to believe the Hoorn chronicler, who continues his favourable report on the trial of the instrument with the words: "I really do not know what is the cause that it is not used any further, but I think the age and the lack of means of the inventor are not among the least of the impediments and obstructions".

These suppositions on the part of Velius were probably wrong. The reason will rather have to be sought in the technical sector. Indeed, although the instrument may have been suitable for use as an azimuth compass, it naturally failed and disappointed as a means for the determination of longitude at sea. There is another point as well. The necessity of constantly moving and readjusting the whole bowl was a drawback of which the observer must have been aware. Improvement on this point was quite possible. In fact, such movement is superfluous with an azimuth compass equipped not only with a single magnetic needle but also with an ordinary compass card, while the position given by the bearing-instrument is read on a graduation along the circumference of the card. In that case it is only the azimuth quadrant which is constantly directed towards the sun by being turned against the movement of the ship. The reading on the card does not change in consequence of the movements of the ship. A compass of this type, which also amounted to a considerable improvement on that of Barlow again, must have supplanted the "Golden Compass" of Reynier Pietersz as well as that of Barlow very readily indeed.

Finally the reader should realize what it must have been like to work with such compasses, in those days when the "compass-maker" was an artisan rather than an instrument-maker, hardly any knowledge existed as to the most suitable materials, and the requirements which a properly working mariner's compass was to satisfy had not yet been defined, *viz.* steadiness, sensitivity, light weight, and powerful directive force, while finally science had not yet prescribed the means which led to the satisfaction of these requirements. At that time the compass card was heavy. The cap in the card was imperfectly constructed. The pivot on which the card turned was not made of the most suitable material, nor was its point sufficiently sharp. The friction between pivot and cap accordingly was considerable. The needle, which had been touched with loadstone, was weakly magnetized, so that the magnetic moment was small. Owing to these causes the violent movements to which a sailing-vessel is liable dragged along the card and made it unsteady. In spite of the warning to be found in the textbooks, the fact that the bowl was not air-tight and the glass cover was insufficiently sealed resulted in air being blown in and pressure being exerted on the card and the needle, which caused deviations. The card oscillated and did not easily return to the original position. This applied all the more in high latitudes, where the directive force is less than in low latitudes. Steering on such a compass as well as reading a bearing must have been a difficult task, apt to involve considerable inaccuracy. Under difficult conditions it may even have become impossible [53]).

Here again it is evident why the task of the navigator in those days was arduous and extremely dangerous. Small wonder therefore that many of them, in spite of eminent seamanship, fell victims to the primitive level of navigation. Its development, both theoretical and technical, was ardently wished for, but it was not realized until the second half of the eighteenth century, when great possibilities were opened up in the matter of the determination of longitude and latitude at sea. The octant, the chronometer, the determination of longitude by lunar distances, the nautical almanac, the method of determining latitude invented by Cornelis Douwes, increased mathematical knowledge, etc., brought the solution. Those who paved the way for all this were good servants of their country.

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[53]) The unsteadiness of the compass was so familiar a thing that it had even become known outside nautical circles. The great seventeenth-century Dutch poet Joost van den Vondel, who celebrated navigation in his work and who borrowed numerous expressions from its idiom in his poetry, was also acquainted with the phenomenon. In his *Gijsbrecht van Aemstel* (1637) he alludes to the oscillation of the compass-needle and the unreliability of its indications. In the Prologue he says that the armorial bearings of the city of Amsterdam will in future be a reliable guide for the seaman and, leading him on like

.... a celestial symbol
On high, unsoiled by fog or earthly mist,
Give heart to the heroic helmsman
Where, frighted by the needle's swinging, he
Drifts in the Arctic Sea.
(transl. James S. Holmes)

§ 5

THE DETERMINATION OF LONGITUDE AT SEA BY MEANS OF THE VARIATION OF THE COMPASS

BRIEF SUMMARY

a. *THE EARLIEST VIEWS.*

As an introduction to this chapter, it may be recalled without a detailed discussion that the Chinese literature of the first centuries of the Christian era refers to the attraction of an iron needle by a loadstone and to the directive force of a magnetized needle. From a Chinese work dating from between 1111 and 1117 Balmer [54]) cites the mention of the fact that the magnetic needle does not point exactly in the direction north-south, but always shows a small declination to the west. From this it is evident that the variation had been discovered at that time. It was attributed to errors in workmanship and in the manipulation of the magnet [55]).

It has been proved that even before 1269 the variation was recognized as a real phenomenon in Europe. Its amount was measured, as is evident from Italian portolani. On a sun-dial made at Nuremberg in 1451, which is equipped with a compass, the magnetic north and south line is shown [56]).

What was the state of affairs at sea? It must be considered impossible that the Portuguese mariners on their voyages of exploration of the fifteenth century should not have watched the behaviour of their compass intently. A mariner pioneering in unfamiliar waters, who has to consider the direction in which he is sailing — if only because at some future time he must be able to return to his home port — cannot be expected to do otherwise. What means of verification was at his disposal? It was nature herself which furnished him with it, as one among the numerous valuable clues for which the seaman of those days used to keep a sharp lookout [57]) and which he interpreted with much greater ease than the present-day navigator, who has become alienated from those signs because so many technical aids are at his disposal. The seaman of those days was able to compare the uncertain direction shown by the needle with the certain direction which the Pole Star furnished. Thus he was induced to take his bearings from the Pole Star, a method which was recorded by Medina in his *Arte de navegar* of

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[54]) H. Balmer, *Beiträge zur Geschichte der Erkenntnis des Erdmagnetismus.* Aarau 1956, p. 36.

[55]) For further details reference may be made to: E. O. von Lippmann, *Die Geschichte der Magnetnadel bis zur Erfindung des Kompasses* (*gegen* 1300). Quellen und Studien zur Geschichte der Naturwissenschaften und der Medizin, Band 3, Heft 1 (Berlin 1932).

[56]) *Comptes rendus du Congrès Int. de Géographie.* Amsterdam 1938, Tome II (Géographie historique), p. 55: H. Winter, *Die Erkenntnis der magnetischen Missweisung und ihr Einfluss auf die Kartographie.*

[57]) These valuable clues include: the discoloration of the sea and the change in appearance of the waves as the ship approaches shallows, the change in the temperature of the sea-water, plants or weeds floating about the ship, clouds and lightning above remote land, the scent of vegetation, which can be detected at a great distance, the booming of the surf on the shore, which can be heard from afar, birds of particular species and in certain numbers flying round the ship, etc.

1545 [58]). During such operations the fact that there was a constant declination to one side — *i.e.* the variation — must undoubtedly have struck these mariners.

The first to record such an observation in his logbook was Columbus. The fact that he took bearings from the Pole Star on his first outward voyage on 13th, 17th, and 30th September 1492 in the early and the latter part of the night is famous in the history of navigation and has been repeatedly referred to. Let us follow the conclusions reached by Franco after an exhaustive study of these observations and of the later reports of Columbus in this field [59]).

The three observations convinced the admiral that the observed difference in azimuth pointed to a movement of the Pole Star about the pole. The observations do not create the impression of being a nautical discovery. Columbus on his voyage saw the variation change in value and he was the first to observe the passage from easterly to westerly variation. He knew that in different places of the world the declination of the magnetic needle had certain values and he was convinced that these values were constant for each place. He had a dim suspicion of a relation between the change of longitude and the change in the variation of the compass.

The fact that seamen exchanged their observations and findings gave rise to a manuscript written by Joao de Lisboa and dating back to 1514. This is now the oldest known document that contains a treatise about the declination of the magnetic needle from the meridian and about the determination of longitude by means of this declination. The manuscript in question formed part of the library of the Marquis De Castello Melhor. Because it was reprinted in 1903 it became accessible and known. It was included in the *Livro de Marinharia de Joao de Lisboa* [60]). The chapter bears the title: "Here begins the treatise about the compass-needle made by Joao de Lisboa in the year 1514, through which one can know from the declination of the magnetic needle — wherever one may be — how far one is away from the true meridian".

The treatise is subdivided into ten short sections and opens with a paragraph in which the Genoese and French compasses are discussed, the needles of which were attached to the card at an angle equal to the local declination relative to its north and south line.

The first section describes how a compass has to be made if it is to be suitable for the purpose. The needle must be as large as possible and must be attached to the card in such a way that it coincides with the north and south line, not deviating from it. The card must be large and the space between the card and the inside of the bowl must be narrow, in order to make accurate readings possible. The second section describes the bowl, on the inside of which a division into 32 points was marked. The third section reveals that a bearing instrument was fixed on the bowl and warns the navigator that the bowl has to be horizontal when he takes a bearing. The description is not sufficiently clear to give an idea of the bearing instrument.

The fourth section teaches in a clear way how the true north is found by an observation of Polaris. The observation has to be taken when the "guardians" —

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[58]) This method was already cited in § 4, p. 382

[59]) Salv. Garcia Franco, *Historia del arte y ciencia de navegar*. Madrid 1947, Tomo I, p. 49.

[60]) *Livro de Marinharia, tratado da agulha de marear, de Joao de Lisboa, copiado e coordenado por J. I. de Brito Rebello*. Lisbon 1903.

i.e. the stars β and γ of the Little Bear - are northeast or southwest of the Pole Star. In the former case the Pole Star is in the meridian, beneath the celestial pole, in the latter it is in the meridian 3½° above the celestial pole. The position of the compass-needle relative to the bearing instrument is the declination of the needle. This observation could also be taken with other stars besides the "guardians".

South of the equator the observation could be taken in a similar way by reference to the Southern Cross, as is taught in the fifth section. This constellation was shown in a star map made by the author.

The next section contains some directions concerning the accurate taking of the observation. The seventh section is a very important one. It says that the meridian on which the needle points exactly towards the north pole of the world, *i.e.* the one on which the variation of the compass is nil, passes through the island of Santa Maria and towards the island of San Miguel of the Azores, thus dividing the world into two equal parts. The meridian passes *via* the Cape Verde Islands to the island of St. Vincent, said meridian lying between the Cape of Good Hope and Cape Frio. The author had invariably found that on this meridian the needle pointed in the direction of the terrestrial pole. He had never met with any other meridian on which the same phenomenon occurred.

Finally the tenth section dicusses the relation between the declination of the compass-needle and terrestrial longitude. This passage is not easy to understand, but its purport is as follows. When one moves between latitudes 30° and 45° round the earth, starting from the prime meridian, for every 250 *leguas* the variation will be found to increase by one point, to a maximum of four points to the east or to the west, in longitude 90° E. and W. It then decreases again to zero in longitude 180°. In this way it was possible to compute the longitude of any place from the variation observed in that place. "With the greatest ease" are the words used elsewhere.

Again, during the first circumnavigation of the world (1519-1522), a Spanish undertaking under the command of the Portuguese Magellan, certain particulars were noted in the direction indicated by the needle in various places, which the Italian Ant. Pigafetta mentions in the journal he kept during the voyage. Pigafetta was of the opinion that the variation of the compass furnished a convenient means of finding one's distance from the prime meridian. He stated that the method had already been proved by experience [61]).

In his textbook Faleiro (see note 33) devotes a chapter to the variation of the

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[61]) *Premier voyage autour du monde par le Chevalier Ant. Pigafetta sur l'escadre de Magellan* 1519-1522, Paris, an IX (1801), p. 281, quoted in Linschoten Vereeniging, *Tweede Schipvaart*, Vol. XLIV, p. XXXII.

J. A. Robertson, *Magellan's Voyage Around the World by Ant. Pigafetta*. Cleveland 1906. Vol. I, p. 89 and p. 253, Note 229.

Linschoten Vereeniging, *Reizen van Barents, Heemskerck, Rijp, en anderen.* Vol. XV, p. XXI, Note 2: reference to *Relazione di Ant. Pigafetta sul primo viaggio*, Rome 1894.

J. Bensaude (*Histoire de la science nautique portugaise*, Résumé, Geneva 1917, pp. 12-22) is of the opinion that Pigafetta's remarks are based on the work of the Portuguese Ruy Faleiro—brother to Francisco—who studied the problem of the determination of longitude. He had proposed three methods, *viz.* two with the aid of the moon and the third by means of the variation of the compass. The latter method is identical with that of Joao de Lisboa, 1514, according to which the longitude is proportional to the amount of the variation.

compass-needle (Chapter VIII, p. 79). He states the problem clearly and writes: "The north-easting of the needles causes navigators many doubts, from which they may be freed by knowing precisely how much the needles northeast or northwest. In addition to this, other advantages will follow, such as knowing exactly in what direction they are sailing. Knowing this, they will follow exactly their courses without error or wandering and also it will help much to a knowledge of the longitude in which they are navigating". He describes the relation between longitude and variation as follows.

On the meridian through Corvo the variation is nil, apart from small discrepancies to be acccounted for as being due to differences between the compasses. When the ship sails westward, the needle will get westerly variation; when she sails eastward, the variation will be easterly, while the variation increases as the ship moves further away. Maximum values are reached at a difference of longitude of 90°. After this the variation decreases again to the meridian of 180° from Corvo, where the variation is nil again. The principle accordingly is the same as that described by Joao de Lisboa in 1514. It thus appears that this knowledge, gained at sea, mentioned by a practical man, subsequently elucidated in a scientific manuscript, now benefits navigation thanks to a nautical textbook. This process took more than fifty years.

Let us now proceed — remaining in the field of Portuguese science — to discuss the learned mathematican and astronomer Pedro Nunez (1492-1577), royal cosmographer and professor in the university of Coimbra. His work includes a treatise entitled

Tratado em defensam da carta de marear, 1537 [62]),

which he dedicated to the Infante Dom Luys, who was his pupil. The determination of the magnetic declination with the aid of the instrument known to us is clearly explained (p. 141). The method of taking observations at two equal altitudes of the sun is followed. Nunez writes (p. 140) that he is convinced of the existence of the variation of the compass, but wonders what is its amount. He had no confidence in the navigators, who gave different values for the same place, sometimes greater, sometimes smaller. In the absence of data he was unable to study the possible relation between the variation of the compass and longitude. He aspired to carry out an inquiry that would benefit the development of the nautical chart and of navigation. This learned theoretician and mathematician, whose investigations and writings were of fundamental importance for the evolution of navigation, calls in the aid of practice. This aid was to be given presently by one of his pupils, Joao de Castro (1500-1548), who in 1538 set sail for Goa in India as master of the ship Gryfo in a fleet under the command of his uncle Garcia de Noronha. His training and scientific bent induced Castro to observe the variation of the compass very carefully on many occasions. In the modern edition of his logbook [63]) a list is given of 43 variations obtained during the voyage from Lisbon to Goa, along the coast of India and in the Red Sea. On these data he bases his own conclusion, *viz.* that it is certain from these observations that there is no relation between the variation of the compass and the longitude of a place. The fact that

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[62]) It forms part of: Pedro Nunez, *Tratado da sphera*, 1537, an extremely rare book, which was published in facsimile by J. Bensaude, *Histoire de la science nautique portugaise*, Vol. 5, Berne/Munich 1915.

[63]) Joao de Castro, *Roteiro de Lisboa a Goa.* Annotado por Joao de Andrade Corvo. Lisbon 1882.

the variation could be different for places situated on the same meridian puzzles him, "from which we are justified in supposing that such variations are caused by particular and inherent mysteries concealed by mighty Nature in her vast and secret workshops". It was due to Castro's conclusion that in Portugal an important phase in the development of scientific navigation was closed.

In other countries the problem continued to occupy the minds of scholars as well as seamen. Naturally the question also used to be asked as to what caused the directive force of the needle. Towards what point did it point? How did the declination arise?

The point towards which the needle pointed was sought outside the earth by some scholars. We are not going into this question now, for it falls outside the scope of this introduction.

It was long thought that the needle ought to point towards the geographical pole, and that, if it did not, this was due to poor workmanship. Wishing to remedy this, Bartolomeo Crescentio [64]) tried to give the needle a special shape, and Fournier [65]) asserts that he had put a needle of this type to the test at Paris and La Rochelle, in order to verify whether it really pointed to the pole.

The suspicion about the existence of a magnetic mountain on the earth dates far back to antiquity and seems to have arisen wherever the mysterious tendency of the magnetic needle to point in a particular direction was discovered. In Europe as well as among the Arabs (Arabian Nights' Entertainments) this mountain struck terror in the hearts of seamen, and the loss of innumerable ships was attributed to it. Balmer [66]) quotes a thirteenth-century Italian poet who spoke about a magnetic moutain [67]). The map of the world of Andreas Walsperger, of 1448, bears a legend in the north to the effect that no navigation takes place in that big sea because of the magnets. A warning against the great danger of magnetic attraction is sounded in a legend on the map of the world of Ruysch (Rome, 1508), north of Iceland: "ships having iron in them cannot return" [68]). The existence of such a mountain in the extreme north was also assumed by the great geographer and cartographer from the Southern Netherlands, Gerhard Mercator (1512-1594). In his chapter devoted to magnetism and marine compasses Coignet [69]) clearly says that Mercator at 16° 30' from the north pole "puts a very prodigious rock and mine of loadstone, towards which all other loadstones in the whole world are attracted". There is no longer any reference to its being a danger to navigation. It is now only the loadstones and the compasses which are attracted by this rock.

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[64]) Bartolomeo Crescentio, *Nautica Mediterranea.* Rome 1607.

[65]) Georges Fournier, *Hydrographie.* Paris 1643, p. 550.

[66]) Balmer, *Beiträge zur Geschichte der Erkenntnis des Erdmagnetismus.* Aarau 1956, p. 533.

[67]) *ibid. Die Sage vom Magnetberg*, p. 651. Hennig, *Terrae Incognitae*, Vol. III, Chapter 149: *Die Seefahrt eines Oxforder Geistlichen in den Atlantischen Norden und die Frage des Magnetbergs.* 1360.

[68]) The map of Ruysch is to be found in the edition of Ptolemy's atlas of 1508. The map is discussed and reproduced by Nordenskjöld, *Facsimile Atlas*, p. 65, and map XXXII. The legend reads: *hic incipit Mare Sugenum. Hic compassus navium non tenet, nec naves quae ferrum tenent revertere valent.*

Nordenskjöld utters the suspicion that these words might perhaps point to the experience that the compass becomes useless in the extreme north, *i.e.* in the neighbourhood of the magnetic pole.

[69]) Michiel Coignet, *Nieuwe onderwijsinghe op de principaelste puncten der Zee-vaert.* Antwerp 1580, p. 5 *verso.*

b. *MERCATOR (1512-1594)*

Mercator, being convinced that the magnetic needle - irrespective of the place on the earth where it was put up - was drawn in the direction of the magnetic pole, thus regarded the magnetic meridian of a place as a great circle passsing through that pole. This pole was the point of intersection of all magnetic meridians, and its position on the earth might be found if for two points the positions of which were known the variation had been determined by measurements. In a letter of 1546 to Ant. Perrenot, Bishop of Arras, Mercator writes that near Walcheren the variation was 9° east and at Danzig presumably 14° east [70]). For the point of intersection of the magnetic meridians he had found a point in latitude 79° N. The prime meridian had thus at the same time become known. With the aid of further data he afterwards obtained improved results. The system was to the effect that for any place on the earth the variation could be calculated and that conversely the longitude could be found from the amount of the variation.

Mercator's famous chart of the world, destined for navigation and dating from 1569 [71]), illustrates his views at that moment. The magnetic pole is indicated in the top lefthand corner in the form of a steep rock with a mark to show its exact position. It lies on the meridian of longitude 180° and in latitude 73°30' N. This meridian passes through the Strait of Anian, which was assumed to connect the Arctic Ocean and the Pacific between Asia and America. Because the chart covers more than 360 degrees of longitude, the 180th degree of longitude appears a second time on the righthand side. On this meridian and in the above-mentioned latitude the same magnetic pole is shown once more, with the following legend: "from sure calculations it is here that lies the magnetic pole and the very perfect magnet, which draws to itself all others, it being assumed that the prime meridian be where I have placed it". Not far from this rock a kind of island is shown, again with a point marked in the middle, the position of which is latitude 77° N. and longitude 174° E. A legend states that the magnetic pole will be found in this place if the meridian through the island of Corvo is taken as the magnetic prime meridian.

In an elaborate legend, printed near these poles, Mercator accounts for his procedure, saying that the very capable seaman François van Dieppe states that the magnetic needle points due north on the Cape Verde Islands Sal, Boa Vista, and Mayo. The legend continues: "This is closely supported by those who state that this occurs at Terceira or S. Maria (which are isles of the Azores). Some believe that this is the case at the most westerly of these islands, which is called Corvo. Now, since it is necessary that longitudes of places should, for good reasons, have as origin the meridian which is common to the magnet and the World, in accordance with a great number of testimonies I have drawn the prime meridian through the said Isles of Cape Verde; and as the magnet deviates elsewhere more or less from the pole, there must be a special pole towards which magnets turn in all parts of the world, therefore I have ascertained that this is in reality at the spot where I have placed it by taking account of the magnetic

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[70]) In 1539 Rheticus had found 13° at Danzig. See: Balmer, p. 103. Letter reproduced in Balmer, p. 313.

[71]) *Drei Karten von Gerhard Mercator. Europa, Britische Inseln, Weltkarte. Facsimile-Lichtdruck nach den Originalen der Stadtbibliothek zu Breslau. Herausgegeben von der Ges. für Erdkunde zu Berlin.* 1891.

declination observed at Ratisbon. But I have likewise calculated the position of this pole with reference to the Isle of Corvo in order that note be taken of the extreme positions between which, according to the extreme positions of the prime meridian, this pole must lie until the observations made by seamen have provided more certain information" [72]).

In the bottom lefthand corner of the chart of the world there is an inset, *viz.* of the arctic regions, which could not be shown in the big chart, since the latter was one on Mercator's projection, in which the pole cannot be present. On the north pole of the earth a steep black rock has been marked. The rock on the magnetic pole lying on the meridian through the Cape Verde Islands and the magnetic pole without a rock in longitude 174° E. on the meridian through Corvo are both shown.

In the chart of the world in two hemispheres, published in 1587 [73]) by Rumoldus Mercator, the latter followed his father's example. On the 180th meridian, being the prime meridian through the Cape Verde Islands, and in latitude 73° 30' N. the magnetic pole is to be found.

After a new edition of the above-mentioned chart of the arctic regions had appeared in 1595, Rumoldus prepared a new edition of it, brought up to date, which is easily recognized because it includes, among other things, *"het behouden huis"*, — a name calling to mind the wintering of 1596/7 — and names given by Barents to capes on the coast of Novaya Zemlya [74]). Rumoldus maintained the four polar islands which his father had imagined, and the two possible positions of the magnetic pole in latitudes 73° 30′ and 77° N. He only altered the longitude of both. For the meridian through the Cape Verde Islands he now gives longitude 178° E. and for the meridian through Corvo 172° E., thus a displacement of 2° [75]).

In the chart of the arctic regions made by Willem Barents [76]) and published in 1598 — the year after his death — the polar islands no longer appear, which amounts to an important correction. However, the author still stuck to the *"polus magnetis"*, shown as a tall rock in latitude 74° N. and lying on the meridian corresponding to that through the Cape Verde Islands in the arctic chart of Mercator.

c. *PLANCIUS (1552-1622)*

Plancius entirely followed Mercator in this matter. In the map of the world in two hemispheres which he appended to the Dutch bible published by Laurens Jacobsz he marked the magnetic pole in latitude 74° N. And when he compiled his big map of the world, dating from 1592 [77]), it was again the map of his

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[72]) This text has been taken from: *The Hydrographic Review*, Vol. IX, No 2, Nov. 1932, p. 7. Text and translation of the legends of the original chart of the world by Gerhard Mercator, issued in 1569.

[73]) It occurs in: *Galliae tabulae geographicae per Ger. Mercatorem.* Duisburg, 1585.

[74]) It appears on p. 73 of: Mercator, *Atlas sive cosmographicae meditationes de fabrica mundi et fabricati figura.* Ed. decima. Henr. Hondius. Amsterdam 1628.

[75]) It is curious that the atlas mentioned in Note 74 has on p. 65 a map of Asia, in which the magnetic pole has been maintained on the 180th meridian, which clearly proves the uncertainty prevailing with respect to the position of the magnetic mountains.

[76]) Reproduced in Vol. XV of *Werken uitgegeven door de Linschoten-Vereeniging: Reizen van Willem Barents, Jacob van Heemskerck.*

[77]) See: F. C. Wieder, *Monumenta Cartographica, reproductions.* Vol. II. 1926. Text and map pages 27, 30, and 35.

learned predecessor which formed his principal source and starting point, though Plancius did not adopt the *rete* employed by Mercator and made ample use of Portuguese data wherever he found them to be more up to date. Owing to his critical method Plancius gave the best representation of land and sea that was possible at the time.

Just as in Mercator's map, in the top righthand corner the small sketch is to be found of the big, steep rock in the Strait of Anian, which indicates the magnetic pole and which lies in latitude 73° 30' N. and longitude 180°. A legend is added to the effect that it lies on the meridian through the Cape Verde Islands. The prime meridian appears to fall through the island of San Thiago. The other possible position of the magnetic pole, referred to the meridian through Corvo, is to be found in latitude 77° N. and longitude 173° E. The legend occurring on Mercator's map of 1569 as to the position of the two poles is repeated verbally, the source being mentioned at the end in the form of the name Ger. Merc. Analogously to Mercator's map we again find in the top lefthand corner the former of these two poles, and in the bottom lefthand corner the inset of the arctic regions.

Finally attention may be drawn to the map of the world by Plancius, engraved by Josua van den Ende and dating from 1604 [78]). The two magnetic poles are to be found in the corresponding places, and so is Mercator's legend of 1569, with the initials. On the globe of Jacob Florentius van Langren, dating from 1589, the two magnetic poles are marked, with the legend stating that they refer to the meridians through Corvo and the Cape Verde Islands respectively. On the later specimen of this globe, of 1612, the fact that the same legends are used shows that the view concerning the poles was maintained by the author.

The geographers of the early seventeenth century begin to think differently of it. The tide had turned. The map of the world in two hemispheres of Willem Jansz. Blaeu of 1605 [79]) no longer contains a magnetic mountain, nor does the map of the world on Mercator's projection by Jodocus Hondius of 1608 [80]). The same applies to the latter's map of the world of 1611 [81]). The prime meridian in this map falls through the Cape Verde Islands.

This changed view is accounted for on the wonderful globe of 1613 by Jodocus Hondius Jr. and Adriaan Veen. In the place where the poles used to be marked, the following legend in Latin is to be found [82]):

> "Gerardus Mercator, and others who followed him in this, had put in this place two magnetic poles, the one referred to the Cape Verde Islands, the other to the islands of Corvo and Flores. However, because there is no cer-

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78) It was described and reproduced by Marcel Destombes. *La mappemonde de Petrus Plancius, gravée par Josua van den Ende*, 1604, *d'après l'unique exemplaire de la Bibliothèque Nationale de Paris*. 1944. Publications de la Société de Géographie de Hanoi.

79) *World Map 1605 Willem Jansz. Blaeu, facsimile of the unique copy belonging to the Hispanic Soc. of America*, text Edw. L. Stevenson, New York 1914.

80) *The Map of the World on Mercator's Projection by Jodocus Hondius*, Amsterdam 1608. From the unique copy in the collection of the Royal Geographical Society with a memoir by Edw. Heawood. London 1927.

81) *World Map by Jodocus Hondius*, 1611. Ed. by Edw. L. Stevenson and J. Fisher, New York 1907.

82) This globe is present in the Netherlands Historical Maritime Museum at Amsterdam. On another, undated specimen, also present there, the same legend is to be found.

tainty at all about this and daily experience teaches us otherwise about the declination of the compass needle, we have omitted both."

The theoretical conception of Mercator had been unable to hold its own against the results obtained in practice.

As long as one magnetic pole, lying in line with the meridian through Corvo, continued to be assumed, this involved the consequence that easterly variation was bound to exist east of this meridian up to longitude 180°, and westerly variation in the western hemisphere. Maximum values for the variation had to be found in longitudes 90° E. and W. This picture had already been sketched in the *Livro de Marinharia* and later by Faleiro, but with them there was no question in these places of a magnetic pole with the property of attracting the needle. Later on Coignet, in his *Nieuwe Onderwijsinghe,* expounds very lucidly that all these phenomena must be due to the presence of one magnetic pole. Moreover he shows by means of a drawing that when two places in the northern hemisphere lie on the same meridian, the variation must be greater for the more northerly one than for the other. In order to demonstrate that the whole conception is correct, he concludes by stating that according to "Mercator's rule" the variation at Antwerp will have to be 9° to the east, and that he had found this value "by experience" [83]).

Plancius, who diligently tried to solve the mystery of terrestrial magnetism, must soon have found that the data about the variation collected by him did not fit in with this simple scheme. What can have been the source of these data?

Plancius himself provides the answer to this question in a manuscript written by him, which is now kept in the Public Record Office at The Hague. [84]) This manuscript is of the utmost importance to us for three reasons. In the first place because it mentions the author's sources. Further it helps us to understand Plancius' theory concerning the possibility of determining longitude at sea. Finally it appears to be the source of the list of values of the variation, to be found in *The Haven-Finding Art,* a fact which accounts for Stevin's gratitude towards Plancius.

The manuscript in question contains three treatises, which are entitled:

1. *Van de graden der lancte ende het affmeten der selver door het Noordoosteren ende Noordwesteren der naelde*
   (Of the degrees of longitude and the measurement thereof by means of the northeasting and northwesting of the needle).

2. *Van de Oost-Indische zeevaert ende haren eygenschappen ende aenmerkingen*
   (Of the navigation to the East Indies and its character, with notes).

3. *Naerder verclaringe van de Oost-Indische zeevaert*
   (Further comment on the navigation to the East Indies).

The treatises sub 1 and 2 were drawn up after the return of De Houtman about

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[83]) Thus to be found in the first edition of 1580, published at Antwerp, as well as in the fourth edition, Amsterdam 1598.

[84]) Public Record Office, The Hague. Loketkas Admiraliteit No 10.

1598, while that sub 3 is of a later date and must have been written after the return of the fleet of Van Neck, in 1599 or 1600 [85]).

Before restricting our attention to the contents of the treatise sub 1 — the most important for the present purpose — we may remind the reader that an obvious reaction to the question concerning the origin of Plancius' data would have been to think of Jan Huychen van Linschoten. In fact, in his *Reijsgheschrift van de navigatiën der Portugaloysers* [86]), the guide published at Amsterdam in 1595, which was of inestimable value to all contemporary explorers and seafarers, this author included hundreds of values of the variation. In addition he published a series of values for a number of specified places lying on the route from Portugal to the East Indies [87]). This list is more than a mere enumeration of values for particular places. Indeed, with its aid an observed variation gave an indication of the longitude in which the ship was at that moment. Thus we read: if the compass has a northeasterly variation of half a point, "you may know that you are near Cabo de bona Esperança", and if it has a northwesterly variation of 3/4 point, "beware of the Isle of Sant Lourenço, for you will see it presently". Although in our view the way in which the place is indicated is very vague, yet some value has to be attached to this list in the above sense, as appears from the two instances here given as well as from others. Considering the provenance of these data, it is evident that no mention is made of any rule as to the relation between variation and longitude, for the Portuguese had already rejected such a relation some fifty years before.

That Plancius read and knew the writings of Van Linschoten is evident from the criticism uttered by him in his *Naerder verclaringe,* but he does not mention his name in the treatise written after the return of De Houtman: *Van de Oost-Indische zeevaert.* It is thus not certain whether Plancius did or did not make use of the work of Van Linschoten, with whom he sometimes disagreed considerably.

Although a description of the contents of the last-mentioned treatise falls beyond the scope of the present introduction, however valuable it may be in view of the fact that the experiences of Dutch, French, English, and Portuguese seafarers are here set forth for the benefit and instruction of others, one remark has to be made. This treatise contains a considerable number of statements about the amount of the variation in various places, which — just as in the writings of Van Linschoten — were intended by Plancius as aids to navigation. Thus we find the instruction to cross the equator on the outward voyage "at 8°30′, 9°, or 9°30′ of increasing northeasting", followed by the advice to pass Cape St. Augustine on the coast of Brazil in a place where the variation was 6 or 7 degrees of increasing northeasting, "then you will be on the right track to sail successfully with God's aid past the shoals of Abrolho". If one acted on this advice, one would be far enough from the coast to be safe with respect to this notorious point. We find the warning: "who pays no attention to the variation of the needle and to the currents will get further from his course than he thinks, as has befallen many people". From these examples it will be sufficiently evident in what respect Plancius considered it important to know these values, and thus also what importance he

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[85]) The treatises mentioned sub 2 and 3 are included in: De Jonge, *De opkomst van het Nederlandsch gezag in Oost-Indië*, Vol. I, 1862, pp. 184-194 and 194-200.

[86]) See: *Werken uitgegeven door de Linschoten-Vereeniging*, Vol. XLIII. *Het Itinerario van Jan Huygen van Linschoten*, 1579-1592, Vols IV and V, 1939.

[87]) *Idem*, Vol. V, pp. 367-370.

attached to the observation of the variation on board [88]. No reference is made in this passage to any special relation between these values and the longitude at sea.

We will confine ourselves to the treatise mentioned above sub 1, *viz.* "of the degrees of longitude", which forms the source from which we may learn to understand the theory of Plancius concerning the relation between the geographical longitude and latitude of a place on the earth and the amount of the variation in that place. His theory forms a unique and interesting page of the chapter in the book of navigation with which we are here concerned, and it has to be discussed before Stevin's work can be assigned its place. It was published under the title of *Memorie van Plancius* in Vol. XXXII of *Werken der Linschoten-Vereeniging* [89], but with the omission of three pages at the end. It is precisely these pages whose contents are of the utmost importance for our purpose. The main title mentioned above is followed by the words:

> "Extract from the writings of Frederick Houtman, from which it is understood how the ships of Amsterdam on their first voyage to the East Indies took the deviation of the needle of the compass by means of the rising and the setting sun, which can also be measured at all the points and the degrees of the compass and at all hours and minutes of the day."

We will follow the treatise sub 1, devoted to "the degrees of longitude".

Whilst formerly, on his chart of the world, Plancius had taken over from Mercator the two magnetic poles lying close together and the prime meridians, he has now made his choice and dropped the prime meridian through the Cape Verde Islands. The prime meridian, from which the degrees of longitude are reckoned, is the one through the islands of Flores and Corvo. On this meridian "the needle of the compass points due south and north without any deviation". Plancius now assumes that there are four such meridians on which the variation is nil and which might thus be called agonics. They are those of 0, 60, 160, and 260 degrees of longitude — reckoned in the eastward direction — which divide the surface of the earth into four lunes, of which one therefore covers 60 and the other three 100 degrees of longitude each. In the lune from 0 to 60° the variation is easterly, in the second it is westerly, and thus alternately. In each lune the value increases from zero to a maximum lying on the median meridian of the lune. The maxima thus lie at 30°, 110°, 210°, and 310°. We read that a point at 17 German miles — *i.e.* 68 nautical miles — east of Cape Agulhas lies on the meridian of 60°E.L. and that the island of Hjelmsöy at the North Cape too lies on this meridian. On the meridian of 160°E.L. — "or very close to it" — lies Canton in China, on that of 260° the westernmost part of California, on that of 310° "Nombre de Dios, a famous port in America". This name is still found on the northern coast of Panama, slightly to the east of Colón. Considering the difference of longitude from Corvo, this place may be meant.

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[88]) In this context reference may be made to the very rare book of William Barlow, *The Navigator's Supply*, London 1597. The author speaks about a voyage made by Sir Francis Drake from the western tip of Cuba to Virginia: "and found his destination only through the navigator's knowledge of the variation of the compass".

[89]) *Werken uitgegeven door de Linschoten-Vereeniging*, Vol. XXXII. *De eerste schipvaart der Nederlanders naar Oost-Indië onder Cornelis de Houtman*, 1595-1597. Part III, 1929, pp. 411-432.

After this outline of the magnetism of the earth Plancius states his sources. This is what he writes:

> "The foregoing foundations and grounds are found certain and true by frequent experiences and observations, made with great diligence and skill by many learned men, intelligent masters, and good navigators of Spain, Portugal, France, England, The Netherlands, and other nations in many countries and places of Europe, Asia, Africa, Peruana and Mexicana [90]), both south and north of the equator. But of the deviation of the needle in the countries and seas lying between China and Mexico I have not so far been able to get accurate information, confirmed by experience." [91])

These words are followed by the instructions teaching how the longitude is derived from the known amount of the variation in a place whose latitude is given. Although this derivation is obscure to the uninitiated reader, it appears at once that this is a problem of spherical trigonometry. Just as Mercator had done, Plancius assumes a mathematical relation between these quantities, which admits of being expressed by means of this auxiliary science. It is not through calculation, but through measurements in a plane surface that the problem is solved. For this purpose two instruments are employed. The first is the "general astrolabe" *(astrolabium catholicum)*, an instrument which had long been known and widely used to solve problems of spherical trigonometry, and the second is a "longitude-finder" specially designed for this method [92]). Only in this way was it possible for Plancius to make the subject accessible and manageable for the seaman of around 1600. In fact, the latter was unable to perform calculations and it was not until the end of the eighteenth century that his accomplishments included a knowledge of spherical trigonometry and its applications.

Mercator formerly had placed the magnetic pole on the continuation of the prime meridian in a latitude determined by calculation, and Plancius had imitated him. Since the latter now, on the ground of observational data, assumed four meridians on which the variation was nil, he could not mark four magnetic poles in a corresponding way, for in that case no variation zero could have occurred on any of the agonics. The term magnetic pole has disappeared from this treatise, although the concept has not been rejected (in so many words).

The treatise shows that on the ground of the observational data at his disposal — and undoubtedly after endless patient trials — Plancius arrived at a thesis reading as follows:

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[90]) Names used at that time for North and South America: *Mexicanae* and *Peruvianae Pars*.

[91]) Balmer, p. 129, points out that the learned Spanish Jesuit Acosta, basing himself on the reports of Portuguese seafarers, mentions four lines of no variation. The reference is to his *Historia natural y moral de las Indias*, Book I, Chapter 17.

José de Acosta (1539-1600) was a missionary in Peru for many years. His aforesaid book, which appeared at Seville in 1590, was very soon translated into six languages, the Dutch version, *Historie naturael ende morael van de Westersche Indiën*, being published at Enkhuizen in 1598. The translator was Jan Huychen van Linschoten, who laid hands on the book as early as 1594 and who thought so highly of its contents that he was no longer satisfied with his own work.

In the passage in question (p. 37) it is said that "an experienced Portuguese navigator" had furnished him with the information. He did not recall the names of the "places" (*sic*) having no variation, except the "vicinity" of Corvo.

[92]) For the construction and the use of these two instruments, see the Appendix.

the point of intersection of the magnetic meridian through a given place with the nearest agonic falls on the same parallel for all places.

The longitude of a place is calculated in a spherical triangle, the known elements of which are: the complement of the latitude of the parallel just referred to, the complement of the latitude of the place, and the variation in that place [93]. The latitude determined by Plancius for that parallel must have been approximately 65°30' [94].

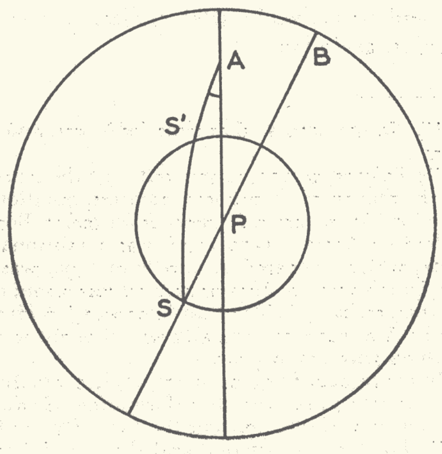
He then starts to apply his rule. On the ground of a Portuguese observation of the variation at Cape St. Augustine in Brazil, amounting to 3° northeasting, he calculates that the longitude of this cape, lying in latitude 8°30' south, must be 5°50'. In the same way he calculates dozens of longitudes, and he does this also for the observational data brought by the ships which completed the first voyage to the East Indies. He had found these observations to be in agreement with those of the Portuguese seafarers. We need not consider the results of the calculations, because the determinations of longitude are based on a relation which does not exist. However, we find that out of the large body of material a number of places, with an appended statement of the pertinent latitude, longitude, and variation, have been collected in two almost identical lists accompanying the treatise. They form the contents of the three pages referred to on p. 401, and these lists are again identical — except for a few minor differences — with the lists which Stevin gives in his *Haven-Finding Art* (pp. 438 - 440).

d. *STEVIN*

Since the reader has become acquainted with the contents of *The Haven-Finding Art* in § 3, while the present section has thrown light on the conceptions of Plancius, Mercator, and others, it is now possible to see Stevin's ideas against the background formed by the work of his predecessors.

The fact that Stevin absolutely rejects a magnetic pole — which impresses the reader all the more because he does so in the opening pages of his treatise —

Fig. 5



[93]) Figure:
Outer circle: equator; inner circle: parallel at latitude 65°30′
PB: agonic; PA: meridian through A
AP: 90° - latitude of A
∠ PAS: known variation at A
S: point of intersection of the magnetic north and south line in A with the parallel SS′
∠BPA: longitude of A in relation to the nearest agonic BPS

[94]) See Appendix, especially fig. 6 and fig. 8 on p. 414 and 417.

implies a definite attitude and a radical change of opinion as compared with his predecessors. On the ground of observations of the variation which had become available — it should be noted that they were taken precisely from the work of Plancius — the impossibility of the existence of a magnetic pole had become established, at least in the sense in which it was taken at that time.

A second and extremely important point of difference from Plancius and Mercator was the fact that no use was made of spherical trigonometry, an auxiliary science of which Stevin, in contrast to Plancius, had perfect command. Stevin rejects the mathematical relation between the amount of the variation and the longitude: a perfectly fundamental difference indeed.

In his *Memorie* Plancius had pointed out briefly that, passing to the east from the meridian of Corvo, the easterly variation increased in the northern as well as the southern hemisphere to a maximum in longitude 30°, after which it decreased again to 0° in longitude 60°. Without any further comment, and as a fully established fact, he mentions the existence of the four agonics, and the increase and decrease of the variation in the three lunes of 100° difference of longitude. He states the latter although he admits he has no data at his disposal about the variation between China and Mexico.

Stevin interprets the material, which he obtained from Plancius and which he presents in *The Haven-Finding Art* (pp. 438 - 440) in his own way. His description of the increase and decrease of the variation in the lunes from 0° to 60° and from 60° tot 160° accords with Plancius, but from that point on he is more cautious than the latter. About the part from longitude 160° to 360°, for which no data are available, he says how "it is suspected to some extent" that the variation will appear to be. Instead of four he assumes six agonics, *viz.* the meridians of longitude 0°, 60°, 160°, 180°, 240°, and 340°, and he calls his system a "conjecture", It has been stated above that in 1608 he expressed himself more cautiously than he had done in the first edition of *The Haven-Finding Art.*

Plancius thought it possible to derive the longitude mathematically from the variation observed. He thus offered a method for the actual determination of longitude.

On the available material Stevin was unable to found anything more than a practical method of "haven-finding", a method which was serviceable and was to prove useful, but which yet was not equivalent to a method for the determination of longitude. In fact, in 1608 he said that the latter had a wider scope and was "worthier". In this way he himself points out the difference.

Summarizing, one may conclude that the allegation that Stevin recorded and put into print the ideas of Plancius with respect to the determination of longitude is by no means true. On the contrary, Stevin held views of his own and he published his own conceptions under an appropriate title. Relying completely on what appeared certain to him, he furnished seamen with two rules, *viz.:*

a. in seeking your destination and sailing along the parallel, note the increase or decrease of the variation of the compass.
b. take plenty of observations, which will make it possible to improve upon the theory and to add to the knowledge of terrestrial magnetism, which will benefit your successors.

Whereas the system of Plancius after some time was called "imperfect and quite frivolous" on the ground of practical experience [95]), so that it soon fell into oblivion, the gist of Stevin's rules proved serviceable and was put into practice.

In view of these considerations Stevin ranks high among the men who attempted to determine longitude on the ground of the variation of the compass. Although the appeal to navigators to take observations was heard abroad as well, he may be considered to have been the first to advance navigation by giving a satisfactory rule and at the same time recommending a suitable instrument for taking the necessary observations. An increased knowledge of terrestrial magnetism resulted from his work.

In *The Haven-Finding Art* Stevin does not account in any way for the curious phenomenon of the deviation of the compass-needle, or for terrestrial magnetism in general. The reader's attention is drawn to the fact that in his *Wisconstighe Ghedachtenissen* of 1605—1608 he discusses terrestrial magnetism in the 2nd Proposition of Book III, Ch. 1 of *The Heavenly Motions* (pp. 253—255) under the heading: "To expound the motion of the earth in its place, and its magnetic rest." In that context he refers to the work of William Gilbert, *De Magnete* [96]) — which appeared in 1600 and is to be mentioned once more at the end of the present section (p. 410) — and says the following about the magnetic condition of the earth: "in the earth there is found such a large amount of loadstone and other substances with magnetic force that, like a big loadstone, it has in itself the properties that are found in small spheres made of loadstone." The earth itself is a magnet. It is this view which was introduced by the above-mentioned English scholar.

Stevin also adopts Gilbert's explanation of the fact that the amount of the variation is not the same in all places of the world, a phenomenon about which, as Stevin says, "many people have wondered and puzzled so long". He compares the earth to a spherical magnet, in which a deep and wide groove has been made. The phenomenon is attributed to the influence of the land, not of the sea. This is to be learned from the following curious and simple explanation: "Since the earth is a sphere of loadstone with deep pits, *viz.* the seas, in whose moving water the above-mentioned magnetic character cannot be present, straight northward pointing indeed is found about the middle of the great seas, such as in the ocean between America and Europe, but when we get to the east, towards Europe, the needle deviates towards the east, and when we get to the west, towards America, it deviates towards the west."

In Gilbert's great work, Book IV, Chapter VII, p. 163, this thesis is set forth in detail. We quote: "It is not easy to determine by any general method how great the arc of variation is in all places and how many degrees and minutes it subtends on the horizon, since it becomes greater or less from diverse causes. For both the strength of true verticity of the place and of the elevated regions, as well as their distances from the given place and from the poles of the world, must be considered and compared; which indeed cannot be done exactly. Nevertheless by our method the variation becomes so known that no grave error will

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[95]) Testimony to be found in the *Journael gehouden bij schipper Jan Cornelisz May, schipper op de Vos*, 3rd July 1611. See: *Werken uitgegeven door de Linschoten-Vereeniging*, Vol. I, p. 16.

[96]) A book devoted to Gilbert and to his main work, its influence, etc., is: Duane H. D. Roller, *The De Magnete of William Gilbert*. Amsterdam 1959.

perturb the course at sea. If the positions of the lands were uniform and straight along meridians and not defective and rugged, the variations near lands would be simple" ... "but the inequalities of the maritime parts of the habitable earth, the enormous promontories, the very wide gulfs, the mountainous and more elevated regions, render the variations more unequal or sudden or more obscure and moreover less certain and more inconstant in the higher latitude." Although Stevin only took over the core of this reasoning, he must have been acquainted with the whole text of Gilbert, who deals extensively with the subject of variation in his Fourth Book [97]).

In the year after the publication of *The Haven-Finding Art* a booklet appeared under the title *Een corte onderrichtinge* [98]); it had been written by a practical seafaring man, *viz.* Albert Haeyen, known as the compiler of a useful book containing charts and sailing directions for the North Sea and its coast from Nieuwpoort to Jutland. As the title says, it is intended to refute the errors daily increasing in the finding of ports "owing to incapable teachers". It is an attack in sailors' jargon. The writer inveighs against haven-finders — by which are to be understood both Plancius and Stevin — who are landlubbers, not versed in nautical terminology, and who in the writer's opinion are not entitled to write about practical nautical matters and to instruct sailors. The system was "built on a straw". The results were poor, a statement which he proves by means of instances taken from the practical experience of the first voyage to the Indies and voyages to the North (p. 21). In one case, when the result was particularly poor, the crew wanted to throw the pilot overboard. The writer's attack is mainly directed against the difficulty of measuring the variation of the compass with sufficient accuracy. The instrument recommended in *The Haven-Finding Art* was no good. "That the taking of a bearing with the compass is a ticklish matter becomes quite clear because several seafaring men, when standing behind a compass, will not agree to half a point in taking a bearing, nay — which is more — not even to a full point, as we have found at sea as well as on the land" (p. 10). It is alleged that navigators, when returning from the first voyage to the Indies, refused to surrender their observational data, obtained with such great risks and difficulties, and that others had faked them, because they were unwilling to put their knowledge and experience, which represented their living, "their field and their plough", readily at the disposal of the scholars on shore. It was in this way that the data for the table of the variation were said to have been collected. We are not going to discuss the further contents of the booklet, which was written with some pretence of learning. Haeyen did not prove himself a man of good education. The disinclination to adopt innovations is after all a familiar phenomenon. When towards the end of the seventeenth century Christiaan Huygens wanted to put his pendulum clock to the test on board, the two navigators who took the observations in accordance with the instructions met with opposition and were heaped with ridicule.

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[97]) The quotations have been taken from the English translation of *De Magnete* by Silvanus P. Thompson, London 1900.

[98]) *Een corte onderrichtinge belanghende die kunst van der zeevaert, waer in ghehandelt wordt hoe men die selfde sullen moghen verbeteren ende oock met een wederleijt, die abuysen die daghelicks door onbequame leer-meesters om havens te vinden, wassen ende toenemen, allen die ter zee handelen tot een ghetrouwe waerschouwinghe door Aelbert Hendricksz, anders Aelbert Haeyen.* Amsterdam 1600.

Another writer who was sceptical of the accuracy of the indications of the compass was Adriaan Metius (1571—1635), mathematical professor in Franeker University. In his *Nieuwe geographische onderwijsinge* [99]) he devotes a chapter to the defectiveness of navigation, in which he shows how in an experiment on shore the compass-needle, after being brought out of its position of equilibrium, frequently fails to return to its original position (pp. 58 and 59). "What can be accomplished on the moving ship by those who pretend that by an observation of the deviation of the needle the longitude for sailing to the east and the west can be found accurately enough?"

A valuable book was that devoted by Keteltas to the azimuth compass, its construction, and its use [100]). In the dedicatory epistle to Prince Maurice the writer expresses his expectation that the mysterious properties of the magnetic needle will be useful for the determination of longitude. He therefore believes in a relation between them. Many navigators, when sailing to the Indies, had intently observed the variation of the compass after having been instructed by Plancius, and they had found the division of terrestrial magnetism by the four agonics, in accordance with "the thesis of the aforesaid master", to be confirmed in actual practice. Attention is drawn to the number four in connection with the "thesis". If it were possible to measure the variation accurately — the writer speaks of an accuracy to within the nearest minute of arc — the problem of longitude would be solved, and for this reason he wrote a treatise about the construction of a number of instruments by means of which the variation could be measured, about the method of checking such instruments and the means for correcting them, in order that by a thorough understanding of the subject one might be able to rely upon one's measurements. His work is comprehensive, clear, and valuable. It includes a fine illustration of an azimuth compass, suitable for use on board and fitted with an azimuth-circle with a cross-bar sight, "an instrument which is in daily common use among navigators".

A repudiating and almost scornful statement is to be found in the book of Robert Hues, translated and annotated by Jodocus Hondius [101]). "There are some people who pretended to derive some measure or certain rule about the variation, as if it occurred in a measurable and proportional way, but it is all in vain. Experience shows that there is no system in it." And somewhat further on he says: "they had better keep silent who think one might make calculations from these deviations in order to find the longitudes of places, which were to be wished, and it would actually occur in this way if the needle pointed always to one and the same point" (Part IV, Chapter 15). It is obvious that this criticism is levelled at Plancius. In fact, at the time it was uttered, *The Haven-Finding Art* had not yet appeared. But Stevin's system was condemned by it too.

The famous Willem Jansz. Blaeu (1571—1638), a nautical expert and a competent judge, also joins the ranks of the opponents, as is evident from the epistle to the reader in the first part of his *Het Licht der Zeevaert* (Light of Navigation) of 1608. He opens this work with *Eene korte onderwijsinghe in de Const*

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99) Adr. Metius. *Nieuwe geographische onderwijsinge.* Amsterdam 1621.

100) Barent Ev. Keteltas, *Het ghebruijck der naeld-wijsinge tot dienste der zee-vaert beschreven.* Amsterdam 1609.

101) Robert Hues, *Tractaet ofte handelinge van het gebruyck der hemelscher ende aertscher globe, in 't Latijn beschreven door Robert Hues en nu in Nederduytsch over-gheset ende met Annotatien vermeerdert door Judocum Hondium.* Amsterdam 1623. First edition 1597.

der Zee-vaert (Brief instruction in the art of navigation), in which everything a seaman needed had been included, but from which learned astronomical matters were omitted. He continues: "we have equally omitted to write somewhat about the finding of longitude, which is commonly called east and west, about which some people claim to have found great things. Nay, even to the extent that one can find one's course towards the east and the west just as certainly as towards the south and the north. But all that has so far been published about this is not only useless, but (if one were to rely thereon) also prejudicial and deceptive, about which we intend to write more fully in the fourth part of this book as also what profit a navigator may gain from the deviation of the needle or variation of the compass, on which these new discoveries have been baselessly founded." We do not know what Blaeu intended to write in support of his criticism and about this "profit", for no copy of Part IV of *Het Licht der Zeevaert* is known. However, his condemnation was clear enough, and Plancius had good reason to be embittered against Blaeu because of his destructive criticism: useless, prejudicial, and deceptive. These terms must have been directed no less against Stevin's work.

We may learn more about Blaeu's adverse judgment from a legend in Latin occurring on a globe made by him, which is preserved in the Antwerp Library and which is dated 1620 [102]). Here the author explains his choice of the place of the prime meridian, and in this connection he refers to the "contemporaries" who wanted to follow an indication given by nature and choose for the prime meridian the meridian on which the magnetic needle shows no declination. But "they err". The needle cannot serve to find the origin of longitude, "since it shows different declinations on the same meridian, according as it is nearer to this or that continent". He thus says that the variation may have different values on the same meridian and that its amount depends on the proximity of land. These are ideas which were no doubt borrowed from Gilbert's work.

Thus in Holland the controversy had not yet been decided. Plancius continued to propagate his doctrine and he induced navigators to take his instruments with them on their voyages. Stevin had *The Haven-Finding Art* reprinted and included in his *Wisconstighe Ghedachtenissen,* in a Dutch and in a Latin edition. Steering the same course as regards this doctrine was Keteltas, but it was condemned by Hondius, Alb. Haeyen, Blaeu, and Metius. It was the data collected at sea which led to a better understanding. Stevin was among those who insisted that frequent observations be carried out. This advice was hammered into navigators both in Holland and elsewhere, and to this day "taking an azimuth" is among the actions performed by them several times a day. The object is no longer to determine the variation, but to compute the declination of the compass-needle from the magnetic meridian — the deviation, which is due to the ship's iron — *i.e.* still in order to be accurately informed about the direction in which the ship is moving.

Though we will not mention further Portuguese, Spanish, French, and English authors who did not believe in a relation between variation and longitude, or who hoped to find it [103]) or believed that it did exist — such an enumeration would

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[102]) A. M. Dermul, *De aard- en hemelgloben van W. Jansz Blaeu in de Antwerpsche Bibliotheek.* Antwerp 1940.

[103]) William Borough, *A Discourse of the Variation of the Cumpas or Magneticall Needle.* 1585. Also in an edition of 1596, Ch. 10.

fall outside the scope of this introduction — we wish to draw attention to two important works, which we cannot pass by in the context of this section.

The first was written by Guillaume de Nautonier, Sieur de Castelfranc en Languedoc, and is entitled *Mécometrie de Leymant* [104]); it is a very extensive work. His system is similar to that of Mercator. He assumes two magnetic poles, one in each hemisphere, in latitude 67°. On the meridian which passes through these two as well as the geographic poles and on which also the island of Ferro is situated, there is no variation of the compass. The tables of the variation given by the author were questioned by some of his contemporaries. Like this author, Kepler long continued to cherish the hope that there would be a simple correlation between the network of meridians and parallels on the earth and the magnetic network.

Fournier, in his voluminous book *Hydrographie* [105]), devotes a long chapter to the determination of longitude, *i.e.* including the method relying upon the variation of the compass (*Livre* XII, *chap.* XXXIV, pp. 606—608). After having uttered his criticism, he concludes: *"c'est donc une folie de s'amuser à chercher les longitudes par telles voyes"*. The matter is thus dismissed with good-natured mockery.

The second of the books referred to above was published in England.

The work in question is that about "the magnet, magnetick bodies also and on the great magnet the earth", by William Gilbert [106]) (born at Colchester in 1544; died 1603), a book that has gained fame in the history of science. It appeared in 1600, *i.e.* very shortly after Stevin's *Haven-Finding Art*. The author was a medical practitioner and a physicist. He engaged for many years in empirical scientific investigations, and he took for granted only those things which had been confirmed by reliable tests. Edward Wright wrote an encomiastic preface for the book, in which he styles the author the "father of magnetick philosophy". It is not surprising that this book contains a chapter entitled "whether the terrestrial longitude can be found from the variation" (Book IV, Chapter IX), and that the name of Stevin and his *Haven-Finding Art* are mentioned in it. Whilst Gilbert calls the values of the variation for the first "segment" from longitude 0 to 60° "in some part true", he disputes the view that the variation is zero all over the meridians through Corvo and Hjelmsöy, or 13°24' over the whole length of that through Plymouth. The value zero does hold at the island of Corvo, but not in other latitudes. He concludes his discussion with the following passage:

> "Consequently the limits of variation are not conveniently determined by means of great circles and meridians, and much less are the ratios of the increment or decrement toward any part of the heavens properly investigated by them. Wherefore the rules of the abatement or augmentation of northeasting or northwesting or of increasing or decreasing of the magnetick deviation, can by no means be discovered by such an artifice. The rules which

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[104]) Guillaume de Nautonier, Sieur de Castelfranc en Languedoc, *Mécometrie de Leymant, c'est à dire la manière de mesurer les longitudes par le moyen de l'eymant .... aussi facilement comme la latitude. Imprimé à Venes chez l'autheur.* 1603.

[105]) Georges Fournier, *Hydrographie, contenant la théorie et la practique de toutes les parties de la navigation.* Paris 1643.

[106]) *Guilielmi Gilberti Colcestrensis, medici Londinensis, De magnete, magneticisque corporibus et de magno magnete tellure; Physiologia nova, plurimis & argumentis & experimentis demonstrata. London* 1600.

follow later for variation in southern parts of the earth investigated by the same method are altogether vain and absurd. They were put forth by certain Portuguese mariners, but they do not agree with the observations and the observations themselves are admitted to be bad." (Translation by S. P. Thompson)

Once again it appears that the values of the variation as determined in actual practice overthrew the theory. The passage ends with the following curious conclusion:

"But the method of haven-finding in long and distant voyages by carefully observed variation (such as was invented by Stevinus and mentioned by Grotius) is of great moment, if only proper instruments are in readiness, by which the magnetick deviation can be ascertained with certainty at sea."

The honourable place assigned to Stevin in the preceding pages is thus confirmed by his contemporary.

e. *THE EVOLUTION OF THE SUBJECT IN THE 17th AND 18th CENTURIES.*

It was to be expected that those who collected and studied the values of the variation as observed by mariners on their voyages should indicate the figures on a chart, in the places where such observations had been taken. This was the method by which a convenient survey could be obtained. The oldest of such "variation charts" is now said to be that of Alonso de Santa Cruz, referred to by him in his *Libro de las Longitudines*. It must therefore date from about 1535 or 1540. Its value was shaken by the findings of De Castro.

Athanasius Kircher (1601—1680) in his great work on magnetism [107]) mentions a "mappa geographico-magnetica", made by Christophorus Burrus or Cristoforo Borri, an Italian Jesuit priest, who lived in Lisbon and in the Portuguese colonies.

Sir Robert Dudley (1573—1649) in his marine atlas *Arcano del mare* [108]) shows the variation in many places in his charts. Five such figures are found on the chart of the North Sea, from Dover and Nieuwpoort to the mouth of the river Weser.

Further a magnificent large chart on Mercator's projection is to be found in the third edition of Wright's *Certaine Errors in Navigation,* published in 1657. It bears the title "A plat of all the world, projected according to the truest rules", and it had been revised and corrected, as had the book, by Jos. Moxon. It is dated 1655. In a cartouche we read the following words:

"The numbers scatteringly dispersed here and there in this sea chart signifie the variation of the compasse. The letters E and W shewe whether it be East or West. The other letters following signifie the observers names, as D Davis, K Kendal, H Hall, L Lynschot, C Candish, CA John de Castro, etc."

On the west coast of Novaya Zemlya is to be found an observation marked WB, which means of course: Willem Barents.

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[107]) Athanasius Kircher, *Magnes sive de arte magnetica.* Rome 1641 (as well as Cologne 1643).

[108]) Robert Dudley, *Arcano del mare.* Florence. - 1st ed. 1646-1647, present in the University Library, Leiden. - 2nd ed. 1661, present in the Netherlands Historical Maritime Museum, Amsterdam.

An unprecedented increase of knowledge was due to the mathematician and astronomer Edmund Halley (1656—1742), who during long ocean voyages carried out observations and published their results — as he had found them in 1700 — in a large map of the world on Mercator's projection [109]). The map was destined for navigators. In the seas it showed the isogonics, with the exception of those in the Pacific. For the benefit of the user, Halley added an elaborate explanation to the map. To navigators a knowledge of the variation is of the utmost importance if they are to be able to determine their course accurately at moments when weather conditions do not permit them to determine the variation for themselves by observation. With respect to the possibility of determining longitude by means of the chart, the explanation says as follows:

"A further use is in many cases to estimate the Longitude at sea thereby: for where the curves run nearly North and South, and are thick together, as about C. Bonne Esperance, it gives a very good indication of the distance of the land, to ships to come from far: for there the variation alters a degree to each two degrees of longitude nearly, as may be seen in the Chart. But in this Western Ocean, between Europe and the North America the curves lying nearly East and West, cannot be serviceable for this purpose."

In view of the slow change to which the variation is subject, which necessitates revision of the chart, Halley requests the cooperation of navigators. Observations will be gratefully acknowledged by him.

The chart had a wide diffusion. In Holland it was included in the *Atlas van Zeevaert en Koophandel* by Louis Renard, Amsterdam 1745. In France it was also known. In England the investigations were continued, and variation charts were compiled by other authors as well. Such charts were also to be found in Holland. Nautical handbooks pointed out the usefulness of these charts.

How Halley's advice about navigation in the vicinity of the southern point of Africa was applied in practice can be learned, as far as Holland is concerned, from the *Zeilage-Ordre, om ten allen tijde van Straat Sunda over Kaap de Goede Hoop naar Nederland te zeilen* (Sailing direction, for sailing at any time from Sunda Strait via the Cape of Good Hope to the Netherlands), issued by the East India Company to their ships, which instructions were approved in 1783. It is stated there:

> "From the longitude and latitude just mentioned, head west by south so as to pass at about 30 miles beyond the shoals lying at the southern end of Madagascar, up to longitude 61° to 62°, where there is now no greater variation than 23° to 24° northwesting.
>
> From there again steer west-south-west, in order to catch sight of the coast of Africa, in the neighbourhood of Punto de Fontes or Algoa Bay. In this channel between Madagascar and Punto de Fontes the greatest variation is found to be between 26° and 27°, and as one approaches the land of Africa or the aforesaid Punto de Fontes, it will decrease to 23°30' to 23°northwesting. Subsequently by repeated sounding try to find the Reef of Agulhas, so as to call at the Cape of Good Hope or Bay False, as you shall have been ordered in the Instructions."

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[109]) *Nova et accuratissima totius terrarum orbis tabula nautica variationum magneticarum index. Juxta observationes anno 1700 habitas constructa per Edm. Halley.*

When in 1754 Cornelis Douwes (1712—1773) published in a treatise [110]) his method of determination of latitude, which became internationally known, he spoke with some reservation about this method of determination of longitude. It was only "in some regions" that by means of the variation one "had roughly some certainty whether one was east or west of a known place", when after a long voyage the estimated longitude had become very unreliable. This was contrasted with the view of Pybo Steenstra, lecturer of mathematics, navigation, and astronomy in the "Atheneum Illustre" of Amsterdam. When in 1770 the latter published his lessons on the finding of longitude at sea, he referred to the method based on the variation of the compass, among the other possibilities, as "so far the best and readiest means of finding the true longitude at sea", provided it be applied in a place where the declination varied rapidly with the ship's movement [111]). None of the existing methods for the determination of longitude produced a reliable result in those days. The development of this problem was very backward as compared with the accuracy of the determination of latitude, which had been enormously advanced by Hadley's invention of the octant in 1731. The result was now reliable to within a few minutes of arc. We can thus sympathize with Steenstra when on 21st November 1763, upon assuming his office as a lecturer, he exclaimed in his inaugural address:

> "Would that it were possible to find the true longitude at all times with the same certainty and ease; then the art of navigation would attain to a degree of perfection which it now with good reason despairs of ever reaching."

The speaker was taking too gloomy a view. At that very time the marine chronometer was being constructed and the results obtained with it were promising. The method of determining longitude by means of lunar distances was being developed. The "Nautical Almanac" needed for the computation was soon to appear. The new instrument for measuring angles permitted a reasonable measurement of distances. The mathematical knowledge of the sailor was brought to a higher level with a view to all this.

For the determination of longitude on the basis of the variation of the compass the knell had sounded. This subject, at which scholars as well as practicians had worked for nearly 300 years, had become a thing of the past.

§ 6

APPENDIX

a. *THE CONSTRUCTION AND THE USE OF THE ASTROLABIUM CATHOLICUM.*

Round about 1600 spherical trigonometry was a subject far above the heads of navigators. And yet they had to do with problems of spherical trigonometry, such as the computation of longitude according to Plancius and the determination of the distance on a great circle between two points on the earth. The ways in which

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[110]) *Verhandeling om buiten den middag op zee de ware middagsbreedte te vinden.* Verhandelingen Hollandsche Maatschappij der Wetenschappen, Haarlem 1754, p. 146.

[111]) Pybo Steenstra, *Openbaare lessen over het vinden der lengte op zee.* Amsterdam 1770, p. 45.

they were enabled to solve the latter problem appear very clearly from a legend on the big map of the world of Jodocus Hondius of 1611 [112]).
There were three methods:

1. by projection and construction of plane triangles
2. with the aid of the *astrolabium catholicum*
3. by measurements on the globe.

Directions for all three methods are given, and a fine and clear illustration of

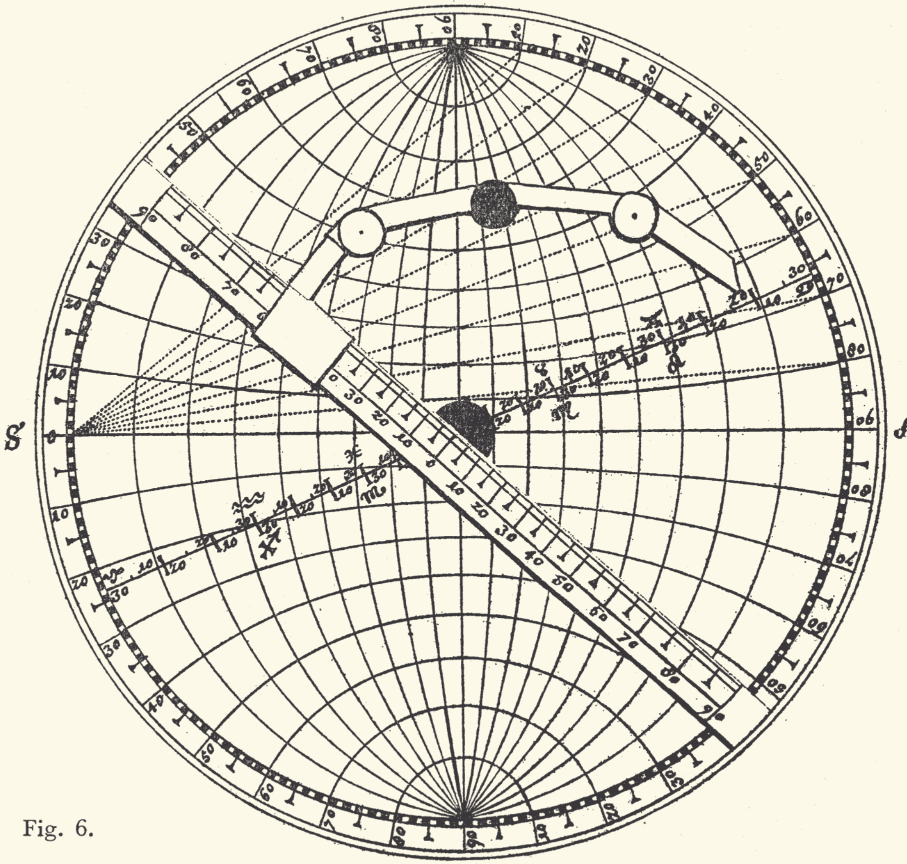


Fig. 6.

Illustration of the network of meridians, parallels, etc., on the astrolabium catholicum, taken from the manuscript of the surveyor Jan Nanninghssoon at Broek-in-Waterland, 1647, present at the State Record Office of North Holland, Haarlem.

[112]) *World Map by Jodocus Hondius* 1611. Edited by Edw. L. Stevenson and J. Fisher. New York 1907.

the *astrolabium catholicum* is included. The instrument generally serves to solve problems of spherical trigonometry, which is done by measurements in a plane. It may be looked upon as a variant of the astronomical astrolabe, which had been known for many centuries past.

The instrument consisted of a flat disk on which was drawn a circular network of lines, being the network of meridians and parallels in equatorio-stereographic projection. Because the centre of projection has been chosen in the equator, the projection of the latter is a straight line, at the same time the diameter in the figure. The meridians meeting in the two poles — at a distance of 90° from the equator — are parts of arcs of a circle, and so are the parallels. In fact, we here have to do with the peculiar property of this projection, that circles on the globe become circles in the projection. Its other special feature is that an angle between curved lines on the globe is equal to the angle between their stereographic projections (Fig. 6).

Adapted to turn about a spindle in the centre of the figure was a rule, the edge of which could be made to coincide with the equator. It carried a division which was identical with that on the equator produced by its points of intersection with the meridians. Along the rule could be moved a small block, which was equipped with a metal point consisting of a number of relatively movable links. By means of this construction it was possible to place the end of this point over any given point of the *rete,* which was necessary in the manipulation.

No specimens of the *astrolabium catholicum* are known. We merely find illustrations of it in various places, such as the title-page of Willem Janszoon Blaeu's atlas *Licht der Zeevaart* of 1608 and the title-page of the marine atlas *De groote lichtende ofte vijerighe colom* (The Lighting Colomne or Sea-Mirrour) of Jacob Aertsz. Colom, Amsterdam 1661. The latter has an illustration of a navigation lesson given in a church, where the teacher from the pulpit — it may be assumed with good reason that it is Plancius who is shown here — hands the instrument to one of his pupils standing at the foot of the pulpit. At the Rijksmuseum, Amsterdam, among the relics of the wintering of Willem Barents in Novaya Zemlya (1596—1597) there is to be found a specimen of the above-mentioned block with the point consisting of links. This proves that an *astrolabium catholicum* formed part of the nautical equipment of the expedition.

For an explanation of the way the instrument was used, further particulars, and illustrations the reader is referred to the present author's article in the maritime review *De Zee,* 1916, p. 180:
„Het gebruik van het Astrolabium Catholicum", and further to the following three works edited by the Linschoten Vereeniging:

Vol. XV *Reizen van Willem Barents,* etc. 1917, pp. XXI ff.
Vol. XXXII *De eerste schipvaart,* etc. 1929, pp. 433 ff.
Vol. XLIV *De tweede schipvaart,* etc. 1940, p. XXXIV.

b. *THE CONSTRUCTION AND THE USE OF THE LONGITUDE-FINDER OF PLANCIUS*

Only one specimen of this instrument has been preserved. It is also present at the Rijksmuseum among the relics of the wintering of Willem Barents in Novaya Zemlya (Fig. 7).

It is further illustrated in the above-mentioned Works of the Linschoten

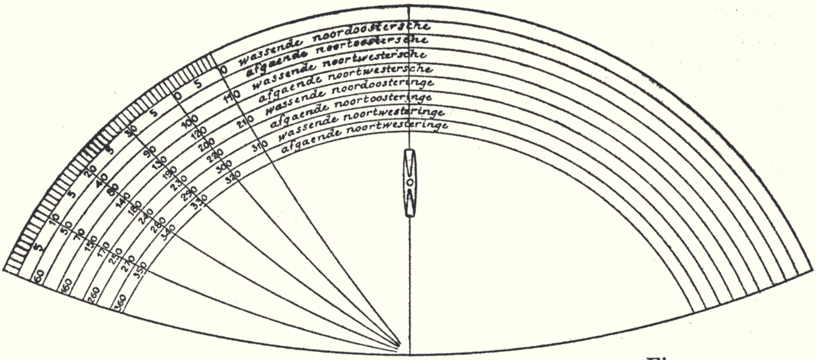


Fig. 7

Longitude-finder of Peter Plancius (Rijksmuseum Amsterdam).

Vereeniging, Vol. XV, p. XXV and Vol. XXXII, p. 436. It is nothing but a flat copper plate, 21.5 cm long, bounded by two arcs of a circle, and it is provided with a number of lines engraved in it. The radius of the circular edge of the

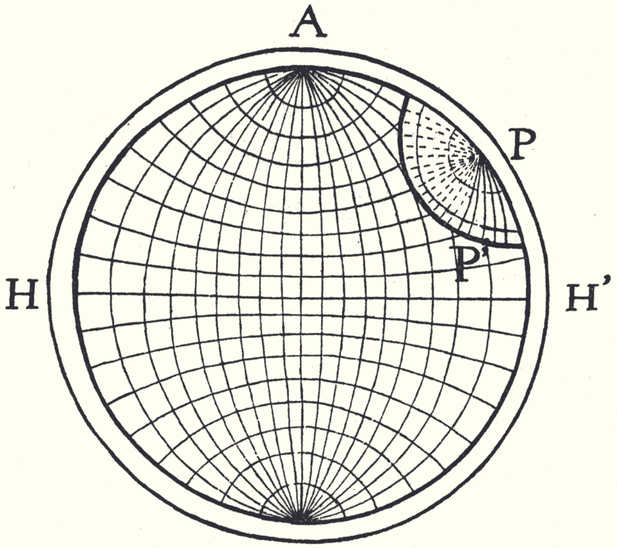


Fig. 8.

This figure shows how the longitude-finder of Plancius has to be used in conjunction with the astrolabium catholicum. A is the place on the earth for which the variation and the latitude are known, while the longitude has to be found. The centre of the division on the longitude-finder is the North Pole. Moreover

$$AP = 90° - \text{latitude } A.$$

On the division of the longitude-finder the point P', where the magnetic meridian through A intersects the edge of the longitude-finder, indicates the difference of longitude between A and the nearest agonic.

plate through construction is found to be 263 mm. The plate was intended to be placed against the edge of the *astrolabium catholicum* and to fit against it (Fig. 8).

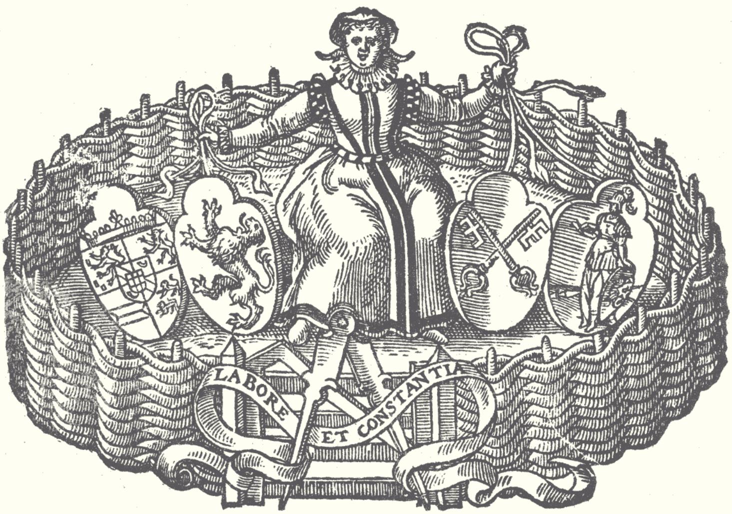
The two instruments accordingly had the same radius. The longitude-finder represents the polar cap of the astrolable and the engraved curved lines are merely parts of meridians on the *rete* of the astrolabe. The second arc-shaped edge of the plate is a parallel of latitude, *viz.* the one on which Plancius assumed all points of intersection of agonics and magnetic meridians to fall. The terrestrial latitude of this locus was an important feature in his theory. Through construction it appears that this latitude must have been slightly less than 65°30'. This value can be confirmed as follows.

The objects at the Rijksmuseum referred to above include a sheet of paper on which a circular *rete* of the same kind as that on the astrolabe is printed. This sheet of papier lay for nearly 300 years in the ramshackle hut in Novaya Zemlya; it was brought to the Netherlands and smoothed as well as possible, and is now preserved between cardboard and a glass plate. It is obvious that the paper has been subject to warping. Nor is the *rete* complete. In spite of all this, in spite of folds and tears, the edge of the longitude-finder still fits tolerably well against the edge of the *rete* on the paper. The other edge falls on a parallel of 65° to 65°30'. How did Plancius arrive at this figure? Did he choose it because the results of his work were thus most satisfactory, or did he take it perhaps from William Borough, who in his *A Discourse of the Variation of the Cumpas or Magneticall Needle,* London 1596, in the 8th chapter speaks about the magnetic pole being situated at a distance of 25°44' from the geographic pole?

When L'Honoré Naber, editor of Vol. XV of the Works of the Linschoten Vereeniging, verified the calculations of Plancius, he assumed that the latter had taken the polar circle for this parallel. As a locus, the polar circle in his opinion was "in a sence cosmically hallowed". Although Naber's results agreed satisfactorily with those of Plancius, the dimensions of the plate show that a small correction has to be made. This does not of course affect the existing view of the theory advocated by Plancius.

Plancius was not the only one to teach the use of these two instruments and to equip navigators with them in order that they might use them on board. Yet another teacher of navigation may be mentioned on whose programme the same subject figures. We are referring to " Jan van den Brouck, a professional navigator in the famous commercial town of Rotterdam", writer of *Instructie der Zee-Vaert,* a small book published in 1610, which was entirely adapted to the needs of the simple would-be navigator. He devotes a good deal of attention to the *astrolabium catholicum,* the practical instrument "by means of which one can do anything that one could do with a celestial or terrestrial globe". By way of example he works out a great many problems, in all sorts of fields. "In this way one will soon be a master in handling the astrolabe." He also works out examples with the longitude-finder, under the heading: "an example of how by means of the plate one can find the longitude according to the method of D.P. Plancius."

DE HAVEN-VINDING.



TOT LEYDEN,
IN DE DRVCKERYE VAN PLANTIIN,
BY CHRISTOFFEL VAN RAVELENGHIEN,
Gesworen drucker der Vniuersiteyt tot Leyden.
cIↄ. Iↄ. IC.
Met Priuilegie.

Priuilegie.

Opden achthienden Martij anno XV^c negenentnegentich, hebben die Staten generael der Vereenichde Nederlanden geconsenteert, ende gheoctroyeert, Consenteren, ende Octroyeren mits desen, Christoffel Raphelengius Bouckdrucker tot Leyden, alleene binnen den tijdt van zesse jaeren naestcommende, inde voors. Vereenichde Nederlanden te mogen drucken, doen drucken, uytgeuen, ende vercoopen, zeker bouck geintituleert Haven-vindingh, welcken hy van meyninghe is, soo int Latijn als int Franchois, ende Duytsch, oft oyck in ander spraken te laten uytgaen, Interdicerende, ende verbiedende allen ende een ygelyck den voors. bouck, int geheel oft int deel, in enigerhande spraken na te drucken, noch oyck buyten de Vereenichde Nederlanden nagedruckt, inde selue uyt te geuen, oft vercoopen, sonder consent vanden voors. Raphelengius, opte verbeurte vande naghedruckte Exemplaria, ende de boete in't Originael verhaelt, &c.

Sloeth.^{vt.}

Ter ordonnantie vande zelue
Heeren Staten generael.

C. Aerssen.

PRIVILEGE.

On the eighteenth of March of the year 1500 and ninety-nine the States General of the United Netherlands consented and granted a patent, and they consent and grant a patent by the presents, to Christoffel Raphelengius, printer at Leyden, only within the time of the next six years, to print, have printed, publish, and sell in the aforesaid United Netherlands a certain book entitled *Haven-vindingh,* which he intends to publish both in Latin and in French and Dutch, or also in other languages, prohibiting and forbidding one and all to reprint the aforesaid book, wholly or in part, in any language, or, if reprinted outside the United Netherlands, to publish or sell it in the said country without the consent of the aforesaid Raphelengius, on pain of confiscation of the reprinted copies and the penalty mentioned in the original, etc.

Sloeth.

At the decree of the said States General.

C. Aerssen.

DE HAVENVINDING.

TIS kennelick datmen over langhen tijt, voornaemlick ſedert dat de groote zeevaerden op Indien en America begoſten, middel geſocht heeft, waer deur den Stierman op zee mocht weten, de eertrijcxlangde der plaets daer teghenwoordelick ſijn ſchip is, om alſoo te commen totte havens daer hy begeert te weſen, ſonder datmen alſnoch tot ſulcke ghewiſſe kennis der langde heeft connen ghecommen: want ſommighe verhopende die te vinden deur de verſcheenwyſing der zeylnaelde, hebben de ſelve verſcheenwyſing een * aſpunt toegheſcreven, die noemende † ſeylſteens aſpunt. maer men bevint na wyder ervaringhen, dat die afwijckinghen ſich na gheen aſpunt en ſchicken. Doch ſo heeft nochtans het ſoucken van dien, middel veroirſaect om tot een begeerde hauen te gheraken, niet teghenſtaende des havens en ſchips ware langden beyde onbekent ſijn. Om twelck eerſt by voorbeelt te verclaren, en daer na te beſchrijven de omſtandighen deſen handel aengaende, waer deur de ghebruyck noch gemeender en ſekerder ſal worden, ſoo is voor al te weten, datmen deur ervaring bevint, de zeylnaelde tot verſcheyden plaetſen (hoewel op gheen ſeylſteens aſpunt reghel houdende ghelijck gheſeyt is) ſeer verſcheydelick te wijſen, als tot ſommighe oirten recht

* *Polum.*
† *Polum magnetis.*

THE HAVEN-FINDING ART.

It is known that for a long time past, principally since the great voyages to the Indies and America began, a means has been sought by which the navigator might know at sea the longitude of the place where his ship is at the moment, in order thus to get to the harbours to which he wishes to go, but that hitherto it has not been possible to arrive at such accurate knowledge of the longitude. For some people, hoping to find it through the variation of the compass, ascribed a pole to the said variation, calling it magnetic pole, but it is found upon further experience that these variations do not obey a pole. Nevertheless the search for this has furnished a means for reaching a desired harbour, even though the true longitudes of both the harbour and the ship are unknown. In order that this may first be explained by means of an example and then the circumstances of the method may be described, as a result of which the application will become even more general and certain, first of all it is to be noted that it is found by experience that the magnetic needle (though it does not obey a magnetic pole, as has been said) points very differently in different places, to wit, in some places due North, in

Noort, tot ander wyckſe na t'Ooſten, elders na t'Weſten, welcke veranderinghen alſmen van t'Ooſten na t'Weſten treckt, op kleyne weghen ſeer merckelick ſijn; als bij voorbeelt t'Amſterdam wyckſe na t'Ooſten 9. * trappen 30. Ⓘ. An t'voorlant van Enghelant 11. tr. Te Lonnen 11. tr. 30. Ⓘ. By Timouth in zee 12. tr. 40. Ⓘ. en ſoo voorts.

* *Gradus.*

Hoemen een haven of landt vindt, daer af de breede en naeldwijſing bekent is.

SVLCKE naeldwyſing, metſgaders de breede der plaetſen bekent ſijnde, deur ervaring der ghene diet metter daet alſoo bevonden hebben, men can daer me ſonder langde te weten de plaets vinden. Als by voorbeelt, an een Stierman bekent ſijnde, dat de breede van Amſterdam is 52. trappen 20. Ⓘ. met naeldwycking na t'Ooſten van 9. tr. 30. Ⓘ. ende dat hy hem vindt op zee inde ſelve breede van 52. tr. 20. Ⓘ, mette voorſcreven Ooſterſche naeldwycking van 9. tr. 30. Ⓘ. Hy weet dat hy ontrent Amſterdam moet weſen, t'ſij mette langde van Amſterdam hoet wil. Aengaende ymant mocht ſeggen; datter wel noch ander plaetſen ſijn vande ſelve breede en naeldwycking, nochtans niet Amſterdam: Tis waer, maer ſij vallen ſeer verre van daer, ende can uyt dander onderkent worden, deur ſeker omſtandighen, van welcke wy hier na ſegghen ſullen. Merckt noch dat hoewel de Stierlieden Amſterdam anders connen vinden deur omligghende others it declines towards the East, elsewhere towards the West, which differences are very noticeable for small distances in going from the East to the West; thus, for instance, at Amsterdam it declines 9°30′ towards the East. Off the Foreland of England 11°, at London 11°30', off Tynemouth in the sea 12°40′, and so on.

How a Harbour or Land is found whose Latitude and Needle-Pointing are known.

The needle-pointing as well as the latitude of the place being known through the experience of those who have found it so in practice, the place can be found by this means without the longitude being known. Thus, for instance, if a navigator knows that the latitude of Amsterdam is 52°20′, with an easterly variation of 9°30′, and he is at sea in the said latitude of 52°20′, with the aforementioned easterly variation of 9°30′, he knows that he must be off Amsterdam, whatever may be the longitude of Amsterdam. If anyone were to say that there are also other places having the same latitude and variation, which yet are not Amsterdam, this is true, but they are very far away from it and it can be distinguished from the others by certain circumstances, which we shall discuss hereafter. Note also that though navigators are able to find Amsterdam ghende landen, giſſing, diepten, ſanden, en ander teyckens, ſonder acht op naeldwyſing te nemen, nochtans hebben wy dat voorbeelt van die bekende plaets gheſtelt, om daer deur te opentlicker te verclaren de ghemeenheyt vande reghel op verre ſeylagen, daermen op langhe tijt gheen landt en ſiet: Als, neem ick, een Stierman begeerende van hier te ſeylen tot Cabo Sant Auguſtijn in Braſilie, ende wetende dat de naeldwycking daer is, van (ghelyckmen ſegt) noort na t'ooſten 3. tr. 10. ⊙, en de zuyderſche breede 8. tr. 30. ⊙. als hy derwaert varende tot ſulcken naeldwycking en breede ghecommen is, hy weet hem ontrent Cabo Sant Auguſtijn te weſen: Ende hoewel giſſing hem anders dede vermoeden, ſal die verlaten, als deur ooſterſche of weſterſche verborghen ſtroomen bedroghen ſijnde, of qualick ghegiſt hebbende: Want dat de naeldwycking die eertijts tot Cabo Sant Auguſtin was 3. tr. 10. ⊙. nu daer niet weſen en ſoude, de reden en laet niet toe ſich ſulcx voor te ſtellen om daer op te werck te gaen: Of dat ymant op zee een ander naeldwijſing vindt dan de voorſcreuen, die hy weet tot Cabo S. Auguſtin te ſijn van 3. tr. 10. ⊙. ende nochtans willende d'ervaring der naelde verlaten, en giſſing volghen, ſich ſeyde ontrent Cabo S. Auguſtin te weſen, wie en verſtaet niet ſulcx ſonder reden te ſijn? als van een die ſich ſelfs teghenſpreect, ſeggende die naeldwijcking aldaer te ſijn van 3. tr. 10. ⊙. ende ſoo niet te weſen.

Merckt wijder wel ghebeurt te ſijn, dat eenen ſeylende na het Eylant van Sint Helena, ende gecommen weſende tot des ſelven Eylants breede, nochtans dat in other ways, from surrounding lands, by conjectural reckoning, depths, sands, and other signs, without paying heed to the needle-pointing, we have nevertheless given this instance of a known place in order thus to set forth more manifestly the general applicability of the rule during long voyages, when no land is seen for a long time. Thus, if a navigator, desiring to sail from hence to Cape St. Augustine in Brazil and knowing that the variation is there (as is said) 3°10′ to the east of the true north and the southern latitude 8°30′, in sailing in that direction has come to this variation and latitude, he knows he is off Cape St. Augustine. And even if conjectural reckoning made him suppose otherwise, he must disregard this, assuming that he has been deceived by unknown eastern or western currents or that he has guessed wrongly. For reason does not permit him to imagine that the variation which was previously 3°10′ off Cape St. Augustine should not have this value now [1]), and to proceed accordingly. Or if a man finds at sea another variation than the above, which he knows to be 3°10′ off Cape St. Augustine, and nevertheless, wishing to disregard the observation of the needle and to rely on conjecture, were to say that he was off Cape St. Augustine, who does not deem this to be an unreasonable procedure, like that of a man who contradicts himself, saying that this variation is 3°10′ there and that it is not.

Note further that it has sometimes happened that a man, sailing to the Island of St. Helena and, having come to the latitude of this island, yet not finding this

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[1]) Stevin here assumes that the variation holding good for a given place is invariable. However, it is now well-established that the variation at any one place is slowly changing with time.

Eylant daer niet vindende, oock niet wetende of hyder ooſt of weſt af was, heeft al ramende ooſtwaert gheſocht, dat weſtwaert lach, ende hoe hy verder alſoo voer, hoe hy verder vande begeerde plaets gerocht: Denckt nu eens, ſoo dien Stierman (die wel ettelicke weken lanck dat Eylant ſocht, ende ettelicke mael daer rontom voer eer hyder in gherocht) had bekent gheweeſt hoe de naelde op Sint Helena wees, ende daer beneffens wetenſchap ghehadt vande naeldwyſing op zee te vinden, of hy moetwillichlick na een grooter naeldwijcking ſoude ghevaren hebben, wetende dat de plaets daer hy begeerde te weſen een kleender hadde?

Hierby machmen verſtaen hoe noodich de kennis der naeldwijſing is: Te meer dat de gene die met wetenſchap der * ſeylſtreken wil varen (twelck den Stierman op groote ſeylaghen niet en behoort onbekent te ſijn) over al het recht noort moet weten, welck noort op zee deur kennis der naeldwijcking ghevonden wort.

** Rumbi wordenſe byde Portugeſen ghenoemt.*

Soomen hier beneffens noch inſiet de onſekerheyt vande ware plaetſen der landen, die na tſegghen der Stierlieden op de eertſclooten gheteyckent worden, ſpruytende daer uyt, datſe het wijſen der leli die elck van huys brengt, altijt voor noort houden, ſtreckende daerenboven noch tot meerder onſekerheyt in haer ſeyling: Men ſal verſtaen het gaſlaen der naeldwijſinghen daer in oock ſeer oirboir te weſen, wantmen deur zeecompaſſen daer toe bereyt, de leli al ſeylende altijt recht noort can doen wijſen, midts de naelde of

t'beſtreisland there nor knowing whether he was to the east or to the west of it, by conjecture sought to the east what lay to the west, and that the further he thus sailed, the further he got from the desired place. Now just consider whether this navigator (who sought this island for several weeks and sailed several times around it before he got there), if he had known how the needle pointed off St. Helena and in addition had known how to find the variation at sea, would deliberately have sailed to a place where the variation is greater, though he knew that the place to which he desired to go had a smaller one?

From this it may be understood how needful the knowledge of the variation is, especially since those who wish to be certain of the course they are following (which ought not to be unknown to the navigator during long voyages) have to know everywhere the true north, which is found at sea by knowledge of the variation.

If further the uncertainty is also recognized of the true positions of the lands which are drawn on the globes according to the information of navigators, which uncertainty results from the fact that they always think the point indicated by the fleur-de-lys which each of them brings from home to be the true north, which moreover leads to greater uncertainty in sailing, it will be understood that the observation of the needle-pointing is also very useful in this respect because it is possible by means of mariner's compasses [1]) prepared for this purpose to make the fleur-de-lys point always due north during the voyage, provided the

[1]) For the construction of such compasses, see the Introduction, § 3b, p. 369.

t'bestreken ijser, soo veel vande leli te verdraeyen, als de saeck vereyscht.

Dit alles wel anghemerct, ende toegelaten wesende verscheyden landen sulcke verscheyden naeldwijsinghen te hebben, gelijck deur ettelicke betuycht wort, het schijnt dat de ghene die niet toe en staen, deur t'behulp der selve naelwijsing de seylage te connen bevoordert worden, of datse de saeck niet en verstaen, of wat anders daer teghen weten dat yghelick niet bekent en is.

Nu alsoo Sijn EXCELLENTIE de voorgaende saken rijpelick overdocht hadde, ende sich inghebeelt meughelick te sijn, de bovescreven voordering der seylaghe hier deur merckelick te connen geschien, heeft als Admirael vander zee, ande Admiraliteyt seker oirden ghestelt, ende onderwijs ghegheven, om te weghe te brengen dat de Stierlieden op sulcke reysen varende, hun daer na ghevougen: Namelick datse van nu voortaen tot veel plaetsen daerse commen, metter daet ende wel sorchvuldelick, ondersoucken de afwijckinghen der seylnaelde vant noorden, nemende daer toe reetschap wel bequaem: Ende van haer reysen weerghekeert sijnde, daer af ghetrouwelick verwitting doen ande voorscreven Admiraliteyt, welcke de selve ervaringen sullen doen in oirden stellen, ende ten ghemeenen oirboire an yghelicken openbaer maken.

Maer op dat elck die wil noch opentlicker verstaen mach alle omstandighen dese saeck aengaende, soo sullen wy hier stellen een begin, van t'ghene men

deur

needle or the magnetized iron is turned away from the fleur-de-lys as much as is required.

All this being considered and it being admitted that different countries have such different needle-pointings, as is testified by many people, it seems that those who do not admit that navigation can be advanced with the aid of the said needle-pointing either do not understand the matter or know something else to the contrary, which is not known to everyone.

When therefore His Excellency had thoroughly considered the above matters and conceived that it was possible for the above-mentioned advancement of navigation to be appreciably effected by this means, as Lord High Admiral he gave order and instruction to the Admiralty to see to it that navigators going on such voyages should act accordingly, namely, that henceforth in many places where they come they should find out actually and very carefully the variations of the needle from the north, using very suitable instruments for this, and upon their return from their voyages should faithfully report the results to the aforesaid Admiralty, which will cause these observations to be listed and published for the use of all.

But in order that anyone who wishes may understand more clearly all the circumstances relating to this matter, we shall here set down the principle of that deur breeder ervaringhen in wille is voorder te vervolghen, tafelwijs vervatende de naeldwijsighen dieder alree gagheslaghen sijn, welck den hoochgeleerden * Eertrijcxschrijver Heer Petrus Plancius, deur langdeurighen arbeyt, en niet sonder groote costen by een vergaert heeft, uijt verscheyden houcken des eertbodems, soo wel verre als na gheleghen: Sulcx dat als de Stierlieden int ghemeen deur dese manier landen en havens sullen vinden, soo wel als eenighe int besonder die alree ghevonden hebben, den selven Plancius ghehouden mach worden voor een der voornaemlickste oirsaken van dien. De voornoemde tafel waer af breeder verclaring ghedaen sal worden is als volght.

Verclaring op de navolghende Tafel.

EER wy commen totte verclaring, willen voor al segghen, dat by aldien namals deur nauwer en sekerder ervaringhen, der plaetsen naeldwijsingen, breeden en langden, anders bevonden wierden dan inde tafel staet, ende datmen alsdan ander manier van verclaring en bepaling van woorden behoufde, dan de volghende, dat ons sulcx van t'voornemen deser ondersoucking niet en behoort af te keeren, maer veel eer daer toe te trecken, als allencx gerakende tot meerder en sekerder kennis eens handels ghesticht op sulcken gront als vooren verclaert is. Dese meyning volghende, wy sullen mettet waerschijnlickste dat ons nu be-which it is desired to continue further by means of wider experience, listing in a table the variations that have already been observed, which the learned geographer Mr. Petrus Plancius has collected by protracted labour and not without great expense from different corners of the earth, both far and near, so that, if navigators shall find lands and harbours generally in this way, as some in particular have already found them, the said Plancius may be considered one of the principal causes of this. The aforesaid table, a more detailed explanation of which will be given, is as follows.

Explanation of the Following Table.

Before we come to the explanation, we wish to say first of all that if afterwards, by more accurate and more exact observations, the needle-pointings, latitudes, and longitudes of the places should be found to be different from those in the table, and if in that case another way of explanation and definition of words should be required than the following, this ought not to keep us from undertaking this investigation, but rather to incite us to it, so that we may gradually attain to greater and more exact knowledge of a method based on the foundation explained above. Following this opinion, we shall proceed with the most probable knowledge nu bekent is voortvaren, al oft warachtich waer; want elck tsijnder tijt der ghelijcke doende, men sal twarachtichste dat inde nateur daer af is, allenx naerder en naerder meughen gheraken.

Dit soo sijnde, ende om nu tot verclaring der tafel te commen, soo sietmen voor al datter sijn drie pilaren, deerste van der plaetsen naeldwijsingen, de tweede vande breede, waer by noch ghevoucht is de derde vande gheraemde langde, op dat de plaetsen inde eertsclooten te lichtelicker ghevonden worden, oock mede om de ghedaenten der naeldwijsinghen daer deur int volghende opentlicker te verclaren. De letter N bediet inde tweede pilaer overal noordersche breede, maer Z zuydersche breede.

Voort wantter gheseyt wort van naeldwijcking, oostering, westering, vergrootende ende verkleenende, oock van eerste en tweede perck, welcke als eygen constwoorden haer bepalinghen vereysschen, soo is voor al kennelick, de zeylnaelde seker eyghenschap te hebben, datse op een selve plaets een selven oirt wijst, maer niet den selven oirt over al, want tot sommighe plaetsen wijstse recht noort, tot ander wijckse na t'oosten, elders na t'westen, daerom segghen wy by manier van bepaling als volghen sal:

siet het 12. blat.

we now have as if it were true; for if everyone does the same in due time, it will be possible to come gradually nearer and nearer to that which is most true in the nature of things.

This being so, and in order to come now to the explanation of the table, it is seen first of all that there are three columns, the first of the variations of the places, the second of the latitude, to which has been added the third, of the estimated longitude, in order that the places may be found more easily on the globes, and also in order to explain the character of the variations more clearly in what follows. The letter *N* in the second column everywhere designates northern latitude, and *S* southern latitude.

Further, because mention is made of variation, easterly variation, westerly variation, increasing and decreasing, and also of first and second segments [1]), which as special technical terms require a definition, it is to be known first of all that the magnetic needle has the particular property that in the same place it points in the same direction, but not in the same direction everywhere, for in some places it points due north, in others it declines to the east, elsewhere to the west; for this reason by way of definition we say as follows:

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*) This term will here be used to render the Dutch "perck", which stands for the part of the earth's surface that is bounded by two half-meridians.

TAFEL DER NAELD-WIISINGHEN.

			Oſtering. tr.	①	Breede. tr.	①	Langde. tr.	①
Eerſte percx opde noortſijde	Vergrootende oosteering	Een der Vlaemſche Eylanden Corvo.	0.	0.	N 37.	0.	0.	0.
		Opt Vlaemſch Eylant Sancta Maria.	3.	20.	N 37.	0.	8.	20.
		Neſſens het Eylant Maio.	4.	55.	N 15.	0.	11.	20.
		By t'Canariſche Eylant Palma.	6.	10.	N 28.	30.	16.	20.
		By Cabo de Roca by Lisbona.	10.	0.	N 38.	55.	24.	30.
		Het weſterlickſte van Yrlandt.	11.	0.	N 52.	8.	24.	12.
		Engelants eint.	12.	40.	N 50.	21.	28.	0.
	Verkleenende oosteering	Een mijl ooſtwaert van Pleymouth.	13.	24.	N 50.	18.	30.	0.
		By Timouth in zee.	12.	40.	N 55.	0.	33.	0.
		Londen in Engelant.	11.	30.	N 51.	24.	34.	6.
		Het voorlant van Engelant.	11.	0.	N 51.	8.	35.	40.
		Amſterdam.	9.	30.	N 52.	20.	39.	30.
			Weſtering.					
Tweede percx opde noortſijde	Vergrootende weſtering	Helmſhuy by weſten de Noortcaep in Finmarck.	0.	0.	N		60.	0.
		Noortcaep in Finmarken.	0.	55.	N 71.	25.	61.	30.
		Noorkin.	2.	0.	N 71.	10.	63.	30.
		Sint Michiel in Ruſſia genaemt Archangel.	12.	30.	N 64.	54.	83.	30.
		De zuyderlicke ſtraet van Vaygats.	24.	30.	N 69.	30.	103.	0.
		Langenes in Nova Zembla.	25.	0.	N 73.	20	100.	30.
		Willems Eylant by Nova Zembla.	33.	0.	N 75.	35.	110.	0.
	Verkleenende weſtering	Yſhouck in Nova Zembla.	27.	0.	N 77.	12.	120.	30.
		Het winterhuys in Nova Zembla.	26.	0.	N 76.	0.	120.	30.

			Ostering. tr.	Ostering. ①	Breede. tr.	Breede. ①	Langde. tr.	Langde. ①
Eerste percx op de zuytsijde	Vergrootende oostering	Op 105. Spaensche mijlen westwaert van Cabo Sant Augustin en Brasilie.	0.	0.	Z.		0.	c.
		By Cabo S. Augustin in Brasilia.	3.	10.	Z. 8.	30.	6.	0.
		Zuyt en noort met Cabo das Almas in Guinea.	12.	15.	Z. 0.	0.	29.	0.
		Noortwest wel soo noordelick vande Eylanden van Tristan da Cuncha.	19.	6.	Z. 31.	30.	30.	0.
	Vercleenende oostering	Noortwest wel soo westelick vande voorscreven Eylanden.	15.	0.	Z. 31.	30.	36.	0.
		Zuyt en noort met Cabo de Bona esperance.	2.	30.	Z. 35.	30.	57.	0.
			Westering.					
Tweede percx opde zuytsijde (uytgenomen Goa Cochin en Canton)	Vergrootende westering	Op 17. duytsche mijlen van Cabo das Aguillas oostwaert.	0.	0.	Z.		60.	0.
		Ontrent 5. mijlen in zee vant lant Natal	4.	30.	Z. 33.	0.	66.	0.
		Byde Baixos da India.	11.	0.	Z. 22.	0.	79.	30.
		Mosambique.	11.	0.	Z. 14.	50.	81.	40.
		Inden inwijck van S. Augustin in Madagascar.	13.	0.	Z. 23.	30.	83.	0.
		Zuyt van Cabo Sant Romain.	16.	0.	Z. 28.	0.	86.	20.
		Inden inwijck van Anton Gil in Madagascar.	15.	0.	Z. 16.	20.	91.	0.
	Vercleenende westering	34. Duytsche mijlen zuytooft van S. Brandaon.	22.	0.	Z. 19.	20.	110.	0.
		Goa een vermaerde coopstat in India.	15.	10.	N 15.	30.	120.	0.
		Cochin.	15.	0.	N 9.	45.	121.	0.
		25. Duytsche mijlen west ten noorden vande zuytwesthouck van Samatra.	6.	0.	Z 5.	28.	147.	0.
		Bantan een coopstadt in India.	4.	45.	Z 6.	0.	150.	0.
		Het Eylant Lubock.	2.	25.	Z 6.	10.	155.	0.
		De zuytwesthouck vant Eylant Balij.	1.	30.	Z 8.	40.	157.	0.
		De mont der Rivier van Cantan in China.	0.	0.	N 23.	0.	160.	0.
		Bunam 46. Duytsche mijlen van het oostende van Iava na het oosten.	0.	0.	Z		160.	0.

TABLE OF THE VARIATIONS.

Segment	Variation	Place	Easterly Variation deg. min.	Latitude deg. min.	Longitude deg. min.
First segment on the northern hemisphere	Increasing easterly variation	Corvo, Azores	0 0	*N* 37 0	0 0
		On the Island of St. Mary, Azores	3 20	*N* 37 0	8 20
		Off the Island of Maio 1)	4 55	*N* 15 0	11 20
		Off the Island of Las Palmas, Canary Isl.	6 10	*N* 28 30	16 20
		Off Cabo da Roca, near Lisbon	10 0	*N* 38 55	24 30
		The westernmost part of Ireland	11 0	*N* 52 8	24 12
		Land's End	12 40	*N* 50 21	28 0
	Decreasing easterly variation	One mile eastward from Plymouth	13 24	*N* 50 18	30 0
		Off Tynemouth 2) in the sea	12 40	*N* 55 0	33 0
		London in England	11 30	*N* 51 24	34 6
		The Foreland of England 3)	11 0	*N* 51 8	35 40
		Amsterdam	9 30	*N* 52 20	39 30
			Westerly Variation		
Second segment on the northern hemisphere	Increasing westerly variation	Hjelmsöy to the west of the North Cape in Finnmark 4)	0 0	*N*	60 0
		North Cape in Finnmark	0 55	*N* 71 25	61 30
		Nordkinn 5)	2 0	*N* 71 10	63 30
		St. Michael in Russia, called Archangel	12 30	*N* 64 54	83 30
		The strait to the south of Vaygach Isl.	24 30	*N* 69 30	103 0
		Langenes on Novaya Zemlya 6)	25 0	*N* 73 20	100 30
		"Willems Eylant", off Novaya Zemlya 7)	33 0	*N* 75 35	110 0
	Decreasing westerly variation	"Yshouck" on Novaya Zemlya 8)	27 0	*N* 77 12	120 30
		"Het winterhuys" on Novaya Zemlya 9)	26 0	*N* 76 0	120 30

(For notes, see p. 441).

			Easterly Variation deg. min.	Latitude deg. min.	Longitude deg. min.
First segment on the southern hemisphere	Increasing easterly variation	At 105 Spanish miles westward from Cape St. Augustine in Brazil [10]	0 0	S	0 0
		Off Cape St. Augustine in Brazil	3 10	S 8 30	6 0
		To the south of Cabo das Almas in Guinea [11]	12 15	S 0 0	29 0
		Slightly more northerly than northwest from the Islands of Tristan da Cunha	19 0	S 31 30	30 0
	Decreasing easterly variation	Slightly more westerly than northwest from the aforesaid Islands	15 0	S 31 30	36 0
		To the south of the Cape of Good Hope	2 30	S 35 30	57 0

(For notes, see p. 441)

		Place	Westerly Variation deg. min.	Latitude deg. min.	Longitude deg. min.
Second segment on the southern hemisphere (except Goa, Cochin, and Canton)	Increasing westerly variation	At 17 German miles eastward from Cape Agulhas 12)	0 0	*S*	60 0
		About 5 miles in the sea from the land of Natal	4 30	*S* 33 0	66 0
		At the Baixos da Judia 13)	11 0	*S* 22 0	79 30
		Mozambique	11 0	*S* 14 50	81 40
		In S. Augustin Bay, Madagascar 14)	13 0	*S* 23 30	83 0
		South of Cape St. Romain 15)	16 0	*S* 28 0	86 20
		In Antongil Bay, Madagascar 16)	15 0	*S* 16 20	91 0
	Decreasing westerly variation	34 German miles south-east from St. Brendan 17)	22 0	*S* 19 20	110 0
		Goa, a famous market-town in India	15 10	*N* 15 30	120 0
		Cochin	15 0	*N* 9 45	121 0
		25 German miles west by north from the southwestern corner of Sumatra	6 0	*S* 5 28	147 0
		Bantam, a market-town in the East Indies 18)	4 45	*S* 6 0	150 0
		The Island of 'Lubock' 19)	2 25	*S* 6 10	155 0
		The southwestern corner of the Island of Bali	1 30	*S* 8 40	157 0
		The mouth of the River of Canton in China	0 0	*N* 23 0	160 0
		Bunam, 46 German miles to the east from the eastern end of Java 20)	0 0	*S*	160 0

(For notes, see p. 441).

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[1]) Cape Verde Islands.
[2]) Northumberland, east coast of Great Britain.
[3]) In view of the latitude given, South Foreland near Dover must be meant.
[4]) The island of Hjelmsöy, to the west of the North Cape.
[5]) Nordkyn, Finnmark, a cape to the east of the North Cape.
[6]) Langenes (Capo de Prior), Sukhoy Nos, a cape on the west coast of Novaya Zemlya, in about latitude 73°30′ N.
[7]) Berg Island, one of the Gorbovi Islands, near the west coast of Novaya Zemlya.
[8]) Cape Bolshaya Ledyanoi, north point of Novaya Zemlya.
[9]) "Het behouden huis", where Heemskerck and Barents passed the winter of 1596/97.

[10]) East coast of Brazil, near Pernambuco.
[11]) What is meant is: in the meridian of Cape Palmas, coast of Liberia, in latitude 4°25′N.

[12]) Cape Agulhas, south point of Africa, in latitude 34°50′ S.
[13]) Baixos da Judia, Mozambique Channel. This shoal is called Judia, after the ship „de Jodin" (the Jewess), which got into trouble there.
[14]) West coast of Madagascar, latitude 23°30′ S.
[15]) Cabo Sant Roman, Cape Andavaka, south coast of Madagascar, situated south-west of Fort Dauphin.
[16]) East coast of Madagascar, latitude 16° S.
[17]) A mythical island, imagined east of Madagascar.
[18]) Java.
[19]) Lubock is the island of Bawean, situated north of Surabaya, in about latitude 6° S. The name is a corruption of the Malay word *lubuk*, which means harbour basin. The island owes this name to the big inlet Sangkapura, on the southern side of the island.
[20]) For Bunam or Bima, see page 370, note 17.

1. *Bepaling.*

DE afwijcking der naelde vant noorden na t'ooſten, heet ooſtering, maer na t'weſten, weſtering, ende int ghemeen naeldwijcking: Maer naeldwijcking en rechte noortwijſing, int ghemeen naeldwijſing.

AENGAENDE de woorden van vergrootende en verkleenende ooſtering, en weſtering, oock van eerſte en tweede perck, die vereyſchen eer wy totte bepaling commen, wat breeder verclaring, tot welcken einde wy aldus ſegghen: Men ſiet inde tafel, dat de naelde in Corvo recht noort wijſt, maer van daer ooſtwaert commende, datſe begint te ooſteren allencx meer en meer, tot een mijl ooſtwaert van Plemouth, alwaer de afwijcking ten grootſten is van 13. tr. 24. ①. Ende van daer voorder commende, ſij begint te vercleynen tot Helmſhuy by weſten de Noortcaep van Finmarcken toe, alwaerſe weerom recht noort wijſt. Voort is de langde van Corvo tot Helmſchuy van 60. tr. Twelck ſoo ſijnde, het blijct dat de voorſ. grootſte naeldwijcking van 13. tr. 24. ①. by Plemouth wiens langde 30. tr. ghreſchiet int middel der twee plaetſen daer de naelde recht noort wijſt, want den 30. tr. is int middel tuſſchen t'begin en den 60. tr.

Tghene.

1st Definition.

The declination of the needle from the north to the east is called easterly variation, but to the west, westerly variation, and in general variation; but variation as well as north-pointing of the needle are called needle-pointing.

As regards the words increasing and decreasing easterly and westerly variation, and also first and second segment, these require some further explanation before we come to the definition, to which end we say as follows: It is seen in the table that in Corvo the needle points due north, but that, when from there one comes to the east, it gradually begins to decline to the east more and more, up to one mile eastward from Plymouth, where the variation is greatest, namely 13°24′. And when from there one gets further, it begins to decrease up to Hjelmsöy to the west of the North Cape of Finnmark, where the needle points due north again. Further the longitude from Corvo to Hjelmsöy is 60°. This being so, it appears that the aforesaid greatest variation of 13°24′ off Plymouth, whose longitude is 30°, is midway between the two places where the needle points due north, for 30° is in the middle between zero and 60°.

Tghene wy hier ghefeyt hebben, vande verandering der naeldwijfing op de noortfijde des eertrijcx, dergelijcke bevint fich deur ervaring oock opde zuytfijde, want op 105. Spaenfche mijlen weftwaert van Cabo Sant Auguftin opt begin der langde, wijft de naelde recht noort, alfoofe oock doet ter plaets inde tafel ghefeyt, op 17. Duytfche mijlen van Cabo das Aguillas, wefende op 60. tr. der langde, ende int middel tuffchen beyden, dats op den 30. tr. valt aldaer oock ghelijck opde noortfijde de grootfte oostering, dats ter plaets inde tafel ghenaemt, noortweft wel foo noorderlick vande Eylanden van Triftan da Cuncha, doende die wijcking 19. tr.

Hier uijt wilmen befluyten, dat de naelde recht noort wijft tot alle plaetfen ghelegen inde twee halfmiddachfronden deur Corvo en de Helmfhuy, van deen * afpunt tot dander. Oock mede dat de naeldens oostering ten grootften is, tot alle plaetfen int halfmiddachfront ftreckende deur de plaets ghelegen een mijle ooftwaert van Plemouth. * *Polo.*

Inder voughen dat in fulck anfien, het eertrijcxdeel begrepen tuffchen die twee halfmiddachfronden, 60. tr. in langde van malcander, is een perck int welck de naelde over al vant noorden na t'ooften wijckt, ende inden helft van dien, dat is het eertrijcx deel begrepen tuffchen de twee halfmiddachfronden, teerfte deur t'begin, het ander deur den 30. tr. foude over al fijn vergrootende oostering: Ende in dander helft verkleenende oostering: welverftaende alfmen vant weften na t'ooften treckt, dats na t'vervolg vande tr. der langde.

A similar thing to that which we have here said about the change of the needle-pointing on the north side of the earth is also found by experience on the south side, for at 105 Spanish miles westward from Cape St. Augustine in longitude 0° the needle points due north, as it also does in the place given in the table, at 17 German miles from Cape Agulhas, which is in longitude 60°, and in the middle between the two, *i.e.* at 30°, as on the north side, falls the greatest easterly variation, *i.e.* in the place mentioned in the table, slightly more northerly than northwest from the Islands of Tristan da Cunha, said variation being 19°.

From this it is concluded that the needle points due north in all places situated in the two meridian semi-circles through Corvo and Hjelmsöy, from one pole to the other. Also that the easterly variation of the needle is greatest in all places in the meridian semi-circle passing through the place situated one mile eastward from Plymouth.

Thus, considering the above, the part of the earth contained between those two meridian semi-circles, 60° in longitude distant from each other, is a segment in which the needle declines everywhere from the north to the east, and in one half of it, *i.e.* the part of the earth contained between the two meridian semi-circles, the first through longitude 0°, the other through 30°, there is everywhere increasing easterly variation, and in the other half decreasing easterly variation, that is to say: when one goes from west to east, *i.e.* in the order of the degrees of longitude.

Deur tgene tot hier toe ghesseyt is vant eerste perck met oostering, ende sijn twee deelen, t'een met vergrootende oostering, t'ander met vercleenende, machmen lichtelick verstaen dergheli jcke ghedaenten vant tweede perck met westering, ende sijn twee deelen, t'een met vergrootende westering, t'ander met verkleenende: Want inde mont der Rivier van Cantan in China, ligghende in langde 160. tr. van Corvo, daer wijst de naelde de derdemael recht noort, daerom aldaer ghetrocken een derde halfmiddachsront, soo is het eertrijcxdeel begrepen tusschen dat tweede halfmiddachsront ende dit derde 100. tr. van malcander, een perck int welck de naelde overal vant noorden na t'westen wijckt, ende int middel van dese twee, dats int halfmiddachsront 50. tr. vant tweede, ende oock soo veel vant derde, oft andersins 110. tr. vant eerste door Corvo, daer heeftmen oock de grootste afwijcking der naelde, soot inde tafel tot twee plaetsen blijckt, deen op Willems Eylant by Nova Zembla, alwaer de grootste westering op die breede bevonden is van 33. tr. dander 34. Duytsche mijlen zuytoost van S. Brandaon, alwaer de grootste naeldwijck op die breede bevonden is van 22. tr. wesende de langde van elck dier twee plaetsen 110. tr. Sulcx dat inden helft van dit tweede perc, dats het eertrijcxdeel begrepen tusschen de twee halfmiddachsronden, t'eerste deur den 60. tr. t'ander deur den 110. tr. soude overal sijn vergrootende westering, in dander helft verkleenende westering.

Van dese 160. tr. der langde, twelck op 20. tr. na, den

From what has so far been said about the first segment with easterly variation and its two parts, the one with increasing, the other with decreasing easterly variation, it can easily be understood that the second segment with westerly variation and its two parts, the one with increasing, the other with decreasing westerly variation, is of the same nature. For in the mouth of the River of Canton in China, situated in longitude 160° distant from Corvo, the needle points due north the third time; consequently, if a third meridian semi-circle is drawn there, the part of the earth contained between the second and the third meridian semi-circle, 100° distant from each other, is a segment in which the needle declines everywhere from the north to the west, and in the middle of these two, *i.e.* in the meridian semi-circle 50° distant from the second and as much from the third, or in other words 110° distant from the first through Corvo, the greatest variation of the needle is to be found, as appears in the table in two places, the one in "Willems Eylant", near Novaya Zemlya, where the greatest westerly variation in that latitude is found to be 33°, the other at 34 German miles southeast from St. Brendan, where the greatest variation in that latitude is found to be 22°, the longitude of each of those two places being 110°. Thus in the one half of this second segment, *i.e.* the part of the earth contained between the two meridian semi-circles, the first through 60°, the other through 110°, there would be everywhere increasing westerly variation, in the other half decreasing westerly variation.

den helft des eertrijcx is, heeft de voorſ. Plancius de naeldwijſinghen becommen ghelijckſe hier vooren beſchreven ſijn: Maer vande reſt des eertrijcx, te weten van Cantan ooſtwaert, of van Corvo weſtwaert, en overcommen de ervaringhen niet die hem van Spaengjaerden, Engelſchen en onſe zeevaerders ter handt ghecommen ſijn, als ghedaen weſende ſonder bequamen tuych, en ghenouchſaem wetenſchap: Doch verwacht hy van daer alle daghe nieuwe zeker ervaringhen, deur ſchepen die meer dan veerthien maenden uijtgheweeſt ſijn. Maer daerentuſſchen ſullen wy ſegghen tgene men van dat deel eenichſins vermoet als volght: By aldien de eyghenſchap der rechtnoortwijſing, niet alleen en is inde voorſ. drie boghen, diemen meent halfmiddachſronden te weſen, ghelijck wy vooren gheſeyt hebben van deen aſpunt tot dander, maer inde heele ronden, ſoo ſoudender opt eertrijck in als ſulcke ſes halfronden ſijn, vervanghende ſes percken.

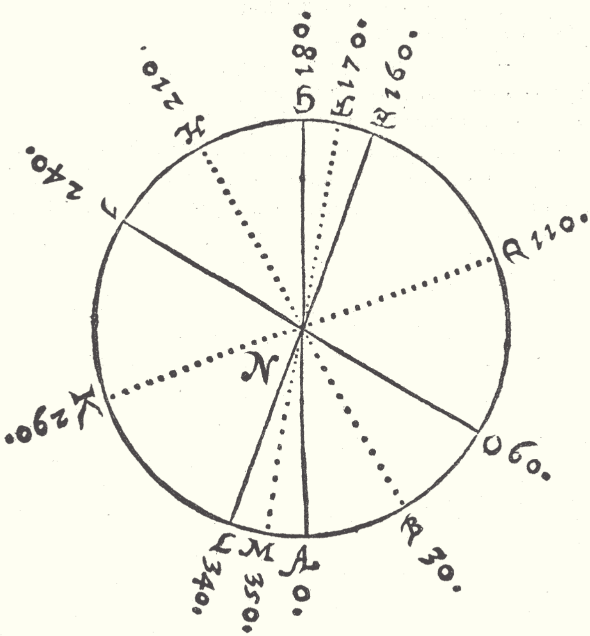
Teerſte met ooſtering lanck 60. tr.
Het tweede met weſtering lanck 100. tr.
Het derde met ooſtering lanck 20. tr.
Het vierde met weſtering lanck 60. tr.
Het vijfde met ooſtering lanck 100. tr.
Het ſeſte met weſtering lanck 20. tr.

Om tgene voorſeyt is deur een form noch opentlicker te verclaren, ſoo laet A B C D E F G H I K L M, het middelront des eertcloots beteyckenen, diens aſpunt N. Voort ſij N A den helft van teerſte halfmiddachſront deur Corvo, N C het tweede, N E het derde, N G het

Of these 160° of longitude, which is 20° short of one half of the earth, the aforesaid Plancius obtained the variations as they have been described above. But for the rest of the earth, to wit, from Canton eastward or from Corvo westward, the observations he obtained from the Spaniards, the English, and our own navigators do not agree, because they have been made without suitable instruments and sufficient knowledge. But he expects any day to receive new and more exact observations about that part through ships that have been away for more than fourteen months. Now in the meantime we shall say what is assumed about that part, as follows. If the property of the north-pointing applied not only in the aforesaid three semi-circles, which are thought to be meridian semi-circles, as we have said above, from one pole to the other, but in the whole circles, there would on the earth be six such semi-circles in all, containing six segments.

The first with easterly variation 60° long
The second with westerly variation 100° long
The third with easterly variation 20° long
The fourth with westerly variation 60° long
The fifth with easterly variation 100° long
The sixth with westerly variation 20° long

In order to explain the above even more clearly by means of a figure, let *ABCDEFGHIKLM* designate the equinoctial circle of the earth, its pole *N*. Further let *NA* be the half of the first meridian semi-circle through Corvo, *NC* het vierde, N I het vijfde, N L het seste, ende alsoo dat de booch A C doe 60. tr. C E 100. tr. dats A E 160. tr. E G 20. tr. dats A G 180. tr. G I 60. tr. dat A I 240. tr. I L 100. tr. dats A L 340. tr. L A 20. tr. dats tgeheel rondt 360. tr. Voort sijn de ses punten B, D, F, H, K, M, middelen tusschen A C, C E, E G, G I, I L, L A. Dit aldus wesende soo bediet,



A N C het 1^{e} perck met oostering.
A N B des 1^{e} percx vergrootende oostering.
B N C des 1^{e} percx verkleenende oostering.
C N E het 2^{e} perck met westering.
C N D des 2^{e} percx vergrootende westering.
D N E des 2^{e} percx verkleenende westering.

ENG

of the second, *NE* of the third, *NG* of the fourth, *NI* of the fifth, *NL* of the sixth, and such that the arc *AC* makes 60°, *CE* 100°, and so *AE* 160°; *EG* 20°, and so *AG* 180°; *GI* 60°, and so *AI* 240°; *IL* 100°, and so *AL* 340°; *LA* 20°, and so the whole circle 360°. Further let the six points *B, D, F, H, K, M* be the mid-points between *A* and *C, C* and *E, E* and *G, G* and *I, I* and *L, L* and *A*. This being so,

ANC	designates the 1st segment with easterly variation
ANB	the increasing easterly variation of the 1st segment
BNC	the decreasing easterly variation of the 1st segment
CNE	the 2nd segment with westerly variation
CND	the increasing westerly variation of the 2nd segment
DNE	the decreasing westerly variation of the 2nd segment

ENG het 3^e^ perck met oostering.
ENF des 3^e^ percx vergrootende oostering.
FNG des 3^e^ percx verkleenende oostering.
GNI het 4^e^ perck met westering.
GNH des 4^e^ percx vergrootende westering.
HNI des 4^e^ percx verkleenende westering.
INL het 5^e^ perck met oostering.
INK des 5^e^ percx vergrootende oostering.
KNL des 5^e^ percx verkleenende oostering.
LNA het 6^e^ perck met westering.
LNM des 6^e^ percx vergrootende westering.
MNA des 6^e^ perx verkleenende westering.

MERCT. Hoewel het te vermoeden is, datmen de drie laetste halfronden niet vinden en sal van ghedaente als de voorgaende gissing inhoudt, maer misschien in menichte meer of min, en van ander ghestalt; doch soo is hier me voorbeeltsche wijse verclaert de manier hoemen de weerelt int gheheel sal meughen deylen, met sulcke halfronden alsser alsdan ghevonden sullen worden: Boven dien is deur tvoorgaende ghenouch te verstaen, wat bedien vergrootende en verkleenende oostering, en westering, oock eerste en tweede halfmiddachsront, met eerste en tweede percken: Om welcke by manier van bepaling te vervaten, men soude meughen aldus segghen:

ENG the 3rd segment with easterly variation
ENF the increasing easterly variation of the 3rd segment
FNG the decreasing easterly variation of the 3rd segment
GNI the 4th segment with westerly variation
GNH the increasing westerly variation of the 4th segment
HNI the decreasing westerly variation of the 4th segment
INL the 5th segment with easterly variation
INK the increasing easterly variation of the 5th segment
KNL the decreasing easterly variation of the 5th segment
LNA the 6th segment with westerly variation
LNM the increasing westerly variation of the 6th segment
MNA the decreasing westerly variation of the 6th segment

NOTE. Though it is to be expected that the three last semi-circles will not be found to be such as the preceding conjecture implies, but perhaps in a quantity either more or less and of a different form, yet it has thus been explained by way of example in what manner the whole world may be divided by such semi-circles as shall be found. Moreover it can be sufficiently understood from the foregoing what is the meaning of increasing and decreasing easterly and westerly variation, also of the first and the second meridian semi-circle, with the first and second segments. In order to summarize this in the form of definitions, it might be said as follows:

2. *Bepaling.*

VERGROOTENDE oostering of westering, is die welcke de naelde van westen na oosten voortghebrocht zijnde, vergroot: Ende verkleenende, die alsdan verkleent.

3. *Bepaling.*

DE halfmiddachsronden daer de naelde recht noort in wijst, heeten wy eerste, tweede halfmiddachsront, en soo oirdentlick voort na tvervolgh vande trappen der langde soo veel alsser sulcke ronden zijn, beginnēde vant halfmiddachsront deur Corvo.

4. *Bepaling.*

TVLACK begrepen tusschen teerste en tweede halfmiddachsront, noemen wy eerste perck, en dander oirdentlick vervolghende tweede, derde perck, tottet laetste.

2nd Definition.

Increasing easterly or westerly variation is that which increases when the needle is carried from west to east; and decreasing variation, which then decreases.

3rd Definition.

The meridian semi-circles in which the needle points due north we call the first and the second meridian semi-circle, and so on in the order of the degrees of longitude, as many such semi-circles as there are, starting from the meridian semi-circle through Corvo.

4th Definition.

The surface contained between the first and the second meridian semi-circle we call the first segment, and the others in due order the second segment, the third, up to the last.

De ghedaenten der naeldwijſinghen aldus beſchreven ſijnde, wy ſullen nu deur voorbeelt verclaren, ghelijck t'voornemen was, dat hoewelder op een ſelve breede evegroote naeldwijckinghen ſijn tot verſcheyden plaetſen des eertrijcx, dat nochtans den Stierman can weten in welcke der ſelve hy is. Laet tot deſen einde een ſchip andermael moeten varen van Amſterdam na Cabo Sant Auguſtin, in Braſilie, wiens breede inde tafel beſchreven ſtaet van 8. tr. 30. ①, ende de naeldwijſing vergrootende ooſtering des eerſten percx van 3. tr. 10. ①. Tſelve ſchip afvarende, ende commende voorby Engelandt, bevindt ſijn naeldwijſing daghelicx meer en meer te ooſteren, tot by Pleymouth toe, alwaerſe ten grootſten weſende van 13. tr. 24. ①, het verſekert hem dat hy tot daer toe ghevaren heeft in verkleenende ooſtering des eerſten percx, ende dat hy van daer voort ſeylt inde vergrootende ooſtering, welcke hy bevindende van 10. tr. opde breede van 38. tr. 55. ①. weet hem te weſen ontrent Cabo de Roca by Liſbona: Van daer af, ontrent zuytweſt anvarende, ſal daghelicx bevinden de breede te minderen, ende de naelde noordelicker te keeren: Oft anderſins ſoo die daghelickſche noordering niet en bleecke, maer dat de naelde een ſelve ſtreeck weſe, oft ooſtlicker keerde, ſalt daer voor houden dat onbemerckelicke ſtroomen ſijn ſchip al varende ooſtwaert drijven: Om twelck te voorcommen, ſalt ſoo veel weſtelicker an ſetten, dat hy daghelicx de naeldens behoirlicke noordering crijghe. Maer ſoo hy quaem totte ooſtering van 3. tr. 10. ①. eer hy

The character of the needle-pointings thus having been described, we shall now explain by means of examples, as was our intention, that although at the same latitude there are equal variations in different places of the earth, the navigator may nevertheless know in which of these places he is. For this purpose let a ship have to sail once more from Amsterdam to Cape St. Augustine in Brazil, the latitude of which is given in the table as 8°30′ and the variation is increasing easterly variation of the first segment of 3°10′. When this ship puts off and sails past England, the variation will be found to become more and more easterly every day up to Plymouth, and since it is there at its greatest, namely 13°24′, this assures the navigator that he has so far sailed in the decreasing easterly variation of the first segment and that from there he will further sail in the increasing easterly variation, and when he finds this to be 10° in latitude 38°55′, he knows that he is off Cabo da Roca near Lisbon. When from there he sails about southwest, he will daily find the latitude decreasing and the needle returning to the north [1]). Or else, if this daily return further to the north did not become apparent, but the needle pointed in the same direction or declined further to the east, he will assume that imperceptible currents are driving his ship eastward; and in order to prevent this, he will direct it so much further westward that he may daily obtain the proper return of the needle to the north. But if he comes to

[1]) The ship, starting from Amsterdam, navigates first in the section BNC ("decreasing easterly variation of the 1st segment"), then in the section ANB ("increasing easterly variation of the 1st segment"). See p. 451.

gherocht totte zuyderlicke breede van 8. tr. 30. ①. hy ſal maken ſoo veel hem meughelick is, die naeldwijcking int zuytwaert varen te behouden, ſoo veel ooſtelicker of weſtelicker ſeylēde als de ſaeck vereyſcht. Ende hoewel hem na giſſing docht anders te behooren, en ſal die nochtans niet volghen, om redenen hier vooren breeder verclaert, want commende alſoo totte zuyderſche breede van 8. tr. 30. ①. met vergrootende ooſtering van 3. tr. 10. ①. hy moet (tmach mette langde dier plaets ſijn ſoot wil) ontrent Cabo Sant Auguſtin weſen, ende dat met ſekerheyt; daermen anders op giſſing betrouwende, ettelicke hondert mijlen vande begeerde plaets gheraeckt, ſonder te weten ofmender ooſt of weſt af light, ghelijck op ſulcke reyſen metter daet ghenouch ghebleken heeft. Daerom tot allen houcken des weerelts de naeldwijſing en breede wel ghenomen ſijnde, ende an alle man bekent ghemaeckt, men ſal de weerelt anders connen beſeylen danmen ghedaen heeft.

Tot hier toe ſijn beſchreven de ghedaenten der naeldwijſinghen, volghende uijt het gheſtelde des tafels: Soo ander ſekerder ervaringhen in toecommenden tijt anders weſen, men ſal daer uyt anders meughen beſluyten, ende inde zeyling ſich na t'beſte altijt ghevoughen.

Hoe-

the easterly variation of 3°10′ before he has reached latitude 8°30′ South, he must do all he can to keep this variation in going to the south, sailing so much more towards the east or the west as required. And though by conjecture he thinks it ought to be otherwise, he must not proceed on this, for the reasons set forth more fully above, for if he thus comes to latitude 8°30′ South with increasing easterly variation of 3°10′, he must be (whatever may be the longitude of the place) off Cape St. Augustine, with certainty, whilst otherwise, relying on conjecture, one gets many hundreds of miles from the desired place, without knowing whether one is to the east or the west of it, as has appeared often enough in practice during such voyages. If therefore the needle-pointing and the latitude are duly observed in all corners of the world and made known to everybody, it will be possible to sail the world in another way than hitherto.

Thus far the character of the variations following from the data of the table has been described. If other, more exact observations should prove different in the future, other conclusions can be drawn from them, and in navigation the best must always be used.

Hoemen het noortpunt en naeldwijsing vindt.

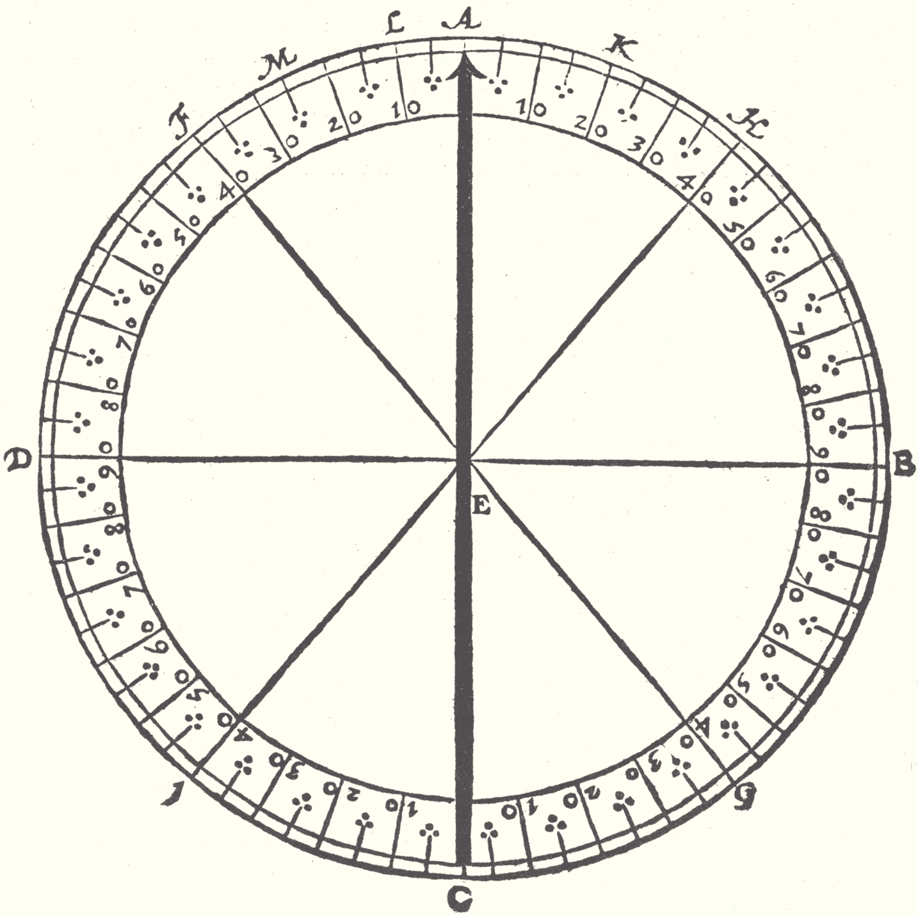
HOE wel het vinden der naeldwijsing (daer af wy hier vooren dickwils ghesyt hebben) an velen bekent is, nochtans sullen wy daer af schrijven voor de ghene diet niet en weten.

Anghesien men hier begeert te vinden de afwijcking der naelde vant noorden, soo souctmen eerst het noortpunt, om de naeldwijsing daer by te verlijcken. De manier der vinding vant selve noortpunt in een beweghende schip op zee, heeft groote ghemeenschap mette manier der vinding vant noortpunt, of vande middachslijn opt vast lant, ende mach onder anderen aldus uijtgherecht worden: Men doet int zeecompas de leli recht overcommen mettet noortende vant stael, of vande zeylnaelde daer onder ligghende: Of noch beter machmen in plaets vande leli, een naelde self boven opt papier vast legghen, deelende t'rondt van tselve papier in sijn 360. tr. beginnende ande naeldens noortpunt als hier onder het rondt A B C D, waer in de naelde beteyckent is met A C, vastghemaect wesende opt selve papier, E is tmiddelpunt: Tgebruyck hier me is dusdanich: Ghelijck den Stierman int soucken der breede, wacht tot dat de middach ghecommen is, te weten tot dat de schaeu van een hangsnoer of rechtsnoer, overcomt mette lini die hy in sijn compas voor de middachslijn houdt, alsoo sal hy hier doen, uijtghenomen dat hy begint 3. 4. of 5. uijren of

How the True North and the Variation are found.

Although the finding of the variation (about which we have often spoken above) is known to many people, we shall write about it for those who do not know it.

Since it is desired to find the variation of the needle from the north, first the true north is sought, in order that the needle-pointing may be compared with it. The method of finding the said true north in a moving ship at sea greatly resembles the method of finding the true north or the meridian line on the land, and can be carried out, among other things, as follows. The fleur-de-lys in the mariner's compass is made to correspond exactly with the north end of the steel or of the magnetic needle lying underneath. Or, better still, instead of the fleur-de-lys a needle itself can be fixed on top of the paper and the circumference of the said paper can be divided into its 360°, starting at the north-point of the needle, *e.g.* the circle *ABCD* below, in which the needle is denoted by *AC,* being fixed on the said paper, *E* being the centre. The use of this is as follows. Just as the navigator, when seeking the latitude, waits until noon has come, to wit, until the shadow of a plumb-line coincides with the line which he regards as the meridian line in his compass, so he must do here, except meer voor middach alſdan, acht nemende op wat trap en ghedeelte van dien de ſchaeu des hangſnoers wijſt, bevint die, neem ick, opden 40. tr. gheteyckent F, ſulcx dat G E F, de ſchaeu bediet, ende nemende alſdan de Sonnens hooghde, bevint die, by voorbeelt, van 25. tr. welcke hy, metſgaders de 40. tr. tot ghedachtnis opteyckent: Wachtende voorts ſoo lang na



middach,

that he begins 3, 4 or 5 hours or more before noon, noting at what degree and part of it the shadow of the plumb-line points. Let us assume that he finds this at 40°, designated by *F*, so that *GEF* denotes the shadow. Then, taking the Sun's altitude, he will find this to be *e.g.* 25°, which he notes down, together with the 40°, as an aid to memory. Then, waiting so long after noon until the middach, tot dat de Son weerom ghedaelt is tot op de selve hooghde alsvooren van 25. tr. sal sien waer de schaeu vant hangsnoer alsdan opt papier wijst, twelck sij, neem ick, 40. tr. over dander sijde, als an H, sulcx dat I E H, de schaeu bediet. Dit soo sijnde, t'middel des boochs F H, als A, is tbegeerde noortpunt, ende want de naelde daer recht op wijst, soo en heeftse in dat voorbeelt gheen wijcking, dan wijst recht noort. Maer soo inde voors. ervaring na middach de schaeu vant hangsnoer niet ghewesen en hadde 40. tr. over dander sijde van A, maer by voorbeelt alleenelick 20. tr. tot K; In sulcken ghevalle deeltmen den booch F K, doende 60. tr. door tghedacht in tween an L, sulcx dat L F, L K, elck doen 30. tr. Twelck soo sijnde, L ist noortpunt, ende de begeerde naeldwijcking daer af is oostering van L tot A 10. tr.

Maer byaldien inde voors. ervaring na middach, de schaeu vant hangsnoer ghewesen hadde op L, dats 30. tr. van F, soo deeltmen den booch F L, doende 30. tr. door tghedacht in tween an M, sulcx dat M F, M L, elck doen 15. tr. twelck soo sijnde, M is tnoortpunt, ende de begeerde naeldwijcking daer af, wesende oostering van M tot A 25. tr. ende alsoo met alle voorbeelden. Maer soo de naelde alleen draeyde, sonder an een papier ghehecht te sijn als hier vooren, ende dat de trappen op den cant vande casse gheteyckent waren, ghelijck wel ghedaen wort: Tghebruyck is daer me alsvooren, midts datmen ten tijde der ervaring, de casse keert tot dat de naelde opt begin der trappen wijst.

Ander

Sun has descended again to the same altitude as before, namely 25°, he must find where the shadow of the plumb-line then points on the paper. Let us assume this to be 40° on the other side, namely at *H,* so that *IEH* denotes the shadow. This being so, the middle of the arc *FH,* namely *A,* is the desired true north, and because the needle points straight to it, it has no variation in this example, but points due north. But if in the aforesaid observation after noon the shadow of the plumb-line is not 40° on the other side of *A,* but *e.g.* only 20°, at *K,* in such a case the arc *FK,* which makes 60°, is divided in two in imagination at *L,* so that *LF* and *LK* each make 30°. This being so, *L* is the true north, and the desired variation is an easterly variation from *L* to *A* of 10°.

But if in the aforesaid afternoon observation the shadow of the plumb-line is at *L, i.e.* 30° from *F,* the arc *FL,* which makes 30°, is divided in two in imagination at *M,* so that *MF* and *ML* each make 15°. This being so, *M* is the true north, and the desired variation, which is an easterly variation from *M* to *A,* is 25°, and the same applies to all examples. But if the needle alone is turned, without being fastened to a paper, as above, and the degrees are marked on the rim of the box, as is sometimes done, it is used in the same way as above, provided the box is turned at the moment of the observation until the needle points at the zero point of the graduation.

* *Quadrantem Azimuthalium seu verticulũ cuius planũ horizontale.*

Ander ſijnder die nemen een * ſoppich vierendeel ronts, wiens ſichtemderſplat, niet teghenſtaende de beweeghlicheyt des ſchips, altijt in waterpas blijft, deur ſulcke manier als int volghende gheſeyt ſal worden. Hier me vintmen de Sonnens hooghde met haer ſopbooch beyde tſeſſens: De form daer af mach dusdanich weſen: A B C bediet een vierendeel ronts, ſtaende rechthouckich opt rondt B D C E, ghedeelt in ſijn 360. trappen, twelck het ſichtemderſplat beteyckent, ſijn middelpunt is F, waer op tvierendeelronts draeyen can, ende op dattet alſins rechthouckich blijft opt voorſ. rondt B D C E, ſoo comt van deen en dander ſijde een ſteunſel, als van G tot by D en E, vaſt ghemaect an tvoorſ. vierendeelrõts, om daer me te drayen. Voort iſſer int rondt B D C E een glas, en daer onder ſijn ſeylnaelde, ſoo lanck alſſe ten langſten inde caſſe bequamelick vallen mach, ende heeft de ſelve caſſe van binnen heur 360. tr. daer de punt der naelde ſcherpelick op wijſen mach, overcommende die 360. tr. met dander 360. tr. boven opt ſichtemderſront. Deſen tuych is deur de vondt van Reyner Pieterſz. hanghende ghemaect op twee verſcheyden aſſen, na de manier der zeecompaſſen, op dat alſoo het rondt B D C E, inde beweginghen vant ſchip altijt evewijdich vanden ſichtemder blijve: Ende op dattet ſelve noch meerder ſekerheyt hebbe, ſoo worter onder een ghewicht an vervought gheteyckent H, van 25. of 30. pont, of ſoo veel als de grootheyt vanden tuych vereyſcht.

Tis oock te ghedencken oirboir te weſen, dattet vierendeelronts tſijnder plaets recht overende ſtaende,

over

There are others who take an azimuthal quadrant, the horizontal plane of which, notwithstanding the movement of the ship, always remains level, by the method to be described in the following. By this means the Sun's altitude is found along with its azimuth. The figure relating thereto may be as follows. *ABC* designates a quadrant of a circle, at right angles to the circle *BDCE,* divided into its 360 degrees, which denotes the horizontal plane. Its centre is *F,* about which the quadrant can turn, and in order that it may always remain at right angles to the aforesaid circle *BDCE,* a support is provided on either side, namely from *G* to *D* and *E,* fastened to the aforesaid quadrant, in order to turn along with it. Furthermore there is in the circle *BDCE* a glass, and underneath it the magnetic needle, which has the maximum length possible in the box, and on the inside of this box are marked the 360 degrees, at which the point of the needle can point accurately, these 360 degrees corresponding to the other 360 degrees on top of the horizontal circle. By the discovery of Reynier Pietersz [1]) this instrument has been suspended on two different shafts, in the manner of the mariner's compass, in order that the circle *BDCE* may thus, in spite of the movements of the ship, always remain parallel to the horizon. And in order that this may be even more certain, a weight is fixed underneath it, marked *H,* namely 25 or 30 pounds or as much as the size of the instrument makes necessary.

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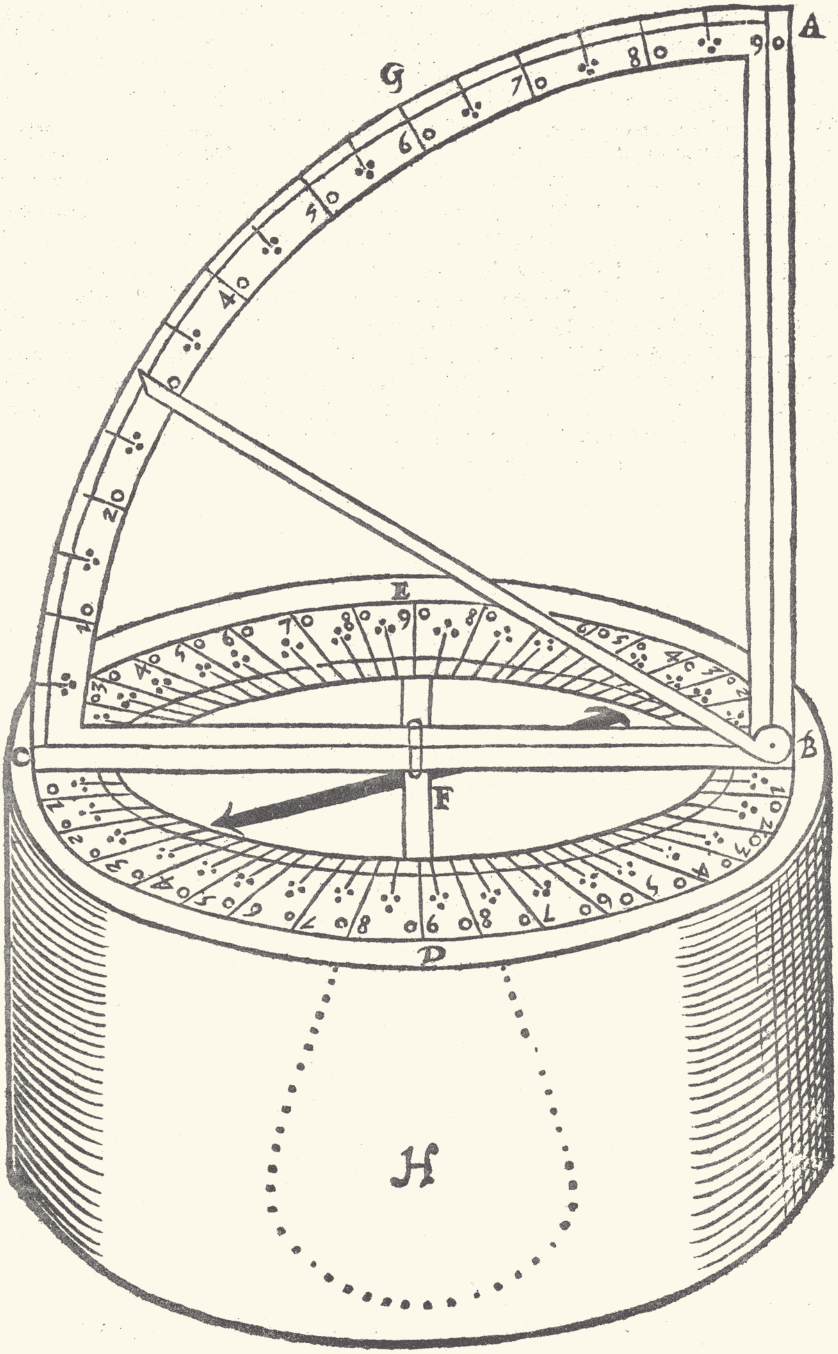
[1]) See the Introduction, § 4 b.: Reynier Pietersz and his "Golden Compass".


A
G
B
C
E
F
D
H

over deen en dander ſijde eveſwaer ſij, dat is de ſijde van F na C, ſoo ſwaer als van F na B, twelckmen weten can mits tvierendeelrondts af te nemen, ende te hanghen met G neerwaert an een draet, vaſt gemaeckt int middel van B C by F, ende alſdan ſalmen vande ſwaerſte ſijde veel af vijlen, tot dat de reghel B C in waterpas hangt.

*Alidada. Angaende ymant mocht dencken, dat de * wijſreghel in verſcheyden plaetſen hoogher of leegher ghedraeyt, te groote verandering int ghewicht mocht geven, daer af en is gheen merckelick feyl te verwachten, om tgroot ghewicht van H, ende de lichticheyt der wijſreghel.

De ghebruyck daer af, om t'noortpunt en naeldwijſing te vinden, is duſdanich: Men begint, gelijck in deerſte wijſe, ettelicke uijren voor middach, draeyende den tuych tot dat de naelde opt begin des ronts wijſt, daer na keertmen het vierendeel ronts ſoo lang herwaerts en derwaerts, tot dat de Son deur de ſichtgaetkens ſchijnt: Twelck ſoo ſijnde, men bevint, neem ick, dat den onderſten cant of wijſer vant vierendeelronts, wijſt int ſichteinderſplat opden 40. trap, ende de hooghde der Son, die int vierendeelronts anghewesen wort van, neem ick, 25. tr. welcke men, mitſgaders de 40. tr. tot gedachtnis opteyckent. Wachtende voort ſoo lang na middach, tot datmen de Son deur den ſeluen tuych ghedaelt vindt tot opde ſelve hooghde alſvooren van 25. tr. men keert alſdan den ſtoel ter eender en ander ſijde, tot dat de Son deur de ſichtgaetkens ſchijnende, de naelde weerom wijſt

opt

It is also to be remembered that it is suitable for the quadrant which is vertical in its place to have the same weight on either side, *i.e.* the side from *F* to *C* to have the same weight as that from *F* to *B*, which can be known if the quadrant is taken off and suspended with *G* downward by a thread fastened in the middle of *BC* at *F*, and then so much must be filed away from the heavier side, until the line *BC* hangs level.

If anyone should think that the pointer might bring about too great a variation in the weight in different places according as it is turned higher or lower, no appreciable error is to be expected from this, because of the great weight of *H* and the lightness of the pointer.

The way in which this instrument is used to find the true north and the variation is as follows. The observation should be started, as in the first case, a few hours before noon, the instrument being turned until the needle points at the zero point of the graduation. Thereupon the quadrant is turned this way and the other until the Sun shines through the sights. This being so, it is found *e.g.* that the lower edge or pointer of the quadrant points in the horizontal plane at 40°, while the altitude of the Sun, which is indicated in the quadrant, is *e.g.* 25°, which is noted down, together with the 40°, as an aid to memory. Then one should wait after noon until by means of the instrument the Sun is found to have descended to the same altitude as before, namely 25°. Then the quadrant is turned this way and the other until, the Sun shining through the sights, the opt begin des ronts: Twelck soo sijnde, t'middelste punt des boochs int sichteindersplat tusschen deerste en tweede ervaring, is tgesochte noortpunt: Ende soo veel de naelde alsdan daer af wijckt, dats de begeerde naeldwijcking, gelijck int eerste voorbeelt wat breeder van sulcx ghesyt is.

Deur tghene hier boven ghesyt is vande ervaring mette Son des daechs, mach derghelijcke verstaen worden ende gheschien met yder vaste sterre des nachts, die ghebruyckende al oft de Son waer: maer niet de Maen, eensdeels om heur rassche eyghen loop, ten anderen om tgroot * verscheensicht datse heeft van weghen sij t'eertrijck soo na is.

* Paralaxim.

Merckt noch datmen voor den middach twee drie vier of meer ervaringhen mach doen: Als by gelijcknis, deerste wesende de Son boven den sichteinder 10. tr. inde tweede 15. tr. inde derde 20. tr. ende doende dergelijcke drie ervaringen op sulcke hooghden na middach, soo bevintmen hoe deen met dander overcomt, ende alsmen alsins een selve noortpunt crijcht, tgheeft den Stierman meerder betrauwen op sijn werck.

Seylende een Stierman van oost na west of van west na oost, t'can ghebeuren dat hy opden tijt van 10. of 12. uijren tusschen deerste ervaring en de laetste, een trap of meer verandering der naeldwijsing crijge, waer uyt wijder volghen can, dattet noortpunt ghevonden deur deerste voormiddachsche ervaring, en de laetste namiddachsche, niet overcommen en sal mettet noorpunt gevonden deur de laetste voormiddachsche needle again points at the zero point of the graduation. This being so, the mid-point of the arc in the horizontal plane between the first and the second observation is the desired true north. And as much as the needle there declines, that is the desired variation, as has been described somewhat more fully in the first example.

From what has been said above about the observation of the Sun in the daytime the same may be understood and done with any of the fixed stars at night, which may be used as if it were the Sun; but the Moon should not be used for this, on the one hand because of the rapidity of its proper motion, on the other hand because of the large parallax it has, because it is so near to the earth.

It is further to be noted that two, three, four or more observations may be made before noon. Thus, for instance, the first when the Sun is 10° above the horizon, in the second 15°, in the third 20°. And if three similar observations are made at the same altitudes after noon, they are found to correspond one with the other, and if the same true north is always obtained, this gives the navigator greater confidence in his work.

When a navigator sails from east to west or from west to east, it may happen that in the interval of 10 or 12 hours between the first observation and the last there is a difference of one or more degrees in the variation, from which it may follow further that the true north found from the first forenoon observation and the last afternoon observation will not agree with the true north ſche ervaring, en deerſte namiddachſche, ſonder nochtans dat den Stierman int werck gheſeylt heeft. Dit hem ſoo ontmoetende, hy can daer uijt ramen hoe veel op ſeker uijren varens de naeldwijſing verandert, ende daer op giſſing maken, om trechte noortpunt en naeldwijſing met noch meerder ſekerheyt te hebben. Tſelve canmen oock weten deur de naeldwijſing ghevonden op voorgaende daghen, ende die verleken mette wijſing des teghenwoordighen dachs.

BYVOVGH.

GHEMERCKT de ghegheven naeldwijſing en breede tſamen een ſeker punt anwijſen, ſoo wel op zee als te lande: Soo volght daer uijt meughelick te ſijn, dat ſchepen op een beſtemde plaets in zee, verre van landt malcander vinden connen. Twelck oirboir is onder anderen, om na ſtorm de ſchepen van een vlote weerom by een te gheraken. Men can daer deur oock ſetten een * ſaemplaets, om aldaer ſchepen van verſcheyden oirten, op een beſtemde tijt te doen vergaren.

* *Rendevous.*

FINIS.

De feylen verbetert aldus.

Inde 11. *ſiide inde cant, voor* Canton, *leeſt* Cantan. *Inde* 24. *ſiide inde cant, voor* Azimuthalium ſeu verticulum, *leeſt* Azimuthalem ſeu verticalem. *Inde* 24. *ſiide, voor* ſichtemderſplat, *leeſt overal* ſichteinderſplat, *ende voor* ſichtemderſfront, *leeſt* ſichteinderſfront. *Inde* 26. *ſiide inde ſeſte reghel, voor* ſvvaerſte ſijde veel afvilen, *leeſt* ſvvaerſte ſijde ſoo veel afvilen.

found from the last forenoon observation and the first afternoon observation, without the navigator having made an error in the work. When he finds this, he may estimate from it how much the variation differs in a given number of hours' sailing, and from this may make a conjecture to have the true north and the variation with even greater accuracy. One can also know this when the variations found on preceding days are noted down and compared with the variation of the day in question.

APPENDIX.

Since the given variation and latitude in combination indicate a definite point, both at sea and on the land, it follows from this that it is possible for ships to find each other at a given point at sea, far from the land. This is useful, among other things, to help the ships of a fleet to reassemble after a storm. By this means it is also possible to fix a rendez-vous where ships coming from different directions may meet at a predetermined time.

FINIS.

VAN DE ZEYLSTREKEN

THE SAILINGS

From the *Wisconstighe Ghedachtenissen* (Work XI, i, 24)

INTRODUCTION

§ 1

THE CONTENTS OF THE TREATISE DEVOTED TO THE "ZEYLSTREKEN"

The treatise about the *Zeylstreken* (The Sailings) is devoted to two special tracks, entirely different in character, along which a ship can move over the earth's surface, *viz.* great circle and loxodrome. It is succinctly and lucidly written. As in *The Haven-Finding Art,* the words express Stevin's meaning in a perfectly clear way. That is why this introduction is no more than a short explanatory commentary on the original text here presented. The reader is assumed to be familiar with nautical terminology. Nevertheless it is necessary to make a few preliminary remarks, in order to point out certain differences in character between *The Haven-Finding Art* and *The Sailings.*

The Haven-Finding Art was intended for sailors, and the author hoped that the practice of navigation might at once reap benefit from the method discovered by him, for which reason it was published in a separate booklet; it included an urgent entreaty — backed by Prince Maurice — to test its value at sea. The publication of translations of the work testify to Stevin's desire that seamen of other nationalities might also profit by it. The treatise on *The Sailings,* on the other hand, is a theoretical discussion of a subject belonging to the practice of navigation indeed, but not one which formed a daily concern of seamen sailing in European waters. In this work Stevin deals with problems which had so far been studied by just a few pioneers of nautical science abroad, a study which had been induced by the fact that seamen who had made ocean-voyages had been confronted with these problems in practice and had been unable to solve them. This treatise drew the attention of the Dutch to a subject which was of fairly recent date. In 1534 Nunes had taken it up, and Mercator and Edward Wright had continued the work. With the aid of their publications — not forgetting those of Apian — Stevin had studied the subject, upon which he continued the work of his predecessors, making use of their writings and calculations.

Here another point of difference between the two treatises becomes apparent. It was possible to speak of a personal conception and an original work of Stevin in the case of the description of terrestrial magnetism given in *The Haven-Finding Art* and the profitable use that could be made of the declination of the magnetic needle. *The Sailings* on the contrary is no original work, as the labour of the pioneers forms the starting-point and the backbone of this treatise. The author, however, managed to produce a systematic, well-arranged and complete summary, of the subject, cast in a clever and instructive form. It is the presentation of the matter that we are entitled to call Stevin's own conception.

Another difference between the two works consists in the language in which Stevin addresses the reader. That of *The Haven-Finding Art* was simple, and the seaman of 1600 could easily follow the argument if he wished to. For *The Sailings* this holds only in so far as the application of spherical geometry and the solution of problems by means of measurements on the globe are concerned. But the subject-matter was beyond the mental range of the contemporary seaman as soon as Stevin began to deal with spherical trigonometry or to solve problems mathematically. In those cases it was only the mathematically trained reader who could follow him. This is the reason why *The Sailings* was not of direct use to navigation at the time of its appearance. It was not until much later and very gradually that this knowledge reached the seaman through the textbooks of navigation and that Stevin's work became of use to practice at sea.

As the "Summary of the Sailings" states, this treatise forms part of Stevin's *Hydrography*. It comprises four definitions, followed by eleven propositions, of which two relate to the great-circle track and nine to the loxodrome. At the end there is an "Appendix" on loxodromes.

The first definition says that a "zeylstreeck" is the line which ships describe when they are sailing, *i.e.* the line a ship follows. The name is identical with the term "sailing track", which is now used to indicate the extended fore-and-aft line. In the special case of steering due east or west Stevin speaks of *"oost en west streeck"* (east and west track). Courses pertaining to other directions bear different names.

The second definition concerns the great-circle track, called *"rechte streeck"* (straight track) by Stevin and defined as the shortest distance between two points on the globe. To those who wonder that arcs are called "straight tracks", he says that one may speak of straightness because these lines do not deviate either to the right or to the left. In present-day terminology he would have said that the great circle on the sphere corresponds to a straight line in a plane, in contrast to the *"cromme streken"* (curved tracks), which are defined in the third definition.

Stevin explicitly excludes the equator and the meridians from the *"cromstreken"*. Why he does so, will be explained presently. He defines the *"cromstreeck"* as the line described by a ship steering a constant course. This line is now called a loxodrome, and this term will be used henceforth. Stevin compares the loxodrome and the great circle. A ship when moving along the former follows a constant course, when sailing along the latter a variable course. Nowadays we say that the loxodrome is a line on the earth's surface which cuts all the meridians at a constant angle. This line does not lie in a plane and accordingly is a curve of double curvature. Loxodromes pass round the earth, through higher and higher latitudes; they never reach the pole, for then they would have to run towards the north, which is contrary to the definition. Stevin speaks of *"slangstreken"* and in the margin of *"spirales"*. Although the equator and the meridians are curves on which the course is constant, so that by the definition they are loxodromes, yet they are great circles and do lie in a plane, which is the reason why Stevin does not include them among the *"cromstreken"*. He is addressing the navigator directly when he gives the advice to become thoroughly familiar with the character of these curves and in cases of uncertainty in the position not to attribute errors too readily to the influence of unknown ocean-currents.

In the fourth definition the loxodromes N. by E., N.N.E., etc., are denominated 1 to 8. The last-mentioned one is the course east, falling along the parallel. In the four quadrants the loxodromes have the same form four and four, such as N. by E., N. by W., S. by E., and S. by W., etc. Whatever holds for one out of a group, applies equally to the corresponding loxodrome in another group.

The definitions are followed by the propositions. In the first of these, two points in a given latitude and longitude are assumed on the earth's surface. With the pole these points form a spherical triangle. Of this triangle the six elements are mentioned, while it is stated that if three of them are known, the other three can be found. As examples Stevin takes the determination of: 1) the distance along the great circle between those two points, and 2) the angles between the great-circle and the meridians through the two places, in other words: the course of the ship in the place of departure and that when she has reached her destination. The problem is solved with the aid of spherical trigonometry, for which Stevin refers to the 40th proposition on spherical triangles occurring in his *Trigonometry.*

The second proposition relates to great-circle sailing. It is taught in two different ways how the courses are determined which have to be steered to follow the great circle. The one method is *"tuychwerckelick"* (mechanical), *i.e.* by means of an instrument, in this case the globe. The other is mathematical. According to the first-mentioned method the place of departure is sought on the globe, after which this place is brought in the zenith by rotation of the globe in its stand. The desired course of departure is read on the horizon between the meridian through the place of departure and a pivoting vertical circle set over the place of destination. Use is thus made of a property of the poles of great-circles, *viz.* that the angle between two great-circles is measured by an arc of a great-circle, a pole of which is the point of intersection of these great-circles. As is evident from this, the reader is required to have some knowledge of spherical geometry. The course found is then followed a certain distance — Stevin speaks of 3 or 4 *"trappen"* (degrees), *i.e.* 180 to 240 nautical miles — upon which in the position thus reached the course is again determined in the same way. Stevin points to the change in the course which comes to light if one proceeds in this way, and to the fact that the displacements of the ship each time take place along a loxodrome. By taking the displacements small, one avoids inaccuracy. It can be checked whether the ship is still on the great-circles, as she should be, by a determination of the latitude from observation of the sun or the stars.

For the mathematical solution of the problem the reader is again referred to the treatise on spherical triangles. If the spherical triangle in question is oblique-angled, the 40th proposition mentioned above can again be applied, but the object can also be attained by making use of right-angled spherical triangles, which is less difficult. Each time, after a given displacement of the ship, the course is determined again. If the change in the course is found to be small, the displacements can be taken larger.

In the third proposition Stevin proceeds to deal with the loxodrome. First he discusses the drawing of it on the globe. As one of the aids with which this can be done, a simple instrument is mentioned, a copper model of the loxodrome fitting on the globe, the idea of which — as Stevin states — had been borrowed from the globe-makers. It is clearly described how such models are made for each track, *i.e.* seven in all, and also how the loxodromes are drawn on the globe, from degree to degree of difference of longitude, with the aid of the models. It is mentioned that in theory the loxodrome cannot reach the pole, though in drawing it seems to do so. But then, drawing does not furnish an accurate result.

With the aid of mathematically calculated tables the determination of the shapes of the loxodromes can take place more accurately than by the method described above. To make it possible to construct a complete table of loxodromes, the latitude has to be determined of the points of intersection of seven loxodromes with the meridians, at differences of longitude of one degree.

Stevin describes two methods by which the object can be attained.

The first of these methods is the result of calculation. In small right-angled spherical triangles, the base of which is one degree of the equator or one degree of a parallel, the hypotenuse is a loxodrome, and the perpendicular side is a part of the meridian, the length of this part of the meridian is calculated each time. For this calculation Stevin again refers to his *Trigonometry, viz.* to the 36th proposition concerning spherical triangles. (The reader will be well aware of the fact that the above mentioned triangles are not spherical, as two of the sides are not great-circles.) The result of the calculations are added together. The bases of these triangles are required to be known. This is the case, for Stevin has at his disposal a table in which the length of one degree of the parallel in a given latitude is expressed in minutes of the equator. It is the table of the *"achtste cromstreeck"* (eighth loxodrome), occuring on pp. 138/9, after the *"Tafels der cromstreken"* (Tables of Loxodromes). It corresponds to the present-day table for the reduction of the difference of longitude to the departure (dep. $= \triangle$ L . cos b), with the only difference that in Stevin's table the interval is 30′, whilst in the modern table the interval is 1° and decreases to 10′ as the latitude increases.

When at the end of the fourth proposition Stevin explains the table of the eighth loxodrome, he says he has taken it without any modifications from the *Cosmographia* [1]) of Peter Apian (Petrus Apianus, 1495—1552) [2]). The table in question is already to be found in the first edition — of 1524 — of this author's widely distributed and well-known book (Book I, pp. 42-43). Apart from a few differences, to be ascribed to printer's errors on both sides, it appears to have been copied faithfully in its entirety from Apian, including the numbers indicating the decrease of the length of the degree of the parallel, from 30′ to 30′ difference of latitude, expressed in seconds. It is most likely that Stevin used a late sixteenth-century edition of the book for his purpose. The edition of 1524 is merely mentioned here to show that the table had existed long before this.

After thus having clearly explained how one is to proceed, Stevin says that it would have taken him too much time to construct in this way a complete table of loxodromes by calculation. He therefore merely pointed the way, without following it himself. Continuing his train of thought in the same direction, he might have pointed out that the numbers mentioned opened up the possibility of interpolation for the latitude, and that interpolation was necessary for the performance of the calculation of the table outlined by him, if accuracy was to be attained. But he omitted to do so.

On the other hand he did take over the *Table of Rumbes* of the English mathematician and nautical expert Edward Wright (1558-1615). His candid

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[1]) *Cosmographicus liber Petri Apiani mathematici studiose collectus* (Landshut 1524). This book was particularly widely distributed. It was translated into many languages and passed through a great many reprints in the sixteenth and the early seventeenth century.

[2]) Peter Apian, whose real name was Benewitz or Bienewitz, born at Leisnig in Saxony, geographer and astronomer, from 1527 Professor of Mathematics at Ingolstadt. Maker of maps and instruments, and famous as an observer of astronomical phenomena.

admission of this opens his explanation of the second method by which the table could be made. His source is Wright's *Certaine Errors in Navigation* [3]), a book which had gained fame in England and had first appeared in London in 1599, the very year in which Wright had translated Stevin's *The Haven-Finding Art,* thus making it known in his country.

Stevin took over the table without any modifications, notwithstanding the fact that he had found "some imperfection" in it. He says he will recur to his objections in the "Appendix". The explanation of its arrangement is preceded, by way of introduction, by the explanation of another table, *viz.* that of the *"versaemde snijlijnen"* (assembled secants), also copied by him from Wright. In the latter's work this table was called "Table for the true dividing of the meridians in the sea-chart". Stevin shows that it is produced by the constant addition of the secants of angles increasing by 10′, as follows:

sec 10′ = 10,000,042
sec 20′ = 10,000,168; the sum is 20,000,210
sec 30′ = 10,000,381; the sum is 30,000,591

After the performance of the additions he drops — without stating his reason for it — the last five digits, so that the numbers in the table correspond to a radius $R = 100$. This explanation is followed by the table, which thus gives the sum of the secants of angles from 0°10′ to 89°50′, the interval being 10′.

Stevin says nothing about the meaning of these numbers. For the benefit of the reader it is here observed that in the Mercator chart the length of one minute of the meridian in a given latitude is equal to one minute of the equator multiplied by the secant of that latitude. That is why in that chart the distance from a given parallel to the equator is equal to one minute of the equator, multiplied by the sum of the secants of angles from 0° to that latitude, taken minute by minute. It is this sum of the secants — though at an interval of 10′ — which the table furnishes. It is thus essentially identical with our present-day table of meridional parts [4]). The latter is no longer calculated by addition of the secants, but from the relation $\int \sec \varphi \, d\varphi = \ln \operatorname{tg} (45° + \frac{\varphi}{2})$. If we compare the table published by Stevin with the modern one, the values to be found in the latter have to be multiplied by 10. If we round off the numbers to integers and disregard differences due to rounding off, etc., the tables are seen to agree up to latitude 38°. Beyond that, Stevin's — or rather: Wright's — values become greater. At 60° the difference is 0.3 minute of the equator and it increases to 2.8 minutes of the equator at 82°, where the modern table ends because the practice of navigation requires no data for higher latitudes. The table in Wright/Stevin goes as far as 89°50′, against which the number 226,223 is mentioned.

Next comes the promised explanation of the second method by which the table of the loxodromes can be constructed. The original text (p. 104, which refers to the figure on p. 95) can easily be followed. As an instance, Stevin takes the

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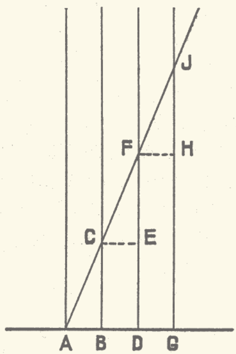
[3]) *Certaine Errors in Navigation, arising either of the ordinarie erroneous making or using of the sea chart, compasse, crosse staffe and tables of declination of the sunne and fixed starres, detected and corrected by Edward Wright* (London 1599). Copies in the British Museum, London, and in the Bodleian Library, Oxford; not in the National Maritime Museum, Greenwich.
[4]) The present-day definition is as follows: meridional parts for any latitude is the length of a meridian, expanded on a Mercator chart, between the equator and that parallel of latitude expressed in minutes of arc of the equator.

determination of the latitudes of the points of intersection of the first loxodrome with the meridians at differences of longitude of 1°, commencing at the equator. In the right-angled triangle, of which the base is an arc of one degree of the equator and the hypotenuse is the first loxodrome (N. by E.) — a triangle which on account of its smallness is regarded as a plane triangle — the latitude of the point of intersection of the loxodrome with the meridian at 1° of longitude is calculated with the aid of the 4th proposition of plane trigonometry. It "is found" that this latitude is 5°1′, and this value is included in the table. In order to find the latitudes of the subsequent points of intersection, Stevin now proceeds as follows. He takes the table of "assembled secants", seeks the meridional parts of 5°1′, and finds 3,014. This number is multiplied by two. Against 6,028 he finds 10°0′, which is the required latitude for a difference of longitude of 2°. Against 3 × 3,014 he finds the latitude for the third point of intersection. We shall recur presently to the calculation of the first point of intersection.

The instructions given by Stevin are indeed perfectly clear, but he does not go into the essence of the matter, nor does he explain why the multiples in question have to be determined. However, this operation becomes clear if we transfer it to the Mercator chart.



In the above figure, which represents a section of a Mercator chart, *AG* is a part of the equator. The points *A, B, D,* and *G* are points of intersection with meridians, at intervals of one degree of longitude. The line *AJ* is the first loxodrome. The latitude of the point of intersection *C* was calculated mathematically by Stevin. The table of "assembled secants" showed him how many minutes of the equator the distance *BC* amounts to. Now △ *CEF* is congruent with △ *ABC*. Consequently *EF* is as many minutes of the equator as *BC*. The distance *DF* in minutes of the equator is thus twice the meridional parts of *BC*. In the table it is looked up what is the latitude corresponding to this number. Thus the latitude of *F* has been determined. The number is included in the table. The congruence applies to all subsequent triangles. In each case, therefore, to find the latitude of a point of intersection, the meridional parts of *BC* have to be added to those of the preceding point of intersection; in other words, the meridional parts of each of the points of intersection form a multiple of the meridional parts of *BC*.

The use of the table of "assembled secants" was a strikingly ingenious idea. Thanks to this, it was possible to reduce the extremely laborious calculation of the shapes of the seven loxodromes to the simple performance of additions and the search of numbers in that table. Although Stevin concludes his argument rather laconically with the words "and so on with the rest of the seven loxodromes", he cannot have failed to realize that the man who originally calculated the table of loxodromes performed an enormous amount of work and made an important contribution to the development of the art of navigation. In Stevin's book the table runs into thirty pages, on each of which the latitudes of 120 points of intersection are given. Stevin was convinced that it would advance both science and cartography. In fact, he emphatically states that it has now become possible to draw the loxodromes with great accuracy on globes and to check their shapes in sea-charts (p. 140); from these words we may infer Stevin's opinion about Wright's work, *viz.* that he had achieved his object. Indeed, the latter had written (page F-2): "the speciall use of this table is for the true drawing of the rumbes in the globe and the chart". Wright had gone no further than the calculation of the points of intersection in question. It was Stevin who made the next move, thus bringing the matter into the nautical sphere, where science is applied in practice.

In his discussion Stevin includes the distance run by the ship. He explains how with the aid of trigonometry the distance along the loxodrome from one point of intersection to the next can be calculated. He performs this calculation only for a very few cases, *viz.* those which he requires in instances to be dealt with later on. The fact that in his opinion Wright's table was still insufficiently accurate — this is the second reference to such inaccuracy — deterred him from completing this calculation. Moreover, he states that he was hindered from doing so by other matters, just as he did before. Thus unfortunately this part of the work remained incomplete from the nautical point of view. Stevin merely pointed the way. Those having a mind and an opportunity for it might complete the table in this respect with the aid of the example.

Next the construction of the table of the eighth loxodrome is explained. In our modern, simple notation it is based on the formula: one degree of the parallel in latitude b is equal to one degree of the equator multiplied by the cosine of b (1° par. in latitude $b = 1^\circ$ eq. $\times \cos b$).

At the end of the fourth proposition Stevin refers again to the "mechanical" method. It may be convenient to have at one's disposal a number of models of loxodromes, made of copper. They could be shaped to correspond to the loxodromes drawn on a globe. Twice seven such models in all were required.

The reader may have wondered why Stevin reckoned with seven loxodromes, *i.e.* with a division of the quadrant into eight parts — corresponding to the division of the compass rose — instead of with a difference in the course of less than one point or 11°15′. The answer lies in the field of the practice of sailing. The ships of the time, being small in size, were difficult to steer. They used to yaw violently and were very unsteady and restless at sea. To be able to steer them with an accuracy to within one point was a reasonable result. A smaller subdivision was senseless. Even small modern sailing craft and yachts, which sail much better and are more manageable than the unwieldy ships of Stevin's day, will do hardly better than this. In consequence the navigator will usually steer on full points and seldom on half points. It is obvious that if the ship is steered on full points, the true course deduced from the compass course need not fall on a full point.

We are now coming back to the passage in which Stevin states that for the point of intersection of the first loxodrome with the meridian at 1° of longitude he "has found" a latitude of 5°1′, in which latitude he found 3,014 for the meridional parts; he continued his explanation with this value. Stated in this way, his words produce the impression that he calculated this value for himself. This, however, is to be doubted, as may be demonstrated by the following.

What were the directions given by Wright in this respect? The following passage is to be found in his book (page F *verso*), where he is referring to his "Table of rumbes", the "Table for the true dividing of the meridians in the sea chart", and to the construction of the former of these tables:

"This table of rumbes is most easily made by addition only with helpe of the table before mentioned shewing how the meridians or degrees of latitude in the nauticall planisphere are to be divided, after this manner. Multiplie the tangens of the angle that the rumb maketh with the equinoctial by 60, the product shall be the first number at the beginning of each table of each rumbe, to bee set over against one degree of longitude [5]), and all the rest are found by perpetuall addition of this number, first to it selfe — for the summe is the number answerable to two degrees of longitude — then to this summe, and so forth in all the rest. These numbers being found out in the table before mentioned did shew at what minute of latitude each rumbe should crosse the meridian for every degree of longitude . . . which being once found, these numbers serve to no further use."

As was the custom in those days, these directions omit to account for the rules given. For the present-day reader this forms no difficulty. He will understand the words and consider their import correct. In fact, the latitude of the first point of intersection is calculated in the Mercator chart — *i.e.* in a flat surface — and is expressed in minutes of the equator. It is of this product that the multiples are taken, upon which the latitudes corresponding to "these numbers" are sought in the Table of meridional parts; and these are the values which have to be included in the table. Still, the rule cannot be said to be formulated very clearly, and it certainly has to be called obscure for the reader of 1600, who was being confronted with a new subject. This reader was bound to expect that the operation would take place in a spherical triangle, and he therefore ought to have been told that it was carried out in a plane triangle. The words "the product shall be the first number . . ." were likely to mislead him. It is true that the last sentence of Wright's argument, which is correct, could cure him of his error; but it is justifiable to speak of a lack of clearness in the text, since for the reader it was hard to realize that "these numbers" also refers to "the first number".

When we now read the corresponding passage in Stevin on p. 535, which refers to the figure on p. 522, we find that Stevin takes as starting-point the spherical triangle *XRQ,* which he treats as a plane triangle, "on account of the smallness of the sides". The side *QX,* or the latitude of the first point of intersection, can be found with the aid of the fourth proposition of plane triangles, to the effect that the side subtending the known acute angle — in this case a part of the meridian — is equal to the base multiplied by the tangent of this known acute angle.

It appears that Stevin has not fully grasped Wright's argument. Possibly he was misled by the insufficient clearness of the text. It was a mistake for him to speak of a spherical triangle. He failed to notice that he was operating with a plane

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[5]) The meaning is: place this number in the table, for the latitude of the first point of intersection, against 1° of difference of longitude.

triangle. Wright's calculation yields the meridional parts of the latitude of the first point of intersection, at the same time the basis for the further operations. If Stevin had understood the directions, he would have had to cipher as follows:

meridional parts b = 60 × tg 78°45′ = 60 × 5.02734 =
301.64 min. of the equator =
3,016.4 units of the table.

His own directions in the form of a formula are:

a) measured in degrees: latitude b = 1° eq. × tg 78°45′ =
1° eq. × 5.02734 = 5°.02734 =
5°1′38″.

b) measured in units of the table: latitude b = 60 × tg 78°45′ = 3,016.4.

We have seen that he made no use of the answer 3,016, nor of 5°1′38″, although it is certain that he knew the latter value, because we meet with it in his criticism of Wright, on p. 581 of the Appendix. The latitude of 5°1′, "found" by him, virtually springs from nowhere, and it has to be assumed that he took it from Wright's table (p. 537 of our edition), and looked up the corresponding number: 3,014.

Stevin's text in some places is incoherent. He understood the operation imperfectly and made the mistake of regarding as latitude the value which really stood for the meridional parts of it. In consequence his directions were incorrect. When we consider this mistake in connection with the imperfections in Wright's text, it becomes evident how difficult it was to attain to the right understanding, although in our eyes it is quite simple. And if Stevin can be said not to have clearly realized the possibilities of the Mercator chart fifty years after its appearance, it is all the less surprising that more than a century had to elapse before this chart with its queer distortion was generally accepted by sailors.

The next seven propositions concern problems which are now called sailing problems. They form the application of the previously discussed theory and an exercise for the student. In each case, a number of elements being known, certain unknowns have to be determined. For all of them Stevin gives three methods of solution, *viz.* one with the aid of the copper models of the loxodromes, one by means of the loxodromes drawn on the globe, and finally the arithmetical method making use of the tables. The text does not call for any further comment. The last-mentioned method was not yet suitable for practical application, since the table was incomplete owing to the absence of one element: the distance. Because Stevin's comprehensive and scientific book in two bulky volumes, of which the treatise on the sailings formed only a modest part, decidedly did not come under the eyes of the practical seaman, the latter derived no benefit from it. No textbook of navigation that was destined for this purpose and contained this information was yet available at that time.

It is only with regard to the problem discussed in the fifth proposition that two remarks have to be made. Here it is required to determine the distance between two places situated in latitude 24°, their difference of longitude being 30°. It is correctly calculated that the distance measured along the parallel — we should now say: the departure — must be 27°24′. But since the circumference of the earth was still unknown, 27°24′ could not be expressed in a linear measure. In this connection Stevin speaks of a variety of miles which are in use in different countries. Miles of different length give different results. In this respect Stevin could not do otherwise than follow the opinion of many people, *viz.* that a degree was equivalent to 18 hours' walk, at 8,000 paces an hour. An hour's walk also used to be equated to 1,500 Rhineland roods, which makes a pace equivalent to 2¼ Rhinelandfeet. The author advises navigators — in the apparent belief that he will be read by them after all — to keep to the result expressed in degrees and minutes, so as to understand one another properly.

In the second place it has to be pointed out that at the end of this proposition (p. 553) Stevin mentions the possibility of the application of interpolation in using the table of the eighth loxodrome.

As announced by Stevin at the beginning, the treatise ends with an "Appendix". This consists of five chapters, in each of which a particular point is dealt with. In the main this amounts to a criticism of the works of Nunes and Wright, the very authors who had formed his sources and the contents of whose works constitute the gist of his treatise. In the Appendix he collects his remarks and objections.

In the first chapter Stevin points out that the method of numbering the loxodromes is not always the same. Thus Hues [6]) reckons from the meridian, calling N. by E. the first loxodrome, whereas Wright commences at the equator, referring to E. by N. as the first loxodrome. Stevin advocates uniformity in this respect and explains why it is preferable to reckon the loxodromes from the meridian rather than from the equator. To him the first way is the natural order, with the "east and west track" or "eighth loxodrome" also classed with the loxodromes, as it ought to be. During the further development of the subject this system was found to be sound and efficient. Wright abandoned his own system.

In the second chapter of the Appendix a remark is made about Nunes (1492—1577) and his treatise on the shapes and the properties of the loxodromes. Stevin disproves a proposition given by Nunes. We shall revert to this criticism in § 2.

As appears from the title, the third chapter treats of inaccuracies in Wright's table of loxodromes. The English, who began to engage in deep-sea navigation after the Portuguese and the Spaniards, also proceeded to study the loxodromes and discovered the mistake made by Nunes. Then Wright's tables were published, which in Stevin's opinion marked a big stride forward. To check them, he made a random test. He figured out the shape of the fourth loxodrome — the fourth involved the smallest amount of figure-work for him — by the method based on spherical trigonometry, described by him in the fourth proposition. He found the point of intersection of this loxodrome with the meridian of 78° to be in latitude 61°26′, whilst in Wright's table he found 61°14′, a difference of "only 12′". He had reason to think that his result was slightly too high, so that he could "assume" — as he says — that the tables in question were "rather accurate".

As to this difference of 12′, or slightly less, it may be observed that the correct value is 61°14′52″, so that the value in Wright's table appears to be

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[6]) Robert Hues (1553-1632) accompanied Thomas Cavendish on his voyage round the world (1586-1588). He was a mathematician and a geographer. Like Hariot and some others, he was patronized by the Earl of Northumberland; he is known in particular for his book *Tractatus de globis et eorum usu* (London 1593), of which an edition appeared at Leiden in 1594.

sufficiently accurate. The difference had arisen because the intervals taken by him in the calculation — *i.e.* 30′, in the table of the eighth loxodrome — were too great. Stevin's procedure consequently was unsound, and so in our opinion was his criticism.

Again Stevin says he had no time to pursue his verification any further. He cannot yet call the table quite perfect, and to justify this statement he derives a proposition with which he had found the numbers not to be in agreement. The operation, which is given on pp. 577 - 583, can be described more simply as follows (See the figure on p. 576 of our edition).

A loxodrome *EI* — the first is meant — commencing at the equator, cuts some meridians at differences of longitude of one degree in the points *K, L,* and *M*. Through these points are drawn parallels, which cut the meridians in *N, P,* and *Q*. *NPQ* is again a first loxodrome. *K* lies in latitude b_1, *L* in latitude b_2. The difference of latitude between *K* and *P* is called $\triangle\, b_1$, that between *L* and *Q*, $\triangle\, b_2$. Because they have very "small sides", Stevin takes the triangles *NKP* and *PLQ* approximately as plane similar triangles and writes:

$$KP : LQ = NK : PL, \text{ or}$$

$$\triangle\, b_1 : \triangle\, b_2 = 1^\circ \text{ eq. } \cos b_1 : 1^\circ \text{ eq. } \cos b_2$$

$$\frac{\triangle\, b_1}{\triangle\, b_2} = \frac{\cos\, b_1}{\cos b_2} = \frac{\sec\, b_2}{\sec\, b_1}$$

Now that this proportion is known and its correctness has been proved, he is going to perform the verification announced by him. He chooses the first loxodrome and will calculate the latitudes of the points of intersection with the 1st and 2nd meridian, *i.e.* *FK* and *GL*. He does so by two different methods, which are here described as methods A and B.

(A) Stevin takes the spherical triangle *KFE*, which to begin with he regards as a plane triangle. Next, by means of his method explained in the fourth proposition (p. 535), he calculates the latitude of the first point of intersection, finding 5°1′38″, a value already referred to. In accordance with the above proposition *NO* becomes 5°0′28″, and thus *GL* becomes the sum of these values, or 10°2′6″, which latitude he says he also finds when ciphering according to the first procedure of the 4th proposition, *viz.* the stepwise calculation. Incidentally it may be remarked that it involved inaccuracy because Stevin was not yet able to make sufficient allowance for the gradual decrease of the cosine of the latitude.

(B) Subsequently determining *GL* by making use of the "table of assembled secants", he finds a different value, in the following way. The meridional parts of 5°1′38″ amount to 3,020; this number, multiplied by 2, gives 6,040, which stands for the meridional parts of 10°1′, a value which is thus found to be 1′16″ less than the value obtained by method A. He thus thinks he has detected an inaccuracy, and states that the difference in question will become greater as the operation proceeds.

If he regarded the triangle *KFE* as a spherical instead of a plane triangle, and then calculated *KF*, he found 5°0′51″ [7]), or 47″ less than the first result, which was at least slightly better. This difference again, though small at first, increases

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7) see: next page.

as the calculation proceeds. Stevin maintains his criticism.

This criticism, however, was not justifiable. Again we find him making the mistake pointed out above: he took for latitude what were meridional parts. In fact, he should have used 3,016 as meridional parts; in that case he would have found the corresponding latitude to be 5°1'.2. This value of 3.016, multiplied by 2, makes 6,032, the corresponding latitude being 10°0'.1. Wright gives 5°1' and 10°0' respectively in his table, so that he was right.

In the fourth chapter Stevin states that the proportion referred to above may serve to construct a reliable table. The interval of longitude is discussed. If the interval of 1° is found too great, the calculation may be performed with 15' or even 10'. But it is doubtful whether the gain in accuracy justifies the additional work. The table of the eighth loxodrome too would have to be calculated for an interval of 1'. Stevin actually seems to have considered this desirability. If he had taken this work in hand himself, some passages of his treatise would have been different and he would not have reached his present conclusions.

The fifth and last chapter deals with a proposal for an improvement of the mariner's compass. This is a purely technical matter, while all the other subjects formed part of the theory of navigation. Some people wanted to replace the usual division of the rose into 32 points by one into 64 parts, although others thought that such small angular differences in the division of the rose could not be read at sea. But Prince Maurice, after having reflected about the matter of accurate steering, had proposed a division into degrees, even into fractions of degrees, provided the compass were large enough in size and were made with great care. The Prince was not aware — nor could he have been — what consequences the use of a large, and consequently heavy, needle entailed: increased friction between the pivot and the cap on which the rose is supported, resulting in unsteadiness of the rose. In the first example a compass is described the needle of which can be moved in relation to the rose, for the adjustment of the variation. A description is also given of the way in which the compass can be mounted accurately on board.

In the second place it is proposed not to make use of a compass-rose to which, as usual, a double magnet in the form of a rhomb had been attached, but to omit the rose and to equip the compass with one magnetic needle. The round bowl was then provided with a division into degrees on the inside. In such a compass the words East and West have to change places. The special difficulties which this apparatus involved in practice with regard to the indication and the reading of courses is clearly described in the text. On such a compass it was also possible to steer true courses. The compass bowl then had to be mounted pivotally on a small pin, so that the lubber line might be made to diverge as many degrees from the longitudinal axis of the ship as the variation amounted to.

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7) This value is correct and is found as the result of one of the formulas for the right-angled spherical triangle: $\text{tg}\ b = \text{tg}\ E \sin \Delta L$ or $\text{tg}\ b = \text{tg}\ 78°45' \sin 1°$.

§ 2

STEVIN'S PREDECESSORS: APIAN, NUNES, MERCATOR, AND WRIGHT

In response to questions which had come up from the practice of navigation, Pedro Nunes (1492—1577) [8]), the learned Portuguese astronomer, mathematician, and nautical expert, in 1534 proceeded to study the line on the earth's surface which is described by ships steering a constant course, *i.e.* the line cutting the meridians at a constant angle. By drawing such lines on a globe he succeeded in appreciating their shape. He found them to be complicated in character. A few years later, in his *Tratado em defensam da carta de marear* of 1537, he gave a definition and description of these lines, which he called *"linhas dos rumos"* [9]). On p. 143 he states: "The rhumb lines are not circles, but irregular curves, which will make equal angles with all the meridians which they cut" [10]). The author in this treatise brings out the difference between sailing on this line and on a great-circle. At first Nunes thought that these lines met at the pole, but later he retracted this statement. In 1566 he published a Latin version of his treatise about the sea-chart, in a more detailed and elaborated form, as part of a longer work, which appeared under the title *Opera* [11]). In this it is described — with figures to illustrate the matter — how the loxodromes are drawn on the globe by mechanical means. The aid which Nunes made use of was a flexible model (*quadrante esférico flexivel*) for each of the eight *"rumos"*, with which to determine the points of intersection with meridians, at equal differences of longitude. Although on a globe of a relatively small size this procedure was defective, so that the line drawn was liable to errors, yet it formed the starting-point for later knowledge about the loxodrome. A second Latin edition of the treatise on the sea-chart is included in the great work of Nunes: *De arte atque ratione navigandi* (Coimbra 1573). In Cap. 26, under the title *Propositum globum rumbis deliniare* (p. 116), the model is illustrated once more and its use is clearly described.

Nunes ascribed a particular property to the loxodrome, *viz.* that the sines of the polar distances of the points of intersection with meridians at equal differences of longitude form a continued proportion. It is this statement which Stevin refutes in the second chapter of the Appendix, making use of the proposition he had

[8]) Nunes, from 1529 onwards cosmographer to King Emanuel of Portugal, Professor of mathematics at Coimbra University, founder of scientific navigation.

[9]) This treatise, together with another treatise and the translation of the famous work of Sacrobosco, forms the *Tratado da Sphera* of Pedro Nunes, published in 1537 at Lisbon. This work was published in facsimile in 1915 by J. Bensaude.

[10]) *Nam serem os rumos circulos, mas linhas curvas irregulares, que vam fazendo com todo los meridianos que passamos angulos iguales.*

[11]) A. Fontoura da Costa in his *A marinharia dos descobrimentos* (Lisbon 1933) says (p. 423) that some authors assume an earlier edition of this work appeared in 1546 at Coimbra. Thus, already Röding in his *Allgemeines Wörterbuch der Marine* (Hamburg 1794) mentions such an edition. But Luciano Pereira da Silva, who described the works of Nunes (*As Obras de Pedro Nunes. Sua cronologia bibliográfica.* Coimbra 1915), denies that such an edition ever existed.

derived in the third chapter. By calculation he shows the unsoundness of Nunes' belief. It is striking that he does so in a cautious and gentle way, and by no means in the form of a reproof.

Stevin's respect for the pioneer in this field is reflected in these words. At the same time it becomes evident that Stevin, when undertaking to study a new subject, goes back to the source. The existence of this source was previously known also to Robert Hues, who in his *Tractatus de globis coelesti et terrestri eorumque usu* (London 1594) in the chapter devoted to the drawing of loxodromes on the globe and to the use of these lines says: *Inventio haec & consideratio delineandi rumbos in globo aliquanto est antiquior. Petrus Nonius Lusitanus multa de his in duobus lib. quos de navigandi ratione conscripsit. Mercator etiam in suis Globis eas expressit* [12]).

To Nunes is due the credit of having raised the art of finding one's way across the ocean — an art which had existed for many centuries before his day — to the rank of a science. Thanks to his work and studies, in Portugal a great height was attained, greater than anywhere else in the sixteenth century. At the same time his work concluded an era in the history of the art of navigation in Portugal. The work of the Portuguese was continued in Flanders, in the first place by Gerhard Mercator (1512—1594) [13]).

Mercator is known to have made a terrestrial globe which carries the date of 1541 as the year in which it appeared [14]). Many compass-roses divided into points appear on it. These directions have been extended so as to form a network of loxodromes. Those who studied and described the globe called this network accurate and admired Mercator's skill in drawing it.

An achievement of unprecedented importance for the art of navigation, owing to which Mercator's name will always be honourably mentioned in the history of navigation, was the compilation of his world-map destined for use at sea. It appeared in 1569. The network of meridians and parallels had been designed in such a way that the loxodromes appeared in it as straight lines. Meridians and parallels are straight lines as well. It is this chart which is known as the Mercator chart, sometimes less properly called the chart on Mercator's projection. It contains a great many legends in Latin. In one of them the requirements which the chart has to satisfy are enumerated. Mercator says: "It is for these reasons that we have progressively increased the degrees of latitude towards each pole in proportion to the lengthening of the parallels with reference to the equator. Thanks to this device we have obtained that . . . no trace will anywhere be found of any of those errors which necessarily be encountered on the ordinary charts of shipmasters, errors of all sorts, particularly in high latitudes" [15]).

It is not known by what method Mercator constructed the network of his chart. He himself has not disclosed this in any posthumous work. The idea that he

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[12]) "This invention and idea of drawing rhumb lines on a globe is rather old. Petrus Nonius from Portugal speaks repeatedly about it in the two books he has written on the subject of navigation. Mercator, too, has represented them on his globes."

[13]) Gerhard Mercator (or Kremer), a Flemish cartographer, nautical expert, maker of instruments, globes, and atlases at Louvain, later Duisburg.

[14]) A specimen of it is to be found in the National Maritime Museum, Greenwich.

[15]) The English translation of all the Latin legends on the chart is to be found in The Hydrographic Review, Vol. IX-2 (1932).

achieved it by mathematical means has to be rejected; probably it has to be assumed that he did it by a graphical method, *viz.* by transferring the points of intersection of loxodromes and meridians, constructed on the globe, to the system of meridians in the chart [16]).

It is to be noted incidentally that although this chart was destined for use at sea, in spite of its useful indications it was not yet very suitable for this purpose. Indeed, the seaman has greater need of detailed charts of limited regions than of a chart comprising all the oceans of the world. Much time had to elapse and a good deal of opposition had to be overcome before the seaman realized the importance of the system on which the chart is based and used it regularly at sea. Claes Heyndericks Gietermaker (1621-1669), writer of the widely used Dutch textbook *'t Vergulden licht der zeevaert* (The Golden Light of Navigation) of 1660, put his finger on the sore spot by ascribing the seaman's failure to promptly adopt improved methods to "inveterate habit, and it is said there is nothing more powerful than habit". In the eighteenth century the octant, which yet made for much greater accuracy in the measurement of the altitude of celestial bodies, met with no better reception. Seamen are conservative, or at any rate they tend to cling to the old familiar ways and methods.

The important problem of determining the distances of the parallels from the equator, as measured in the Mercator chart, was transferred to the mathematical sphere by two English nautical experts of great fame, *viz.* Thomas Hariot (1560-1621; the name is sometimes spelled Harriot) and Edward Wright (1558-1615). In the legend from the Mercator chart quoted above, Mercator states that he had increased the degrees of latitude in the same proportion in which the parallels in his chart have been lengthened, *i.e.* in proportion to the secant of their latitude. These words form the basis of the calculations undertaken by Hariot and Wright.

Of the former a manuscript has been preserved which bears the title: *The Doctrine of Nauticall Triangles Compendious* [17]). It dates from before 1596, in all probability from 1594. The bulk of this work is formed by a *Canon nauticus,* being a "table of meridional parts", from minute to minute, calculated by constant addition of the secants from one minute to the next. Hariot had based these calculations on the table of Clavius (1537-1612) [18]), dating from 1586, which contained the values of the sine, the tangent, and the secant for every minute and to seven places of decimals, as we should now say. Hariot's work was original, for he constructed his table before Wright, but no writings of Hariot have been published, nor was the manuscript in question printed. Thus the credit of having been the first to construct such a table is indeed due to

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16) This is the conclusion reached by D. Gernez in a paper entitled "*Quel procédé Mercator employa pour tracer le canevas de sa carte de 1569 à l'usage des marins?*" published in the Communications of the Académie de Marine de Belgique, Tome I, 1936-37. In this paper the writer reviews the numerous hypotheses about this matter and gives his own opinion.

17) The manuscript was described by Prof. Eva G. R. Taylor in The Journal of the Institute of Navigation, Vol. VI, 1953, p. 131. For Hariot, reference may also be made to her book *The Mathematical Practitioners, Cambridge* 1954.

18) Clavius, whose real name was Christoph Schlüssel, a Jesuit and a teacher of mathematics in Rome as well as Vatican astronomer, was one of the most diligent commentators on Sacrobosco.

Hariot, but since his *Canon* was not published, neither science nor navigation benefited by his work.

It is quite a different matter with Wright. He, too, calculated a table, which is known from the textbook by Thomas Blundevile (1560-1602), entitled: *His Exercises, containing sixe treatises . . . to be read and learned of all young gentlemen* (London 1594). This book was used and esteemed for fifty years, by the educated reader rather than by the simple seaman learning his trade. The treatise devoted to navigation comprises a chapter on the sea-chart, in which Cortes and Coignet are quoted and the loxodrome is illustrated and described. At the end there is a reference to Mercator and his worldmap with its increasing distances between the parallels. How this increase was brought about, "by what rule I knowe not", says Blundevile, "unlesse it be by such a Table as my friende M. Wright of Caius Colledge in Cambridge at my request sent me (I thanke him) not long since for that purpose, which Table, with his consent, I have here plainlie set downe together with the use thereof as followeth". It gives the meridional parts from latitude 1° to 80° in sixtieths of a degree of the equator, at intervals of 1°. For 1°, 30°, 52°, and 80° it gives respectively 60, 1883, 3668, and 8399, as against 60, 1888, 3665, and 8375 in the present-day nautical tables. The difference increases gradually with the latitude. Blundevile explains how the network of the chart is to be made. The parts of the equator are measured along the meridian, after which by means of the table the parallels are drawn at the correct distances from the equator. It is true that Blundevile owes this table to Wright, but it differs from the one which Wright himself published in 1599. This table, in its original form, has also been in the hands of others. It has been used indiscreetly, but it is beyond the scope of this introduction to go into this.

The appearance of the previously mentioned textbook, *Certaine Errors in Navigation . . . detected and corrected by Edw. Wright* (London 1599), forms an important landmark in the development of scientific navigation [19]). The author, who had gained knowledge and experience at sea and who was an eminent mathematician, prepared a second and amended edition, which appeared in 1610. Many years after Wright's death, Joseph Moxon edited the third edition (1657), to which he added the translation of *The Haven-Finding Art.* The fact that the book continued to be sold and used until the end of the seventeenth century bears witness to the great influence of Wright's work on the deepening of nautical knowledge in the seventeenth century.

In his "Praeface to the Reader", Wright enumerates the many imperfections inherent in the art of navigation (which was already "some thousands of yeeres" old), in particular those relating to the chart. His treatise contains the means to improve the subjects in question. In the present context attention will be paid only to his tables of meridional parts and those of loxodromes. Of the latter he says with justifiable pride: "with help of which table, the Rumbes may in any chart, mappe or globe much more truely be described then by those maechanicall wayes long since published by Petrus Nonius, or latily practised by some globemakers in England" (p. ggg 3).

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[19]) The title-page of the book, the "Praeface", and "A table for the true dividing of the meridians in the sea chart" are reproduced in The Hydrographic Review, May 1931, under the title of : Origin of Meridional Parts.

Wright gives a clear exposition of the construction of the network of the Mercator chart and concludes with the following words, which seem to echo those of Mercator: "the parts of the meridian at every poynt of latitude must needs increase with the same proportion wherewith the secantes or hypotenusae of the arke, intercepted betweene those pointes of latitude and the aequinoctiall do increase. Now then wee have an easie way layde open for the making of a table (by help of the Canon of Triangles) whereby the meridians of the Mariners Chart may most easily and truely be divided into parts, in due proportion from the aequinoctiall towards either pole". (p. D) The basis for the table had thus been described. He made it "by perpetuall addition of the secantes answerable to the latitude of each point or parallel unto the summe compounded of all the former secantes, beginning with the secans of the first parallels latitude and thereto adding the secans of the second parallels latitude" (p. D).

It was originally Wright's intention to publish the table of meridional parts with an interval of 1′ of latitude, up to a maximum latitude of 89°59′ and expressed in units of 0.0001 minute of the equator. But on second thoughts, for the sake of simplification, he chose an interval of 10′ and for the unit he took 0.1 minute of the equator, which did not result in any "sensible error". In this form the "Table for the true dividing of the meridians in the sea chart" is to be found in his book. It covers a range from 0°10′ to 89°50′.

He considered it necessary to drop three digits (p. D *verso*) "not onely for the easier, but also for the truer making of the table". "Truer", because he may have doubted the accuracy of the last decimal places. Wright was aware that the values in his table were a little too great, but he considered it sufficiently accurate. If anyone wished to attain greater accuracy and to work with a smaller interval, he might construct a table with he aid of the *Canon magnus triangulorum* of Rheticus (1514-1576) [20].

Next Wright describes (p. F *verso*) how this table has to be used for the construction of the "table of rumbs". In a few words it is said that this is done by "perpetuall addition", but no further explanation is given of why this is so. These words, however, have already been explained by the proof given above (p. 484), that the meridional parts of each of the points of intersection form a multiple of the meridional parts of the first point of intersection.

The table , which takes up twenty pages, now follows. It gives the points of intersection for seven loxodromes with meridians at a difference of longitude of 1°, and goes as far as latitude 89°59′. This great range shows that theory went further than practice. In fact, for practical purposes it was not necessary to carry the calculation thus far. Wright reckons the loxodromes from the equator, so that E by N is the first. The construction of the table must have taken a great deal of time. The number of points of intersection which had to be calculated was 3,600. The calculation of the table of the seventh loxodrome (N by E) alone — the longest of all — necessitated as many as 1,440 determinations of latitude. This loxodrome passes exactly six times round the earth before reaching latitude 89°59′.

It is this table which has been faithfully copied in its entirety by Stevin, with

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[20]) George Joachim Rheticus, whose real name was Joachim von Lauchen, born at Feldkirch (Vorarlberg, Austria), in the former province of Rhätikon and called Rheticus on this account. Mathematician and astronomer: 1537 Professor at Wittenberg; cooperated with Copernicus; 1542 Professor at Leipzig.

the exception of the order in which the loxodromes were taken.

In § 1 it has already been shown that Apian must also be reckoned among Stevin's predecessors in this field.

§ 3

CONCLUSION

It has already been stated that Stevin, when copying Wright's tables, wrote that he had found "some imperfection" in them, words which cannot but be regarded as a kind of accusation. Recurring to his objections in the Appendix, Stevin says that he has made random tests with the fourth loxodrome — not with the others — and that he has found differences. They were not great and not of a nature to prevent him assuming that Wright's tables were reasonably accurate. He therefore drew attention to differences which in his opinion were to be found in them, but he did not go to the bottom of the matter. He was moderate in his criticism and took over the tables, in spite of his objections to them.

It is only natural that Wright, who had seen the *Wisconstighe Ghedachtenissen,* did not leave the matter there, although he esteemed Stevin's work. In the second edition of *Certaine Errors* he included a detailed reply, under the title: "Simon Stevin his errors, in blaming me of error in my tables of Rumbes" [21]), from which it is quite evident how strongly he resented Stevin's criticism. He expresses this in his words: "but we shall (I make no doubt) find a greater fault in his fault-finding. Though I never durst presume to imagine I could set forth anything without some fault. It sufficeth me if the fault be so little that it cannot sensibly be discerned, which I hope I shall hereafter shew." These words at the same time characterize the object his argument was intended to achieve.

Wright evidently considers it exaggerated to criticize a difference in the latitude of no more than 12′ and writes: "surely therefore Master Stevin might have eased me of some trouble and have saved himself some labour and reputation both, if he had spared his pains in seeking to find fault with me for so small a matter, whereby navigators could incur neither dammage nor danger, although mine error were so much as he supposeth. For in the practice of navigation what inconvenience can arise to a seaman by such an error as shall cause him to goe wide of his course no more then 12 minutes, in sailing so far that he must alter his longitude 78 degrees and his latitude 61 degrees and more?" On the enormous distance this difference was very small indeed, from the practical point of view.

Wright went more thoroughly into the matter and recalculated his table of meridional parts twice, aided by two calculators, this time from minute to minute. He found no differences that might result in any "sensible error". He also recalculated the table of the eighth loxodrome, which Stevin had taken from Apian, this time with an interval of 10′ in the latitude. He checked the values for the fourth loxodrome, found by Stevin — who had worked with the table of Apian — and he applied

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[21]) In the third edition this reply to Stevin is again included, in the same words. The only difference is that the tables have now been printed in a more compact form.

the newly calculated table, the result being that the difference of 12′ was reduced to a very small one. "The truth is . . . thinking it would be sufficient if my Table of Rumbs erred not any where so much as a minute in the latitude of any rumb, which by this triall already made, I assure my self they do not." Wright is of the opinion that Stevin's error is in "his own grosse manner of triall, much more then in my table". If he had worked with smaller intervals, he would have found smaller deviations. And thus the duel was ended. Wright openly spoke his mind. He showed the lack of justification of Stevin's criticism and called his checking method too gross. But that Stevin's "reputation" was injured is a statement which is not in keeping with the general tone of his reply. It was just a little too sharp, and certainly not in accordance with his esteem of Stevin when he translated *The Haven-Finding* Art and wrote an introduction to it.

Paragraphs 1 and 2 have clearly confirmed that the substance of *The Sailings* forms no original work, as we already stated at the outset. Stevin, wishing to become informed about a subject which had never before been discussed in Holland and which was of great import to navigation, sought this information in the works of the few foreign scholars who had already studied the matter. He went back to the sources, as scientists are accustomed to do, and he properly mentioned his sources. He thoroughly studied the writings cited above and took over the main points. He acted in the same way with regard to the system of drawing the loxodromes on the globe by means of models — a system which had already become known among the globemakers of his day —, the old reduction table of Apian, and the two new tables of Wright. He incorporated these matters in his treatise and in his own words elaborated the whole into a logical and conveniently arranged discourse. The form into which he cast the treatise commands our admiration. The clarity of his words proves how familiar he had made himself with the subject. The criticism — directed in particular against Nunes and Wright — which Stevin considered himself justified in making, was included in the Appendix. Wright reprimanded him by showing that the argument was unsound and unfounded. He might also have pointed out the error in thought made by Stevin, which the latter might have avoided by following the example. But this was left by Wright to posterity.

Stevin's predecessors had developed the theory in their writings in the form of more or less elaborate expositions, incorporating the definitions in the text, so that they were virtually hidden in it and the reader had to discover them for himself. Moreover, the reader was faced with difficulties because the authors, though they did give indications as to the way in which some of the calculations had to be performed, omitted to give an explanation of the reason why. Stevin, on the other hand, renders his reader the great service of putting definitions foremost, formulating them clearly, and giving them in large print in his book. This made it much easier for the reader to understand the gist of the matter. The systematic composition of the treatise was of great value to students and must have aroused their gratitude to the author. Stevin's work was exceptional, since long after his day the textbooks on navigation were still deficient in point of proper arrangement of the subject-matter, systematic composition, rigour, and briefness of the argument. We refer, for instance, to the two popular textbooks of Gietermaker and Klaas de Vries, which were used in Holland almost to the exclusion of all other books, the former from 1660 to about 1800 and the latter from 1702 to about 1820. It is not surprising that from such books the navigator was unable to learn spherical trigonometry, for instance.

Stevin's book of 1605 was of quite a different nature. The reader who was acquainted with the language of mathematics and the derivation of formulas was able to learn a good deal from it, thanks to Stevin's search in foreign sources, his able and careful elaboration of the subject-matter, and his lucid presentation of it. Here again — as with *The Haven-Finding Art* — it may be said that his work was of great use to his contemporaries. There is more: the practice of Dutch shipping, too, owes him a debt of gratitude. This will become evident from the following pages, in which we shall briefly describe how his work was continued in the Netherlands.

To Willebrord Snellius (1591-1626) [22]), who translated the *Wisconstighe Ghedachtenissen* into Latin [23]), and who was therefore intimately acquainted with their contents, the credit is due of having widened the knowledge of the loxodrome by his book entitled *Tiphys Batavus* [24,25]). It contained a long introduction, a treatise on the loxodrome, another devoted to the determination of the speed and the dead reckoning, and finally two tables. The first of these is entitled *Tabulae canonicae parallelorum.* It is a table of meridional parts from 0° to 70°, to four places of decimals, with an interval of 1′, made by adding up the secants from minute to minute. Verification has shown it to be very accurate. It is not identical with that of Wright/Stevin. The second is a table of loxodromes, entitled *Canones loxodromici.* It has quite a different character from that of Stevin's Table of Loxodromes. The object of Stevin's table was primarily to become acquainted with the true shapes of the loxodromes on the earth's surface. It is true that Stevin had described how sailing problems could be solved by means of his table. However, not only was this method inconvenient, but it could not be followed because, but for an occasional exception, Stevin had not included the calculation of the distance in his table. It was thus incomplete. The table of Snellius was destined for practical use at sea. It is divided into quarter points and with the difference in latitude from minute to minute as argument gives the distance and the departure, the latter two magnitudes in miles of 15 to the degree. In this book Snellius showed how sailing problems are solved by means of the table. His system undoubtedly marked an advance on that of Stevin. Nevertheless this again was not altogether suitable for practice as yet, because the search in the column of the difference in latitude was not satisfactory. This book moreover, being of a scientific character and being written in Latin, was above the head of the seaman.

We shall pass by the achievements in this field of Ezechiel de Decker (*ca* 1595-1667), surveyor and mathematician [26]), and of Adriaan Metius (1571-1635),

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[22]) Willebrord Snellius, mathematican and physicist, Professor at Leiden, famed for his method of measuring a degree of latitude and for his law of refraction.
[23]) *Works* XIb.
[24]) Willebrord Snellius van Royen, *Tiphys Batavus, sive histiodromice, de navium cursibus et re navali* (Leiden 1624).
[25]) Tiphys was the steersman of the ship "Argo", known from Greek mythology through the voyage of the Argonauts. Tiphys Batavus presumably means: the Dutch Tiphys, or the Dutch steersman.
[26]) Ez. de Decker lived at Gouda, later at Rotterdam. He admired Stevin and is known to have published the first Dutch table of logarithms: *Nieuwe Telkonst, inhoudende de logarithmi* (Gouda 1626).

Professor of mathematics and astronomy at Franeker, but direct our attention at Cornelis Jansz. Lastman (died before 1653). The latter had been born in Vlieland, had followed the sea, and around the middle of the seventeenth century at Amsterdam, in the Haarlemmerstraat, ran a nautical college, which was called *In de vergulde Graed-Boogh* (In the Golden Cross-staff) and which was continued after his death by his son Simon.

Lastman compiled a table of meridional parts as well as *Tafelen der Compas-streecken* (Traverse Table), which are to be found in the textbooks of navigation published by him [27]. As with Stevin, this table applies to seven loxodromes, while there is also a table for the eighth loxodrome, which gives the reduction of departure into difference in longitude. It covers a range up to latitude 80° and the interval is 1′.

Whilst Stevin had furnished the latitude of the points of intersection with the meridians for the seven loxodromes and his table was intended to advance science, Lastman's table was so arranged that it was destined and suitable for practical application. Its character, therefore, is different. For the seven loxodromes, commencing at the equator and extending as far as latitude 75°, it gives the longitude and the latitude of the points through which the ship passes as she sails upon the loxodrome, at intervals of one geographic mile (1 geographic mile = 4 nautical miles). The argument is the distance run. The table takes up 50 pages, each of which contains 325 calculated places. The seventh loxodrome alone takes up 18 pages and necessitated the calculation of 5,760 places. The figure-work must have been enormous indeed.

Although an interval of one point in the course was large and consequently inconvenient — because a true course obtained by reduction of the course steered seldom falls on a full point — still it could be used in practical navigation at sea in order to determine the position when the course and the distance run were known, or *vice versa* — though this was slightly more difficult — to determine the course and the distance between two known places. Lastman's table was taken over by others. It is found again in many books, and was used for a long time, even to the early part of the nineteenth century. It was ousted by the traverse table which Cornelis Douwes (1712-1773) [28] included in his *Zeemans-Tafelen* [29]. On the

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27) Cornelis Jansz. Lastman, *Lastmans beschrijvinge van de Kunst der Stuerlieden* (Amsterdam). Many editions are known: 1642, 1648, 1653, 1657, 1661,1675, 1714. In addition: Vlissingen 1659.

28) Corn. Douwes was mathematician to the Admiralty, to the East India Company, and to the city of Amsterdam, examiner of naval officers and navigators, teacher at the "Zeemans-Collegie", on Oude Zijds Achterburgwal, Amsterdam.
Douwes made the seaman independent of the determination of latitude which was based on the observation of the sun at the moment of its meridian passage. He discovered a simple scheme of calculation, which could be applied by the common seaman, with the aid of which the latitude could be figured out from two altitudes of the sun outside the meridian, the time interval between the observations being known. His method was applied from 1750 to about 1850 by all seafaring nations of the world, in the Netherlands to the end of the nineteenth century. The possibility created by Douwes greatly promoted the safety at sea. From the English, Douwes received a remuneration for his discovery (*cf.* my book *Cornelis Douwes, 1712—1773, zijn leven en zijn werk* (Haarlem 1941).

29) *De noodige en bij ondervinding beproefde nieuw uitgevondene Zeemans-Tafelen en voorbeelden tot het vinden der breedte buiten den middag, door Cornelis Douwes* (Amsterdam 1761). Up to 1858 this table passed through 16 editions in the Netherlands. It is met with in numerous tables, *e.g.* in English and American tables.

English model he constructed a table in which, the course and the distance run being known, the difference in latitude and the departure were found. The interval in the course was small. A point was divided into eight parts, thus: 1/8, 1/4, 1/3, 1/2, 2/3, 3/4, 7/8, a division which did not hold its ground and which was not used in England.

We will conclude with two opinions about Lastman's table, pronounced by colleagues of his.

Pieter Rembrantsz van Nierop (died 1708) prepared an amended edition of the textbook written by his uncle, the shoemaker-astronomer Dirk Rembrantsz van Nierop [30]), author of a large number of books on navigation and astronomy. In the preface he says: the art of navigation, as described formerly by the Portuguese, the Spaniards, Medina, Coignet, Zamorano, and William Bourne, "was no more than an *A-B* in comparison with that of the present day, for when our C. J. Lastman in 1621 [31]) first brought forward his Tables of Sines, Tangents, and Secants as well as his Table of Meridional Parts and eight Tables of Loxodromes, for use in navigation on a spherical earth, people made fun of him, saying: What does the man mean by these things? And now see how he has augmented and amended it up to 1640, and indeed, how he is now being followed by other writers, so that it is evident from this how various errors in navigation can be corrected more and more, some of them by the correct application of the art and others by diligent observation."

The second opinion was given by Simon Pietersz (born in 1601), a nautical teacher at Medemblik. In the textbook written by him [32]) he concludes each discussion of a given subject with an interrogation of the pupil. After the discussion of the calculation of the position by dead reckoning we read (p. 111):

"Question: What do you make use of when you want to indicate the position in the chart?

Answer : Now one thing, now another. But in general I have recourse to the miles and degrees which I find in the *8 Compasstreken van Lastman* [33]).

Question : Why, are they so excellent?

Answer : They are so wonderful, so commendable and infallible that they surpass all other means and their use."

In the paragrahps 2 and 3 we have shown that slightly more than one hundred years elapsed between the moment at which the scholar Nunes began to study the loxodrome and that at which the Dutch seaman could avail himself of the knowledge that had been gathered about it and in day-to-day practice at sea was able to perform calculations concerning the distance run or the course to be followed. The way was long. Stevin was one of those who helped to pave it.

This statement applies to the Netherlands. In England this part of the art of navigation developed along different lines. Richard Norwood (1590-1675), who had followed the sea and later became a nautical teacher in London, famous for his method of measuring a degree of latitude (1635), calculated a table of courses

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[30]) Pieter Rembrantsz van Nierop, *Verbeterde en vermeerderde Nieroper Schat-Kamer* (Amsterdam 1697).

[31]) Probably this has to be 1631.

[32]) *Simon Pietersz. Stuermans Schoole, in welcke de navigatie ofte Konst der Stuerluyden seer ordentelick en bequamelijck voorgestelt en geleert wert. Oock heel gedienstigh om in de schoolen der navigatie ghebruyckt te worden* (Amsterdam 1659).

[33]) Traverse Table of Lastman.

and distances, which, the interval in the course being 1° and the distances being expressed in nautical miles, gave the difference in latitude and the departure, the latter two magnitudes not in miles but in tenths of miles [34]). In later editions they were given in miles as well as tenths of miles, so that the construction of this table was identical with the one that is now known and used at sea.

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[34]) This table is to be found in his textbook *The Sea-Mans Practice* (London 1637). The book, which was very popular and the 15th edition of which appeared in 1682, was reprinted and used up to the early eighteenth century (in 1716, for instance).

VIERDE BOVCK DES EERTCLOOTSCHRIFTS, VANDE ZEYLSTREKEN.

Argumentum.

CORTBEGRYP

DER ZEYLSTREKEN.

WAnt de menichvuldighe vvyde zeylagen deser landen verscheyden souckers veroirsaeckt hebben, van vonden streckende tot voordering der groote zeevaerden, die elck verthoonde an sijn VORSTELICKE GHENADE als Admiral, om daer me tot hun voordeel te gheraken: Soo is de stof des * Zeeschrifts een der besonder oirsaken ghevveest, die hem track totte begheerte en oeffening der * VVisconsten: Sulcx dat hy deursien heeft al het oirboirste en diepsinnichste dat van die stof mijns vvetens ghehandelt vvort. Vant'selve Zeeschrift nemen vvy voor ons hier te beschrijven dit vierde bouck vande Zeylstreken, vvaer achter noch volghen sal den handel vande Havenvinding, en oock van Ebbenvloet, overmits vvy daer in vvat besonders hebben, dat in dese vvisconstighe ghedachtnissen sijn plaets vereyscht. Angaende de rest des Zeeschrifts, daer toe verstrecken hem tot ghedachtnis verscheyden boucken van die stof handelende, en door hem oversien.

Hydrographiæ. Mathematicarum artium.

Definitiones.

Dese beschrijving der Zeylstreken, sal na vier noodighe* bepalinghen der eyghen vvoorden, begrijpen 11 voorstellen, vvelcker tvvee eerste sijn van rechte Zeylstreken, d'ander van cromme. VVaer achter noch volgen sal een Anhang der Cromstreken.

BEPA-

SUMMARY OF THE SAILINGS

Since the numerous voyages from these parts to remote countries have induced many investigators to find aids for ocean navigation, which each by himself, striving after his own profit, showed to His Princely Grace as Admiral, the subject of hydrography was one of the causes which urged him especially to study and practise mathematics, so that he investigated all the most useful and subtle things that have to my knowledge been said about that subject. Of this hydrography we here intend to describe this fourth book, of the Sailings, which will be followed by the treatise on the Haven-Finding Art, and also that of Ebb and Flow, since we have made some remarkable discoveries about this, which require their place in these mathematical memoirs. As regards the remainder of hydrography, various books dealing with that subject, which he has examined, serve to aid his memory.

This description of the Sailings is to contain, after four necessary definitions of the special terms, 11 propositions, the first two of which refer to great-circle tracks [1]), the others to rhumb lines [2]). This is to be followed by an Appendix about loxodromes.

[1]) Orthodromes.
[2]) Loxodromes. This is the term that will be used in the translation.

BEPALINGHEN.

1 BEPALING.

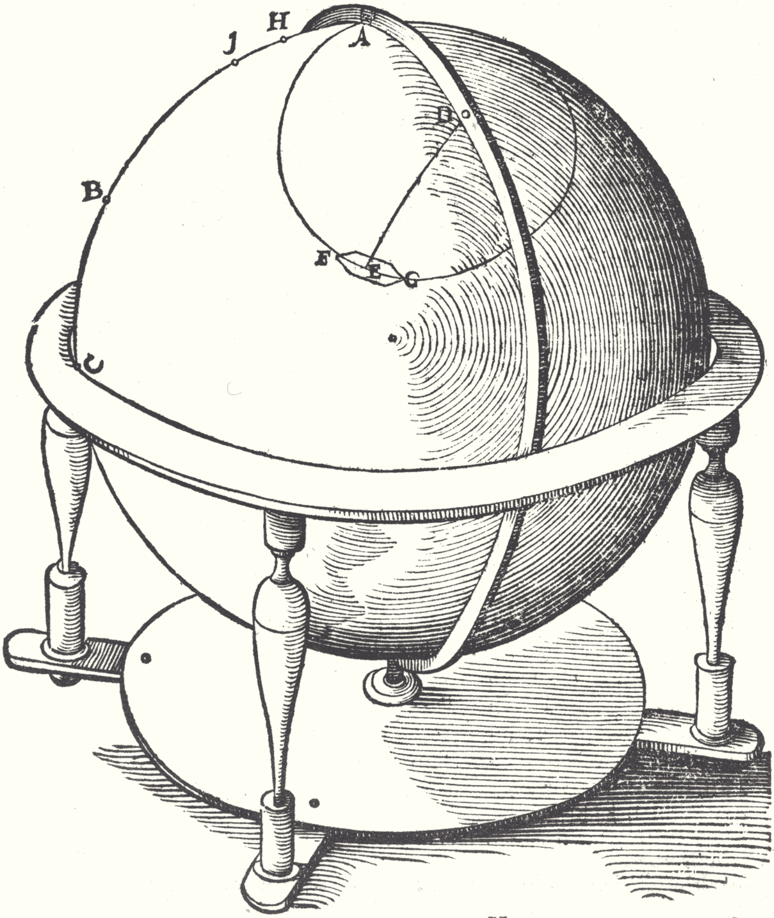
Zeylstrekẽ sijn de linien die seylende schepen beschrijvẽ.

Als een schip seylende van oost na west, de verdochte lini of streeck diet int varen beschreven heeft, heet int ghemeen zeylstreeck: Int besonder oostenwest streeck, en van ander winden of oirten crijchtse ander namen.

2 BEPALING.

Rechte streec noemen vvy des Eertcloots cortste booch tusschen tvvee punten.

Laet op den Eertcloot A B C, tusschen de twee punten A en B, ghetrocken sijn de booch A B, wesende de cortste neem ick die daer tusschen ghetrocken



DEFINITIONS

1st DEFINITION.

Sailing tracks are the lines which ships describe when they are sailing.

Thus, when a ship sails from east to west, the imagined line or track which it has described in sailing is in general called sailing track, in particular east and west track, and according to the other points of the compass or places it receives other names.

2nd DEFINITION.

Great-circle track we call the shortest arc on the earth between two points.

On the earth *ABC* let there be drawn, between the two points *A* and *B*, the arc *AB*, being the shortest, I assume, which can be drawn between them, which must be the arc of a great circle. Such directions are nowadays marked in mariner's can worden, t'welck sijn moet de booch eens grootste rondts: Soodanighe streken worden uu ter tijt beteyckent inde zeecompassen mette 32 linien daer in beschreven, welcke deur t'ghedacht vanden * doender af langs t'vlack des eertcloots voortghetrocken tot inden sichteinder, of anders op de gheboothste eertclooten alsoo gheteyckent, bedien de 32 ghemeene streken, of winden.

Efficiente.

Angaende ymant dencken mocht hoe dat cromme boghen eyghentlick ghenouch rechte streken ghenoemt worden, die sal weten dat dese rechtheyt geseyt wort int ansien datse noch ter rechter noch ter slincker sijde en wijcken, gelijck wel doen de cromme streken, diens bepaling volght.

3 BEPALING.

Een schip buyten t'middelront en middachront so seylende, dat de booch ghetrocken vande kiellini totten aspunt, altijt op de kiellini een selven houck maeckt: De lini die t'schip dan gheseylt heeft noemen vvy Cromstreeck.

Laet inde form der 2 bepaling D des eertcloots aspunt sijn, E een schip, t'welck gheseylt hebbe van A tot E, soo dat de booch E D, getrocken vande kiellini F G als van E totten aspunt D, altijt op de kiellini F G maecke een selven houck als F E D, t'welck soo ghebeuren soude als t'schip altijt op een selve streeck seylde die t'zeecompas anwijst, en dat de leli altijt recht noort wese. Dit soo sijnde, de lini of booch A F E die t'schip gheseylt heeft heet cromstreeck. Nu ghenomen dat den houck F E D recht sy, soo sal het schip altijt recht oost of recht west angevaren hebben, en de booch A E sal deel eens kleenronts sijn: En dattet gheduerlick soo voortseylde, het soude weerom commen ter plaets van A daert begost, volschrijvende het rondt. Hier uyt canmen verstaen dat de rechte oost streeck A B, en de cromme A E, veel verschillen: Om van t'welck breeder verclaring te doen, soo laet C beteyckenen het recht oostpunt, wesende de gemeene sne des sichteinders en middelronts, en t'punt A sy onder het sop, en t'punt B inde booch A C, en de booch A F E sy even met A B. Dit soo sijnde, ghenomen dat een schip seylde van A na C altijt op de booch A C, het sal int ansien des gheens die an A is, altijt recht oostwaert anseylen, maer niet int ansien des gheens die int schip is, welcke geduerlick grooter en grooter verschil sal vinden, ja ten eynde soo groot, dat genomen t'punt A te liggen op de breede van 50 tr. soo sal den seylder ontrent C commende, hem bevinden te seylen oock ontrent de 50 tr. van oosten na zuyden. Wederom, hoe wel B recht oogst light van A, nochtans een schip seylende van A af altijt recht oost an int ansien des seylders, en sal niet geraken tot B, maer verre van daer tot E, welverstaende dat de booch A E even ghenomen wort met A B alsvooren.

Merckt noch dat hoe wel B recht oost light van A, nochtans so en ligt A niet recht west van B, t'welck op groote boghen veel verschillen can. Laet by voorbeelt van een plaets diens breede 45 tr. ghenomen worden een booch van 90 tr. recht oostwaert: De rechte streeck van die oostersche plaets na d'ander, en sal niet sijn recht west, maer so veel noorderlicker als de breede bedraecht, te weten 45 tr. dats recht noortwest. Sulcx datmen om vande westlicker plaets te seylen na de oostelicke, beginnen moet recht oost an, maer vande oostelicke na d'ander (om op een rechte streeck te seylen, want op cromstreken heeft het weerkeeren altijt de naem des teghenoverwint vant wechvaren) noordwest an: En waer de

plaetcompasses by the 32 lines described therein, which, being produced mentally by the observer over the surface of the earth to the horizon, or otherwise drawn in this way on the model globes, designate the 32 common points of the compass or "winds" [1]).

Since one might wonder how arcs can properly be called straight tracks, one is to know that this straightness means that they deviate neither to the right nor to the left, as do the curved tracks, the definition of which follows.

3rd DEFINITION.

When a ship sails outside the equator and the meridian in such a way that the arc drawn from the ship to the pole always makes the same angle with the keel-line [2]), the line on which the ship has then sailed we call loxodrome.

In the figure of the 2nd definition let *D* be the pole of the earth, *E* a ship which shall have sailed from *A* to *E,* so that the arc *ED,* drawn from the keel *FG, viz.* from *E* to the pole *D,* shall always make the same angle with the keel *FG, viz. FED,* which would happen thus if the ship always sailed on the same point indicated by the mariner's compass and if the fleur-de-lys invariably pointed due north. When this is the case, the line or arc *AFE* on which the ship has sailed is called loxodrome. If it is now assumed that the angle *FED* is right, the ship will always have sailed due east or due west, and the arc *AE* will be part of a small circle. And if it sailed continually on in this way, it would come again to the place *A* where it began, completing the circle. From this it can be understood that the orthodromic course east *AB* and the loxodromic course east *AE* are widely different things. To give a fuller explanation of this, let *C* designate the true east, being the common intersection of the horizon and the equator, and let the point *A* be in the zenith and the point *B* in the arc *AC,* and let the arc *AFE* coincide with *AB* in *A.* This being so, if it is assumed that a ship sailed from *A* to *C* always along the arc *AC,* to one who is in *A* it will always appear to be sailing due east, but not so to one who is in the ship; he will find a constantly greater and greater difference, even finally so great that if the point *A* is assumed to lie in latitude 50°, the man in the ship, coming near *C,* will find he is sailing about 50° south from the east. Again, though *B* lies due east of *A,* nevertheless a ship appearing to the man in the ship always to be sailing from *A* due east will not arrive at *B,* but, far from there, at *E,* it being understood that the arc *AE* coincides with *AB* in *A* as above.

Note further that although *B* lies due east of *A,* nevertheless *A* does not lie due west of *B,* which may differ a good deal on large arcs. For example, from a place whose latitude is 45° let an arc of 90° due east be taken: the great-circle course from that more easterly place to the other will not be due west, but so much more to the north as the latitude amounts to, *viz.* 45°, that is due northwest, so that, in order to sail from the more westerly to the easterly place, one first has to sail due east, but from the more easterly to the other place (to sail on a great-circle course, for on loxodromes the home voyage is always called after the point of the compass opposite to that of the departure) due northwest. And

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[1]) Winds = Spanish *vientos,* the word commonly used in sixteenth-century Spanish textbooks, meaning "directions".

[2]) Fore-and-aft line. See pag. 368.

plaetſens breede van 57 tr. ſulck verſchil ſou dan oock van 57 tr. ſijn, dats over de vijf ghemeene ſtreken.

En hoemen naerder den aſpunt ſeylt, hoemen ſulck verſchil op groote ſeylagen merckelicker bevint, en dat om bekende redenen, die op een Eertcloot met haer behoirlicke reetſchappen openbaer ſijn. Inder voughen dat Stierluyden die daer ontrent varen en landen ſoucken, noodich is van deſe ſaeck goe kennis te hebben, want t'ghebeurt den onverdochten wel, datſe haer ſchip op een ander plaets vindende dan hun giſſing me brengt, ſulcx ten eerſten wijten onbemerckelicke afleydende ſtroomen, daer af nochtans d'oirſaeck mach ſijn het bovechreven niet gagheſlaghen te hebben na t'behooren.

Tot hier toe is gheſeyt vande cromſtreeck recht ooſt en weſt angheſeylt, die altijt een ront is, maer d'ander (uytghenomen inde * middachronden en int middelront) ſijn altemael * ſlangtrecken, wiens form en ghedaente deur de volghende voorſtellen openbaer ſal worden. *Meridiano circulo & æquatore. Spirales.*

4 BEPALING.

Eerſte cromſtreeck noemtmen die in yder vierendeel des ſichteinders naeſt het middachront is, d'ander volgende heet de tvveede, en ſoo oirdentlick voort totte achtſte, die altijt een * evevvijdich ront is. *Circulus parallelus.*

Als by voorbeelt int vierendeel des ſichteinders van noort tot ooſt, de cromſtreeck naeſt het middachront, of anders geſeyt naeſt het noorden, diemen oock heet noort ten ooſten, wort d'eerſte cromſtreeck ghenoemt, noornoortooſt de tweede, noortooſt ten noorden de derde, en ſo voort met d'ander totte achtſte, dat is d'ooſtcromſtreeck die altijt een ront is evewijdich mettet middelront: En ſoodanich is oock d'oirden in d'ander drie vierendeelen des ſichteinders.

De reden waerom de cromſtreken benevens de naem die ſy hebben na de winden, noch geſeyt worden eerſte, tweede, derde, &c. is duſdanich: Angheſien vier * lijckſtandige ſtrekē, als by voorbeelt de ſtreec van noort ten ooſten, noort ten weſten, zuyt ten ooſten, en zuyt ten weſten, in form malcander heel gelijck, en van grootheyt heel even ſijn, ſulcx dat deur de leering van een, de ghedaente over alle vier verſtaen wort, ſoo vallet oirboir om oirdentlick van deſe ſtof te handelen, datmen die als * afcomſt een ghemeene naem gheeft, te weten eerſte, als haer * gheſlacht, om niet elcke mael vier winden t'ſamen te moeten noemen, of maer een ghenoemt ſijnde, dat d'ander niet vergheten en ſchijnen. *Homologa. Species. Genus.*

NV DE VOORSTELLEN.

ALſoo ſijn VORSTELICKE GHENADE int leſen van *Coſmographia Petri Appiani & Gemmæ Friſij*, ghecommen was tot *Cap.* 13 *primæ partis:* Daer na tot 7 *Cap. in libello de locorum ſcribendorum ratione*, Alwaer ſtont de manier om deur ghetalen te vinden op wat ſtreeck d'een plaets van d'ander light, heeft het ſelve alſdoen overgheſlaghen, om twee redenen, d'eene dat den gront uyt welcke de wercking ghetrocken was daer niet by en ſtont, ten anderen dat where the latitude of the place is 57°, such difference would then also be 57°, *i.e.* more than five points.

And the nearer one gets to the pole, the more appreciable will this difference be found on long voyages, for familiar reasons, which are clear on a globe by means of its accessories. In such a way that it is necessary for navigators, sailing in those regions and trying to discover unknown lands, to be well acquainted with this matter, for it sometimes happens to those who are not prepared for it that, finding their ship to be in another place than according to their conjecture, they attribute this at once to imperceptible diverting currents, though the cause of it may be that they have not observed the above properly.

Up to this point, mention has been made of the loxodrome on which the ship sails due east and west, which is always a circle, but the others (except in the meridians and in the equator) are all spirals, the appearance and character of which will become clear from the following propositions.

4th DEFINITION.

First loxodrome we call the one which in each quarter of the horizon is nearest to the meridian, the next is called the second, and so on in due order up to the eighth, which is always a parallel.

Thus, for instance, in the quarter of the horizon from north to east the loxodrome nearest to the meridian, or in other words nearest to the north, which is also called north by east, is called the first loxodrome, north northeast the second, northeast by north the third, and so on with the others up to the eighth, *i.e.* the east loxodrome, which is always a circle parallel to the equator. And such is also the order in the other three quarters of the horizon.

The reason why, besides the name they have in accordance with the points of the compass, the loxodromes are also called first, second, third, etc., is as follows. Since four homologous tracks, such as *e.g.* the tracks of north by east, north by west, south by east, and south by west, are quite similar in form and equal in size, so that when one has learned one of them, one understands the character of all four, it is expedient, in order to deal with this subject in the proper way, to give them a common name indicating their genus, *viz.* first loxodrome, so that we need not each time mention four points of the compass together or, if only one is mentioned, it may not appear that the others have been forgotten.

NOW THE PROPOSITIONS.

When His Princely Grace, in reading *Cosmographia Petri Appiani*[1] *&* *Gemmae Frisij,* had come to *Cap. 13 primae partis,* and afterwards to *7. Cap. in libello de locorum scribendorum ratione,* which described the manner in which to find by numbers in what direction one place lies in respect to the other, he skipped this part, for two reasons, the one because the ground on which the operations were based was not mentioned, secondly because at that time he was not yet skilled in

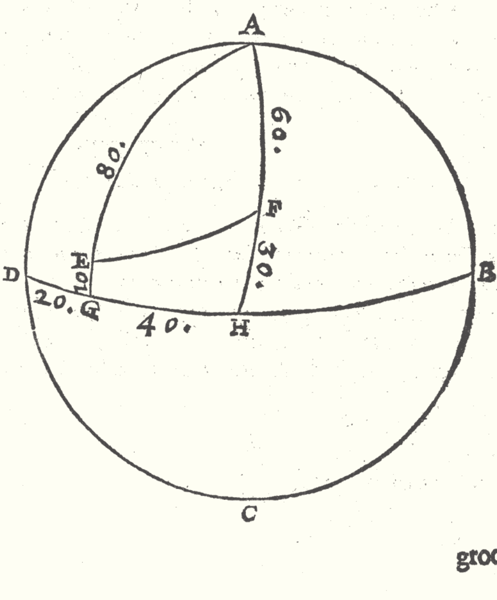
[1]) *Cf. Introduction,* p. 482.

hy doen noch niet ervaren en was inden handel der platte en clootsche driehoucken: Maer hem daer na inde selve gheoeffent hebbende, en ghedachtich sijnde t'gene inde boveschreven hoofrsticken overgheslaghen was, heeft in die plaets ten selven eynde ander manier van wercking ghedaen deur kennis der oirsaken, en dat niet alleen op de vinding der streeck van d'een plaets tot d'ander, maer oock op al d'onbekende palen dieder vallen in sulck voorstel, t'welck hier vervought is als volght.

1 VOORSTEL.

Wesende van tvvee plaetsen ghegheven drie palen deser ses: Rechte streeck van d'eerste plaets totte tvveede: Rechte streeck vande tvveede plaets tot d'eerste: langdeschil: Schilbooch der breede van d'eerste plaets: Schilbooch der breede vande tvveede plaets: En verheyt der plaetsen: Te vinden d'ander drie onbekende palen.

TGEGHEVEN. Laet A B C D den eertcloot sijn diens middelront B D, des selfden begin D, aspunt A, eerste plaets t'punt E, tweede plaets t'punt F, tusschen welcke getrockē is een grootste ronts booch E F, als verheyt: Deur de selve twee plaetsen E, F, sijn getrockē de twee vierdeelenronts A E G, A F H, en des punts E breede E G sy van 10 tr. en sal sijn schilbooch A E doen 80 tr. Voort des punts F breede sy F H van 30 tr. en sal sijn schilbooch A F doen 60 tr. De langde van E sy D G 20 tr. en de langde van F sy D H 60 tr. sulcx dattet verschil der langde vande twee plaetsen E F is G H doende 40 tr. Inder voughen dat hier vande ses palen, ghegheven of bekent sijn de drie, te weten A E, en A F, schilbogen der breede, en G H langdeschil. TBEGHEERDE. Wy moeten de drie onbekende palen vinden, te weten de rechte streeck E F, dat is op wat rechte streeck datmen van E na F moet seylen, of anders de grootheyt des houcx A E F: Ten anderen de streeck F E, dat is de grootheyt des houcx A F E: Ten derden de verheydt, te weten de langde des boochs E F.



Merckt noch tot breeder verclaring der saeck dat de ses palen int voorstel verhaelt, sijn de ses ghemeene palen eens clootschen driehoucx, te weten drie houcken en drie sijden, welcke in dese stof sulcke namen hebben.

TWERCK.

Anghesien de booch G H 40 tr. is, voor de plane and spherical trigonometry. But having afterwards trained himself therein and remembering what he had skipped in the above-mentioned chapters, he proposed instead, for the same purpose, operations of another kind by knowledge of the causes, not only about the finding of the direction of one place in respect to the other, but also about all the unknown elements occurring in this proposition, which is here given as follows.

1st PROPOSITION.

If of two places three out of the following six elements are given: course of the great-circle track from the first place to the second; course of the great-circle track from the second place to the first; difference of longitude; complement of latitude of the first place; complement of latitude of the second place; and distance [1]) between the places: to find the other three unknown elements.

SUPPOSITION. Let *ABCD* be the earth, whose equator is *BD,* its beginning *D,* the pole *A,* the first place the point *E,* the second place the point *F,* between which has been drawn an arc of a great circle *EF,* for their distance. Through the said two places *E* and *F* have been drawn the two quarter circles *AEG, AFH,* and let the latitude *EG* of the point *E* be 10°, then its complement will be *80°*. Further let the latitude of the point *F* be *FH* = 30°, then its complement *AF* will be 60°. Let the longitude of *E* be *DG* = 20°, and let the longitude of *F* be *DH* = 60°, so that the difference of longitude between the two places *E* and *F* is *GH,* making 40°. In such a way that here out of the six elements three are given or known, *viz.* *AE* and *AF,* the complements of latitude, and *GH,* the difference of longitude. REQUIRED. We have to find the three unknown elements, *viz.* the course of the great-circle track *EF, i.e.* what course one has to sail from *E* to *F,* or otherwise the magnitude of the angle *AEF.* Second, the course *FE, i.e.* the magnitude of the angle *AFE.* Third, the distance, *viz.* the length of the arc *EF.*

Note also, as a fuller explanation of the matter, that the six elements mentioned in the proposition are the six common elements of a spherical triangle, *viz.* three angles and three sides, which in this subject are so called.

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[1]) Stevin invariably uses the word "distance", no matter whether it is taken along the great circle or the loxodrome.

grootheyt des houcx E A F, soo heeft de driehouck A E F drie bekende palen, te weten den selven houck E A F 40 tr. Voort de sijde A E 80 tr. en A F 60 tr. deur t'ghegheven: Hier me ghesocht d'ander drie onbekende palen, worden bevonden deur het 40 voorstel der clootsche driehoucken voor t'begheerde, te weten den houck A E F voor de streeck E F 55 tr. 51 ①, wijckende so verre vant noorden na t'oosten: Ende den houck A F E voor de streeck F E 109 tr. 44 ①, wijckende soo verre vant noorden over t'westen na het zuyden: Of anders ghesеyt van westen na zuyden 19 tr. 44 ①. Ende de verheyt E F 42 tr. 15 ①.

VERVOLGH.

Tis openbaer hoemen deur elcke drie ghegheven bekende palen, d'ander drie onbekende vinden sal, sulcx dattet niet noodich en is besonder voorbeelden te beschrijven van die verscheydenheden in menichte seer veel vallende, te weten ses op elcke begheerde pael der ses palen. TBESLVYT. Wesende dan van twee plaetsen ghegheven drie palen deser ses: Rechte streeck van d'eerste plaets totte tweede: Rechte streeck vande tweede plaets tot d'eerste: langdeschil: Schilbooch der breede van d'eerste plaets: Schilbooch der breede vande tweede plaets: verheyt der plaetsen: Wy hebben ghevonden d'ander drie onbekende palen, na den eysch.

2 VOORSTEL.

Op rechte streken te seylen.

Nadien sijn VORSTELICKE GHENADE grondelick verstaen hadde den handel der seyling op cromstreken die hier na beschreven sal worden, en daer by verlijckende de rechte streken datse de cortste wech gheven, soo heeft hem behoirlick ghedocht, en d'oirden te vereysschen, reghelen beschreven te worden hoemen die cortste streken soomen wilde seylen soude: T'welck oirsaeck was dat wy daer op letten, en t'gene ons van dies ontmoete by ghedachtenis stelden, daer af beschrijvende twee voorbeelden, t'eerste * tuychwerckelick, *Mechanice.* t'ander wisconstich.

1 *Voorbeelt tuychwverckelick.*

TGHEGHEVEN. Laet inde form der 1 bepaling A en B twee plaetsen op den eertcloot beteyckenen, A daer t'schip af vaert, B daert sijn moet.

TBEGHEERDE. Men wil een rechte streeck seylen vande plaets beteyckent met A, totte plaets beteyckent met B.

TWERCK.

Men sal van A tot B trecken een verborgen oft uytvaghelicke grootsteronts booch, beteyckenende de rechte streeck die t'schip seylen moet, daer na t'punt A ghestelt sijnde onder het * soppunt, en dan de sopbooch gheleyt over B, sy wijst *Puncto verticuli.* inden sichteinder, neem ick, dat B recht west van A light. Dit soo sijnde, men sal van A na B seylen recht westwaert an, by gissing drie of vier * trappen verre, *Grados.* die commen, neem ick, van A tot H: Alwaer t'punt H gheteyckent sijnde, men salt brenghen int middachront onder het toppunt, den aspunt soo veel verleeghende als de saeck vereyscht, daer na de sopbooch andermael geleyt over t'punt

PROCEDURE.

Since the arc *GH* is 40°, for the magnitude of the angle *EAF*, the triangle *AEF* has three known elements, *viz.* the said angle *EAF* = 40°; further the side *AE* = 80°, and *AF* = 60°, by the supposition. If by means of these we seek the other three unknown elements, the required values are found by the 40th proposition of spherical trigonometry [1]), *viz.* the angle *AEF* for the course *EF* = 55°51′, deviating thus far from the north to the east. And the angle *AFE* for the course *FE* = 109°44′, deviating thus far from the north *via* the west to the south. Or in other words: from the west to the south, 19°44′. And the distance *EF* = 42°15′.

SEQUEL.

It is evident how from any three given known elements we must find the other three unknown elements, so that it is not necessary to describe special examples of those various cases, of which there are a great many, *viz.* six for each required element of the six.

CONCLUSION. If of two places three out of the following six elements are given: course of the great-circle track from the first place to the second; course of the great-circle track from the second place to the first; difference of longitude; complement of latitude of the first place; complement of latitude of the second place; distance between the places; we have found the other three unknown elements; as required.

2nd PROPOSITION.

To sail on great-circle tracks.

After His Princely Grace had thoroughly understood the method of sailing on loxodromes to be described hereafter and, having compared therewith the great-circle tracks, had found that they give the shortest route, he considered it expedient and required by the order of things that rules should be described of how those shortest tracks would have to be sailed, if this were desired. Which induced us to attend to this matter and to make a note of what we had found about it, describing two examples of it, the first mechanical and the second mathematical.

1st Example, Mechanical.

SUPPOSITION. In the figure of the 1st definition let *A* and *B* denote two places on the earth, *A* the place from which the ship sails, *B* the place for which it is bound.

REQUIRED. It is desired to sail on a great-circle track from the place denoted by *A* to the place denoted by *B*.

PROCEDURE.

Draw from *A* to *B* an erasable arc of a great circle, denoting the great-circle track to be sailed by the ship; if then the point *A* is placed under the zenith and the vertical circle is made to pass through *B,* it indicates in the horizon, I assume, that *B* lies due west of *A*. This being so, one has to sail from *A* to *B* due west, by conjecture three or four degrees further, which I assume to be from *A* to *H*. And when the point *H* has been marked there, one has to bring it into the meridian under the zenith, lowering the pole as much as is required. If then

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[1]) Stevin's *Trigonometry* (Work XI; i, 13), p. 295. See Vol. II B, p. 755.

ſy wijſt inden ſichteinder dat B van H light neem ick 3 tr. van weſten nae zuyden, en daerom ſalmen op ſulcken ſtreec van H na B ſeylen weerom by giſſing eenighe 4 of 5 tr. verre, t'welck ſy neem ick tot I, alwaer t'ſchip ghecommen ſijnde, men ſal daer weerom doen ſulcx alſmen an H dede, alwaermen oock bevinden ſal datmen dan noch zuydelicker an nae B ſal moeten ſeylen danmen van H dede: En ſghelijcx doende ſoo dickwils tot datmen ter plaets B comt, men ſal de begheerde rechte ſtreeck A B gheſeylt hebben: Mits welverſtaende dat de ſtucken als A H, H I, en dierghelijcke cleen genouch genomen ſijn, Want hoe wel in plaets van A H weſende een grootſte ronts ſtuck, gheſeylt wiert een cromme ſtreec weſende cleenronts deel wat noordelicker uytcommende, voort dat in plaets van d'ander ſtucken des boochs A B, gheſeylt wierden ander cromme ſtreken weſende ſlangtrecxdeelen, doch met ſulcke ſtucken cleen genouch te nemen, canmen maken dat ſoodanich verſchil van gheender acht en is.

Merckt noch datter vant werck duſdanighe proef can genomen worden: Het ſchip ghecommen weſende tot neem ick H, en datmen dan deur dadelicke ervaring mette Son of ſterren d'eertcloots breede bevint t'overcommen mette breede die H op den ghebotſten eertcloot anwijſt, dat geeft met reden vermoeden het ſchip de rechte ſtreeck wel gheſeylt te hebben, t'welck volghen moet alſmen wel ghegiſt heeft.

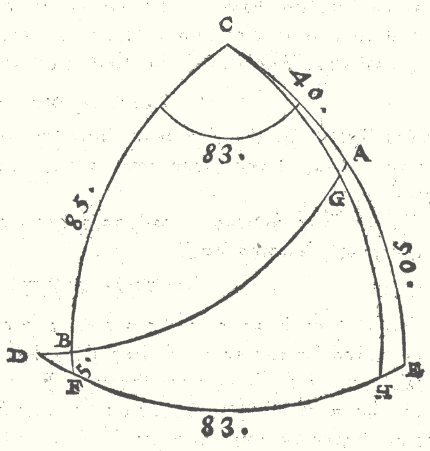
VERVOLGH.

Soo de vaert moeſt ghedaen ſijn opt middelront, t'is kennelick datmen altijt ſoude moeten varen recht ooſt of weſt; Maer moetende op een middachront gheſchien, datmen dan altijt recht zuyt of recht noort ſoude moeten varen.

2 Voorbeelt wiſconſtich.

TOGHEGHEVEN. Laet A en B twee plaetſen op den eertcloot beteyckenen, A daer t'ſchip af vaert, B daert ſijn moet, C den aſpunt, D E het middelront, de breede van A is E A 50 tr. en van B is F B 5 tr. en t'verſchil haerder langden is F E 83 tr.

TBEGHEERDE. Men wil een rechte ſtreeck ſeylen vande plaets beteyckent met A, totte plaets beteyckent met B, en dat wiſconſtelick vinden, te weten deur rekening der clootſche driehoucken.



Bereytſel van t'eerſte deel des wercx.

Ick treck van A tot B een grootſte rondts booch beteyckenende de rechte ſtreeck die t'ſchip ſeylen moet: Daer nae de ſelve A B voorwaert tot datſe het middelront ontmoet, t'welck ſy in D: Daer nae den booch E A voorwaert

tot

the vertical circle is again made to pass through the point *B,* it indicates in the horizon that *B* lies from *H,* I assume, 3 degrees from west to south, and accordingly one has to sail this course from *H* to *B* again by conjecture some 4 or 5 degrees further, I assume as far as *I.* And when the ship has arrived there, one has to do there again as at *H,* where it will be found that one will have to sail to *B* even more to the south than one did at *H.* And if the same is done until one arrives at the place *B,* one has sailed on the required great-circle track *AB,* provided the distances *AH* and *HI* and the like have been taken small enough. For even if instead of on *AH,* which is a part of a great circle, one sailed on a loxodrome, which is a part of a small circle, arriving slightly more to the north, and if further, instead of on the other parts of the arc *AB,* one sailed on other loxodromes, which are parts of spirals, yet by taking these parts small enough one can cause this difference to be of no account.

Note that the procedure may be checked as follows: When the ship has arrived, I assume, at *H,* and when it is then found by observation of the sun or the stars that the latitude on the earth corresponds to the latitude of *H* on the globe, this gives one reason to assume that the ship has rightly sailed on the great-circle track, which has to follow if the conjecture has been right.

SEQUEL.

If the sailing had to take place on the equator, it is obvious that one ought always to sail due east or west. But if it had to take place on a meridian, it is obvious that one ought always to sail due south or due north.

2nd Example, Mathematical.

SUPPOSITION. Let *A* and *B* denote two places on the earth, *A* the place from which the ship sails, *B* the place for which it is bound, *C* the pole, *DE* the equator; the latitude of *A* is $EA = 50°$, and that of *B* is $FB = 5°$, and their difference of longitude is $FE = 83°$.

REQUIRED. It is desired to sail on a great-circle track from the place denoted by *A* to the place denoted by *B,* and to find this mathematically, *viz.* by means of a calculation of spherical trigonometry.

Preliminary of the First Part of the Procedure.

I draw from *A* to *B* an arc of a great circle, denoting the great-circle course on which the ship has to sail. After this I produce the said *AB* until it meets the equator; let this be in *D.* After this I produce the arc *EA* until it meets the pole *C*; then *AC* will be 40°, for when we subtract $EA = 50°$ from $EC = 90°$, tot datse den aspunt C ontmoet,en sal A C doen 40 tr. want van E C 90 tr.ghetrocken E A 50 tr. blijft voor A C 40 tr. S'ghelijcx treck ick F B voorwaert tot datse den aspunt C ontmoet,en sal B C doen 85 tr. want van F C 90 tr.ghetrocken F B 5 tr. blijft voor B C 85 tr. en den houck B C A, diens grootheyt megebrocht wort van F E 83 tr. doet als de selve oock 83 tr.

1 *Deel des wercx.*

Om eerst te vinden op wat streeck men sal beginnen te seylen van A na B,so moet ick weten de grootheyt des houcx C A B, want soo veel salmen moeten seylen van noorden na westen. Om daer toe te commen, soo heeft den selven driehouck C A B drie bekende palen, deur t'bereytsel, te weten den houck B C A 83 tr. de sijde A C 40 tr. en B C 85 tr. Hier me ghesocht de drie onbekende palen, worden bevonden deur het 40 voorstel der clootsche driehoucken, te weten den houck C A B 92 tr. 8 ①, de langde A B van d'een plaets tot d'ander 81 tr. 41 ①,en den houck C B A 39 tr. 45 ①. Nu soo veel als doet den voorschreven houck C A B,te weten 92 tr. 8 ①, soo veel salmen van A af moeten beginnen te seylen van noorden over westen na zuyden, dat is 87 tr. 52 ① van zuyden na westen, welcke seyling gheduert neem ick 4 tr. verre tot G toe, sulcx dat A G doet de selve 4 tr.

2 *Bereytsel dienende tottet 2 deel des wercx.*

Anghesien dat de vinding der onbekende palen eens driehoucx sonder ghegheven rechthouck als de voorgaende,moeyelicker valt dan met een ghegeven rechthouck,so sullen wy een bereytsel stellen om int volgende te wercken deur driehoucken met een ghegheven rechthouck,aldus: De driehouck B F D heeft drie bekende palen, te weten den houck B F D recht, mette sijde F B 5 tr. deur t'ghegheven, en den houck D B F even sijnde metten houck C B A, doet deur t'eerste deel des wercx 39 tr. 45 ①:Hier me ghesocht den houck D, en de sijde D B,worden bevonden deur het 34 voorstel der clootsche driehoucken, te weten den houck D 50 tr. 26 ①, en de sijde B D 6 tr. 27 ①, die vergaert tot A B 81 tr. 41 ①,comt voor A D 88 tr. 8 ①.

2 *Deel des wercx.*

Om te vinden op wat streeck men sal beginnen te seylen van G na B,ick treck van C deur G tot int middelront E F de booch C G H als middachront, waer me G H D een rechthouckich driehouck is,hebbende drie bekende palen,te weten den houck G H D recht, den houck D 50 tr. 26 ① deur het 2 bereytsel, en de sijde G D 84 tr. 8 ①, want A D doet 88 tr. 8 ① deur het 2 bereytsel, daer af ghetrocken A G doende 4 tr. deur t'eerste deel des wercx, blijft alsboven voor G D 84 tr. 4 ①: Hier me ghesocht den houck H G D, wort bevonden deur het 34 voorstel der clootsche driehoucken van 87 tr. 45 ①: En op sulcken streeck van zuyden na westen moetmen van G seylen na B, t'welck 7 ① zuydelicker is danmen van A tot G seylde, want ghetrocken 87 tr. 45 ①, van 87 tr. 52 ①, blijft de selve 7 ①. Nu dan van G aldus gheseylt hebbende soo verre men oirboir verstaet, men sal om voorder te seylen daer weerom doen als an G gedaen wiert,en dergelijcke tot ander plaetsen soo lang datmen tot B comt.

MERCKT ten 1 dat hoewel de boveschreven 7 ① zuydelicker te seylen so weynich is, dattet met een seylende schip niet gageslagen en can worden,doch ver-the remainder is 40° for *AC*. In the same way I produce *FB* until it meets the pole *C*; then *BC* will be 85°, for when we subtract *FB* = 5° from *FC* = 90°, the remainder is 85° for *BC*; and the angle *BCA*, whose magnitude follows from *FE* = 83°, like the latter is also 83°.

1st Part of the Procedure.

In order to find first what course one has to start sailing from *A* to *B*, I have to know the magnitude of the angle *CAB*, for thus much one will have to sail from north to west. To find this, the said triangle *CAB* has three known elements, by the preliminary, *viz.* the angle *BCA* = 83°, the side *AC* = 40°, and *BC* = 85°. When herewith the three unknown elements are sought, they are found by the 40th proposition of spherical trigonometry [1]), *viz.* the angle *CAB* = 92°8′, the distance *AB* from one place to the other 81°41′, and the angle *CBA* = 39°45′. Now as much as the aforesaid angle *CAB* amounts to, *viz.* 92°8′, so much one will have to start sailing from *A*, from the north *via* the west to the south, *i.e.* 87°52′ from south to west, which sailing continues, I assume, 4° further to *G*, so that *AG* is the said 4°.

2nd Preliminary, Serving for the 2nd Part of the Procedure.

Since the finding of the unknown elements of a triangle without a given right angle, like the foregoing, is more difficult than with a given right angle, we shall give a preliminary to operate in the following by means of triangles with a given right angle, as follows. The triangle *BFD* has three known elements, *viz.* the angle *BFD* = 90°, with the side *FB* = 5°, by the supposition, and the angle *DBF*, being equal to the angle *CBA*, by the first part of the procedure is 39°45′. When herewith the angle *D* and the side *DB* are sought, they are found by the 34th proposition of spherical trigonometry [2]), *viz.* the angle *D* = 50°26′ and the side *BD* = 6°27′; when we add the latter to *AB* = 81°41′, we get 88°8′ for *AD*.

2nd Part of the Procedure.

In order to find what course one should start sailing from *G* to *B*, I draw from *C* through *G* to the equator *EF* the arc *CGH* as meridian, in consequence of which *GHD* is a right-angled triangle having three known elements, *viz.* the angle *GHD* = 90°, the angle *D* = 50°26′, by the 2nd preliminary, and the side *GD* = 84°8′, for *AD* is 88°8′ by the 2nd preliminary, and when from this we subtract *AG*, being 4° by the first part of the procedure, the remainder is, as above, 84°4′ for *GD*. When herewith the angle *HGD* is sought, it is found by the 34th proposition of spherical trigonometry [3]) to be 87°45′. And that course from south to west one has to sail from *G* to *B*, which is 7′ more to the south than the sailing from *A* to *G*, for when we subtract 87°45′ from 87°52′, this 7′ is left. Now therefore, after having sailed thus from *G* as far as is considered expedient, in order to continue one has to do again as one did at *G*, and similarly at other places until one arrives at *B*.

NOTE in the first place that although the sailing of the above-mentioned 7′ more to the south is so short a distance that it cannot be observed in a sailing

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[1]) Stevin's *Trigonometry* (Work XI; i, 13), p. 295.
[2]) *ibid.*, p. 245.
[3]) *ibid.*, p. 245.

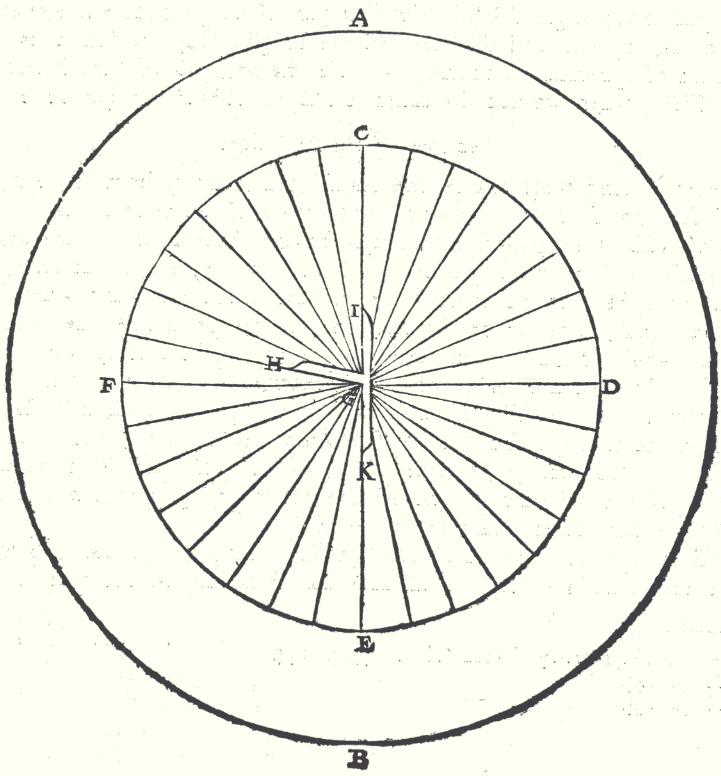
verstaetmen daer deur datmen de volghende booch van G voortwaert grooter mach nemen dan 4 tr. Maer soo d'eerste booch te groot had genomen geweest, sulcx datmen op een minder met sekerheyt seylen can, tis kennelic datmen dan de rekening op een minder booch behoort te maken. Noch staet te gedencken datter in dit voorbeelt op even boghen meerder verandering valt by t'punt B, dan verder daer af, want by B commende, men sal moeten van zuyden na westen seylen alleenelick 39 tr. 45 ①, (deur dien den houck D B F soo groot is) t'welck 48 tr. 7 ① zuydelicker is dan doenmen an A begost, alwaer den houck B A E bevonden wiert van 87 tr. 52 ①.

MERCKT ten 2 dat soomen begeerde te weten de breede van t'punt G, om t'ondersoucken offe deur dadelicke ervaring soo bevonden wort, als van dergelicke int 1 tuychwerckelick voorbeelt ghezeyt is, men soude hier boven benevens den houck D G H des driehoucx D G H, noch vinden de sijde GH, wantse de begheerde breede anwijst. TBESLVYT. Wy hebben dan op rechte streken gheseylt na den eysch.

3 VOORSTEL.

Cromstreken tuychvverckelick te teyckenen.

De dadelicke Eertclootmakers ghebruycken verscheyden middelen en reetschappen totte teyckening der cromstreken elck dat hem best bevalt: Een van



dien

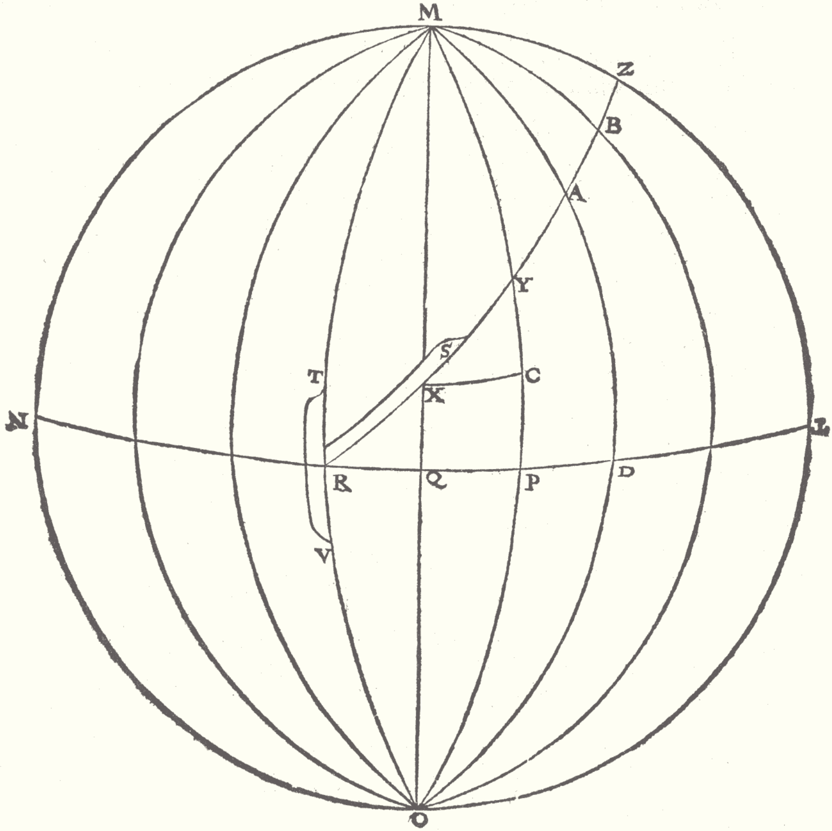
ship, yet it is understood thereby that the next arc from *G* on may be taken larger than 4°. But if the first arc had been taken too large, so that a smaller arc appears necessary in order to sail with accuracy, it is obvious that one must then make the calculation on a smaller arc. It should also be borne in mind that in this example with equal arcs the change is greater at the point *B* than further away, for when one gets to *B*, one will have to sail from south to west only 39°45′ (because the angle *DBF* has this magnitude), which is 48°7′ more to the south than when one started at *A*, where the angle *BAE* was found to be 87°52′.

NOTE in the second place that if it were required to know the latitude of the point *G*, so as to ascertain whether it is found the same by observation as it has been said in the 1st mechanical example, in the above one would have to find, in addition to the angle *DGH* of the triangle *DGH*, the side *GH*, because it designates the required latitude. CONCLUSION. We have thus sailed on great-circle tracks; as required.

3rd PROPOSITION.

To draw loxodromes mechanically.

Practical globe-makers use various means and tools for drawing loxodromes, each taking that which suits him best. We shall here explain one of them, not in dien ſullen wy hier verclaren, niet om inde daet naghevolght te worden, maer om dattet wel uytdruckt den gront van t'ghene begheert is, en daer na beter gedaen moet ſijn. Laet A B een cloot ſijn, hier op beſchrijf ick eenich cleender rondt C D E F, diens middelpunt G, welck ront ick deel in 32 even deelen, treckende van daer tottet middelpunt G 32 boghen, die my de 32 ghemeene ſtreken beteyckenen. Hier in anſien ick C G E voor de booch van noort na zuyt, en F G D daer op rechthouckich voor de booch van weſt na ooſt. Dit ſoo ſijnde ick maeck een coper clootſche ſcheefhouck G H I K op den cloot paſſende, en hebbende de ſcheefheyt van een ſtreeck, want ſoo veel doet den houck F G H. Nu ghelijck hier ghemaeckt is de clootſche ſcheefhouck van een ſtreeck, alſoo ſalmender meer maken tot ſeven toe, te weten voor elcke ſtreeck een die tuſſchen F C commen. Deſe ſeven clootſche ſcheefhoucken bereyt ſijnde, men ſal nemen een ander cloot L M N O vande ſelve grootheyt als A B, alwaer L N het middelront beteyckent, M den noortſchen aſpunt, O den zuytſchen, tuſſchen deſe twee aſpunten ſijn middachronden ghetrocken als M P O, M Q O,



M R O, ſnyende het middelront van trap tot trap inde punten P, Q, R. Hier op teycken ick de cromſtreecken als volght: Ghenomen dat ick eerſt wil hebben de cromme noortooſtſtreeck, ſoo neem ick uyt de boveſchreven ſeven coperen cloot.

order that it may be imitated in practice, but because it shows very well the foundation of what is required, and because hereafter it can be done better. Let *AB* be a globe; on this I describe a small circle *CDEF,* the centre of which is *G,* which circle I divide into 32 equal parts, drawing from there to the centre *G* 32 arcs, which denote the 32 common points of the compass. Herein I look upon *CGE* as the arc from north to south, and *FGD,* at right angles thereto, as the arc from west to east. This being so, I make an oblique spherical angle of copper *GHIK,* fitting on the globe and having the obliquity of one point, for that is the magnitude of the angle *FGH*. Now just as here the oblique spherical angle of one point has been made, in the same way others have to be made, up to seven — *viz.* one for each point — which come between *F* and *C*. When these seven oblique spherical angles have been made, another globe *LMNO* of the same size as *AB* must be taken, where *LN* denotes the equator, *M* the north pole, *O* the south pole; between these two poles have been drawn meridians, *viz. MPO, MQO, MRO,* intersecting the equator from degree to degree in the points *P, Q, R*. Upon this I draw the loxodromes as follows: Assuming that I first want to have the loxodrome of northeast, I take out of the above-mentioned seven oblique clootsche scheefhoucken dien welcke de noortooststreeck beteyckent, de selve sy R S T V, diens sijde T V vervought is op een der middachronden, als op M R O, soo dat den houck des coperen cloothoucx past op R ghemeene sne des middachronts en middelronts L N, en treck van R langs R S een liniken tottet naeste middachront als tot X: vervough daer na den coperen scheefhouck opt middachront M Q O, en also dattet houckpunt R dan comme an X: Treck daer na van X langs de voorschreven scheefhoucx sijde het liniken X Y. En alsoo voortgaende na Z tot datmen den aspunt na ghenouch is, of gheraeckt, men sal de cromme noortooststreeck op den Eertcloot gheteyckent hebben: Wy segghen hier boven tot datmen den aspunt na ghenouch is, of gheraeckt, doch wisconstelick ghesproken en can niet gherocht worden, want de slangtreck soude oneyndelick daer rontom loopen en altijt naerderen sonder gheraken; Maer *Mechanicè.* * tuychwerckelick can een sichtbaer aspunt gherocht worden.

Deur t'ghene wy tot hier toe gheseyt hebben vande teyckening der noortooststreeck, is openbaer de teyckening van al d'ander cromstreken, en kennelick hoemen tot alle plaetsen eens Eertcloots de cromme seylstreken teyckenen sal na sijn wille. TBESLVYT. Wy hebben dan cromstreken tuychwerckelick gheteyckent, na den eysch.

Vande onsekerheyt inde voorgaende vvijse van teyckening.

Want deur de gheduerighe en menichvuldighe versetting van desen coperen scheefhouck R S T V onsekerheyt int werck can volghen, alsoo om dergelijcke redenen oock can in meer ander tuych tot sulcken eynde ghemaeckt, of alwaerder sekerheyt in dat sulcx onbewesen blijft: Soo ist te weten dat wy die manier alleenlick hier ghestelt hebben, eensdeels op datmen daer deur verstae de onsekerheyt dieder is inde cromstreken alsoo op Eertclooten gheteyckent. Ten anderen om dattet wel verclaert de gront en eyghenschappen der cromstreken, daermen op bouwen mach wisconstighe wercking, deur welcke de tuychwerckelicke meerder sekerheyt can crijghen, als blijcken sal, eerst beschreven sijnde de tafels als volght.

4 VOORSTEL.

Tafel der cromstreken te maken.

De somme deses voorstels is, dat wy moeten vinden in ghetalen, hoe lanck dat sijn de boghen als inde form des 3 voorstels Q X, P Y, en dergelijcke, want die ghetalen bekent wesende, en na den eysch van dien punten gheteyckent als X, Y, *a*, *b*, Z, en van t'een punt tottet ander linikens ghetrocken, men crijcht de begheerde cromstreeck. Het vinden deser bogen soude meugen aldus toegaen:

EERSTE MAECKSEL VANDE TAFELS DER CROMSTREKEN.

Laet R Z noch eens de vierde cromstreeck beteyckenen, daer af wy vinden willen de boghen Q X, P Y: Tot desen eynde segh ick dat de driehouck X Q R drie bekende palen heeft, te weten den houck X R Q 45 tr. den houck X Q R recht, en de sijde R Q 1 tr. Hier me ghesocht de sijde Q X, wort bevonden deur het 36 voorstel der clootsche driehoucken van 59 ① 59 ②. Om nu te vinden

de lini

spherical angles of copper the one which denotes the loxodrome of northeast; let this be *RSTV*, whose side *TV* has been placed on one of the meridians, *viz.* on *MRO*, so that the vertex of the oblique spherical angle of copper comes in *R*, the point of intersection of the meridian and the equator *LN*, and I draw from *R* along *RS* a short line to the next meridian, *viz.* *X*. Then place the oblique angle of copper on the meridian *MQO*, in such a way that the vertex *R* then comes in *X*, thereafter draw from *X* along the aforesaid side of the oblique angle the short line *XY*. And proceeding in this way to *Z* until one is near enough to the pole or reaches it, one has drawn the loxodrome of northeast on the globe. We said above: until one is near enough to the pole or reaches it, but mathematically speaking it is impossible to reach it, for the spiral must pass around it infinitely and always approach it without reaching it. But mechanically speaking it is possible to reach a visible pole.

From all that we have hitherto said about the drawing of the loxodrome of northeast it is evident how all the other loxodromes have to be drawn and it can be known how in any place of a globe we shall draw the loxodromes we desire. CONCLUSION. We have thus drawn loxodromes mechanically; as required.

Of the Uncertainty in the Foregoing Method of Drawing.

Because the constant and frequent displacement of this copper oblique angle *RSTV* may result in uncertainty in the procedure, as may also happen for the same reasons with other tools made for this purpose — or even if the procedure were certain, this remains unproved — it is to be noted that we have only given this method here, on the one hand in order that the uncertainty in the loxodromes thus drawn on globes may be understood thereby; on the other hand because it explains very well the foundation and the properties of loxodromes on which mathematical operations can be based, owing to which the mechanical method may become more certain, as will appear when first the tables have been described, as follows.

4th PROPOSITION.

To make a table of the loxodromes.

The gist of this proposition is that we have to find the numerical values of the lengths of the arcs, *viz.* in the figure of the 3rd proposition *QX*, *PY*, and the like, for when these values are known and points have been marked accordingly, *viz.* *X*, *Y*, *A*, *B*, *Z*, and short lines have been drawn from one point to the other, the required loxodrome is obtained. The finding of those arcs might take place as follows.

FIRST METHOD OF MAKING THE TABLES OF THE LOXODROMES.

Let *RZ* once again denote the fourth loxodrome, from which we want to find the arcs *QX* and *PY*. To this end I say that the triangle *XQR* has three known elements, *viz.* the angle $XRQ = 45°$, the angle $XQR = 90°$, and the side $RQ = 1°$. When herewith the side *QX* is sought, it is found by the 36th proposition of spherical trigonometry [1]) to be 59′59″. Now in order to find the line *PY*, I draw the arc *XC* parallel to *QP*; then *PC* will also be 59′59″, like *QX*,

[1]) Stevin's *Trigonometry* (Work XI; i, 13), p. 255.

de lini P Y, ick treck de booch X *c* evewijdich met Q P, en sal P *c* dan oock doen 59 ① 59 ②, gelijck Q X: Sulcx datter vanden driehouck Y *c* X, gevonden moet worden de sijde *c* Y, om die te vergaren tot P *c*, en dan te hebben de booch P Y: Hier toe heeft den selven driehouck Y *c* X drie bekende palen, te weten den houc Y X *c* 45 tr. den houck Y *c* X recht, en de sijde X *c* 59 ① 58 ②, want so veel doet dien langdetrap buyten t'middelront deur de ghemeene tafel diemen daer af maeckt, en hier na oock volghen sal; daer me ghesocht de sijde *c* Y, wort bevonden deur het 36 voorstel der clootsche driehoucken van 59 ① 57 ②, die vergaert tot P *c* 59 ① 59 ②, comt voor P Y 1 tr. 59 ① 56 ②, en soo voort met dan d'ander.

TWEEDE MAECKSEL VANDE TAFELS DER CROMSTREKEN.

Anghesien het maken van volcommen tafels na de voorgaende eerste wijse, langher soude vallen dan my den tijt toelaet, soo sullen wy een ander stellen, beschreven en onlancx uytghegheven deur *Edwart Wright*; want hoewelse eenige onvolcommenheyt hebben daer wy inden Anhang der cromstreken breeder af segghen sullen, nochtans connense tot verclaring des voornemens dienen.

Tottet maken vande volgende tafels der cromstreken wort eerst beschreven als bereytsel een tafel der versaemde snylijnen van 10 ① tot 10 ① aldus:

De snylijn van 10 ① doet.	10000042.
Daer toe de snylijn van 20 ① doende 10000168 comt	20000210.
Daer toe de snylijn van 30 ① doende 10000381 comt	30000591.

En so voort; maer eyntlick salmen overal de vijf laetste letters afsnyen, en sal een tafel sijn als volght:

I TAFEL

so that of the triangle *YCX* the side *CY* has to be found, in order to add it to *PC* and then have the arc *PY*. For this, the said triangle *YCX* has three known elements, *viz.* the angle $YXC = 45°$, the angle $YCX = 90°$, and the side $XC = 59'58''$, for that is the value of this degree of longitude outside the equator by the common table which is made thereof and which is to follow hereafter. When herewith the side *CY* is sought, it is found by the 36th proposition of spherical trigonometry [1]) to be 59′57″. When we add this to $PC = 59'59''$, we get $PY = 1°59'56''$, and so on with the others.

SECOND METHOD OF MAKING THE TABLES OF THE LOXODROMES.

Since the making of complete tables by the foregoing first method would take longer than time permits me, we shall give another method, described and recently published by *Edward Wright* [2]), for although they have certain imperfections, which we shall discuss more fully in the Appendix of Loxodromes, yet they may serve to explain our intention.

With a view to the making of the following tables of loxodromes, by way of preliminary a table is first described of the assembled secants, increasing by 10′, as follows:

The secant of 10′ is	10,000,042
If to this we add the secant of 20′, being 10,000,168, we get	20,000,210
If to this we add the secant of 30′, being 10,000,381, we get	30,000,591

And so on; but finally the five last digits must be discarded, and then the table will be as follows:

TABLE OF ASSEMBLED SECANTS

degrees minutes secants

1) Stevin's *Trigonometry* (Work XI; i, 13), p. 255.
2) *Cf. Introduction*, p. 482-483.

tr.	Ⓘ	ſnÿlinen.
0	10	100
0	20	200
0	30	300
0	40	400
0	50	500
1	0	600
1	10	700
1	20	800
1	30	900
1	40	1000
1	50	1100
2	0	1200
2	10	1300
2	20	1400
2	30	1500
2	40	1601
2	50	1701
3	0	1801
3	10	1901
3	20	2001
3	30	2101
3	40	2201
3	50	2302
4	0	2402
4	10	2502
4	20	2602
4	30	2703
4	40	2803
4	50	2903
5	0	3004

tr.	Ⓘ	ſnÿlinen.
5	10	31
5	20	3205
5	30	3305
5	40	3405
5	50	3506
6	0	3606
6	10	3707
6	20	3808
6	30	3908
6	40	4009
6	50	4110
7	0	4210
7	10	4311
7	20	4412
7	30	4513
7	40	4614
7	50	4715
8	0	4815
8	10	4916
8	20	5018
8	30	5119
8	40	5220
8	50	5321
9	0	5422
9	10	5523
9	20	5625
9	30	5726
9	40	5827
9	50	5929
10	0	6030

tr.	Ⓘ	ſnÿlinen.
10	10	6132
10	20	6234
10	30	6335
10	40	6437
10	50	6539
11	0	6641
11	10	6743
11	20	6845
11	30	6947
11	40	7049
11	50	7151
12	0	7253
12	10	7355
12	20	7458
12	30	7560
12	40	7662
12	50	7765
13	0	7868
13	10	7970
13	20	8073
13	30	8176
13	40	8279
13	50	8382
14	0	8485
14	10	8588
14	20	8691
14	30	8794
14	40	8897
14	50	9001
15	0	9104

tr.	①	snÿlinen.	tr.	①	snÿlinen.	tr.	①	snÿlinen.
15	10	9208	20	10	12358	25	10	15610
15	20	9312	20	20	12464	25	20	15721
15	30	9415	20	30	12571	25	30	15832
15	40	9519	20	40	12678	25	40	15942
15	50	9623	20	50	12785	25	50	16053
16	0	9727	21	0	12892	26	0	16165
16	10	9831	21	10	12999	26	10	16276
16	20	9935	21	20	13106	26	20	16388
16	30	10039	21	30	13213	26	30	16499
16	40	10144	21	40	13321	26	40	16611
16	50	10248	21	50	13429	26	50	16723
17	0	10353	22	0	13537	27	0	16835
17	10	10457	22	10	13645	27	10	16947
17	20	10562	22	20	13753	27	20	17060
17	30	10667	22	30	13861	27	30	17173
17	40	10772	22	40	13969	27	40	17285
17	50	10877	22	50	14078	27	50	17398
18	0	10982	23	0	14186	28	0	17512
18	10	11087	23	10	14295	28	10	17625
18	20	11192	23	20	14404	28	20	17738
18	30	11298	23	30	14513	28	30	17852
18	40	11403	23	40	14622	28	40	17966
18	50	11509	23	50	14731	28	50	18080
19	0	11615	24	0	14840	29	0	18194
19	10	11720	24	10	14950	29	10	18309
19	20	11826	24	20	15060	29	20	18423
19	30	11932	24	30	15170	29	30	18538
19	40	12038	24	40	15280	29	40	18653
19	50	12145	24	50	15390	29	50	18768
20	0	12251	25	0	15500	30	0	18884

tr.	①	*snijlinen.*	tr.	①	*snijlinen.*	tr.	①	*snijlinen.*
30	10	18999	35	10	22565	40	10	26358
30	20	19115	35	20	22688	40	20	26489
30	30	19231	35	30	22811	40	30	26621
30	40	19347	35	40	22934	40	40	26752
30	50	19464	35	50	23057	40	50	26884
31	0	19580	36	0	23180	41	0	27017
31	10	19697	36	10	23304	41	10	27149
31	20	19814	36	20	23428	41	20	27282
31	30	19931	36	30	23552	41	30	27416
31	40	20048	36	40	23677	41	40	27549
31	50	20166	36	50	23802	41	50	27683
32	0	20284	37	0	23927	42	0	27818
32	10	20402	37	10	24052	42	10	27953
32	20	20520	37	20	24178	42	20	28088
32	30	20639	37	30	24304	42	30	28223
32	40	20757	37	40	24430	42	40	28359
32	50	20876	37	50	24556	42	50	28495
33	0	20995	38	0	24683	43	0	28632
33	10	21115	38	10	24810	43	10	28769
33	20	21234	38	20	24938	43	20	28906
33	30	21354	38	30	25065	43	30	29044
33	40	21474	38	40	25193	43	40	29182
33	50	21594	38	50	25321	43	50	29320
34	0	21715	39	0	25450	44	0	29459
34	10	21836	39	10	25579	44	10	29598
34	20	21957	39	20	25708	44	20	29738
34	30	22078	39	30	25837	44	30	29878
34	40	22199	39	40	25967	44	40	30018
34	50	22321	39	50	26097	44	50	30159
35	0	22443	40	0	26228	45	0	30300

tr.	①.	snylinen.	tr.	①.	snylinen.	tr.	①.	snylinen.
45	10	30442	50	10	34902	55	10	39857
45	20	30584	50	20	35058	55	20	40032
45	30	30726	50	30	35215	55	30	40208
45	40	30869	50	40	35373	55	40	40385
45	50	31013	50	50	35531	55	50	40563
46	0	31156	51	0	35690	56	0	40741
46	10	31301	51	10	35849	56	10	40921
46	20	31445	51	20	36009	56	20	41101
46	30	31590	51	30	36169	56	30	41282
46	40	31736	51	40	36330	56	40	41463
46	50	31882	51	50	36491	56	50	41646
47	0	32028	52	0	36654	57	0	41829
47	10	32175	52	10	36816	57	10	42013
47	20	32322	52	20	36980	57	20	42198
47	30	32470	52	30	37144	57	30	42384
47	40	32618	52	40	37308	57	40	42570
47	50	32767	52	50	37473	57	50	42758
48	0	32916	53	0	37639	58	0	42946
48	10	33066	53	10	37806	58	10	43135
48	20	33216	53	20	37973	58	20	43325
48	30	33367	53	30	38141	58	30	43516
48	40	33518	53	40	38309	58	40	43708
48	50	33670	53	50	38478	58	50	43901
49	0	33822	54	0	38648	59	0	44095
49	10	33975	54	10	38819	59	10	44289
49	20	34128	54	20	38990	59	20	44485
49	30	34282	54	30	39162	59	30	44681
49	40	34436	54	40	39334	59	40	44879
49	50	34591	54	50	39508	59	50	45078
50	0	34746	55	0	39682	60	0	45277

tr.	①.	*snijlinen.*	tr.	①.	*snijlinen.*	tr.	①.	*snijlinen.*
60	10	45478	65	10	52030	70	10	59960
60	20	45679	65	20	52269	70	20	60257
60	30	45882	65	30	52510	70	30	60555
60	40	46085	65	40	52752	70	40	60856
60	50	46290	65	50	52995	70	50	61159
61	0	46496	66	0	53241	71	0	61465
61	10	46703	66	10	53487	71	10	61774
61	20	46911	66	20	53736	71	20	62085
61	30	47120	66	30	53986	71	30	62399
61	40	47330	66	40	54237	71	40	62716
61	50	47541	66	50	54491	71	50	63035
62	0	47754	67	0	54746	72	0	63357
62	10	47967	67	10	55003	72	10	63682
62	20	48182	67	20	55262	72	20	64011
62	30	48398	67	30	55522	72	30	64342
62	40	48616	67	40	55784	72	40	64676
62	50	48834	67	50	56049	72	50	65014
63	0	49054	68	0	56315	73	0	65354
63	10	49275	68	10	56583	73	10	65698
63	20	49497	68	20	56853	73	20	66045
63	30	49720	68	30	57124	73	30	66396
63	40	49945	68	40	57398	73	40	66750
63	50	50171	68	50	57674	73	50	67107
64	0	50399	69	0	57953	74	0	67468
64	10	50628	69	10	58233	74	10	67833
64	20	50858	69	20	58515	74	20	68202
64	30	51090	69	30	58800	74	30	68574
64	40	51323	69	40	59086	74	40	68950
64	50	51557	69	50	59375	74	50	69331
65	0	51793	70	0	59667	75	0	69715

tr.	①.	*snÿlinen.*	tr.	①.	*snÿlinen.*	tr.	①.	*snÿlinen.*
75	10	70104	80	10	84354	85	10	108865
75	20	70497	80	20	84945	85	20	110075
75	30	70894	80	30	85546	85	30	111328
75	40	71296	80	40	86158	85	40	112630
75	50	71703	80	50	86781	85	50	113982
76	0	72114	81	0	87415	86	0	115389
76	10	72530	81	10	88061	86	10	116856
76	20	72951	81	20	88719	86	20	118389
76	30	73377	81	30	89389	86	30	119993
76	40	73808	81	40	90073	86	40	121675
76	50	74245	81	50	90771	86	50	123444
77	0	74687	82	0	91483	87	0	125209
77	10	75134	82	10	92210	87	10	127180
77	20	75588	82	20	92952	87	20	129272
77	30	76047	82	30	93711	87	30	131498
77	40	76512	82	40	94486	87	40	133879
77	50	76984	82	50	95280	87	50	136437
78	0	77462	83	0	96091	88	0	139200
78	10	77947	83	10	96923	88	10	142205
78	20	78438	83	20	97775	88	20	145497
78	30	78937	83	30	98648	88	30	149139
78	40	79442	83	40	99544	88	40	153213
78	50	79955	83	50	100464	88	50	157834
79	0	80476	84	0	101409	89	0	163176
79	10	81004	84	10	102380	89	10	169501
79	20	81541	84	20	103380	89	20	177259
79	30	82085	84	30	104409	89	30	187284
79	40	82639	84	40	105471	89	40	201513
79	50	83201	84	50	106565	89	50	226223
80	0	83773	85	0	107696	90	0	000000

Dit bereytsel vande tafel der versaemde snijllnen aldus ghedaen sijnde, en om nu tottet maken vande tafels der cromstreken te commen, soo laet inde voorgaende form R Z beteyckenen d'eerste cromstreeck, sulcx dat den houck X R Q des driehoucx X R Q nu doe 78 tr. 45 ①. Om te vinden de booch Q X, ick aensie de driehouck X R Q voor plat, om de cleenheyt der sijden, en segh datse drie bekende palen heeft, te weten den houck X Q R recht, X R Q 78 tr. 45 ①, de sijde Q R 1 tr. Hier me ghesocht de sijde Q X wort bevonden deur het 4 voorstel der platte driehoucken van 5 tr. 1 ①, die ick inde volghende tafel van d'eerste Cromstreeck stel byde breeden nevens 1 tr. der langde. Om nu al de volghende breeden deses tafels met cortheyt te vinden, ick sie inde voorgaende tafel der versaemde snijlinen wat ghetal datter overcomt mette bovescreven 5 tr. 1 ①, en bevinde 3014, want de 5 tr. hebben 3004, en noch 10 sijn het everedelic deel voor de 1 ①. Dit ghetal van 3014 dient my int gemeen tottet vinden der ghetalen van P Y, *d a*, en al d'ander dierghelicke, t'welck aldus toegaet: Totte 3014, vergaert ander 3014, comt 6028, daer op vinde ick t'overcommen inde voorgaende tafel der versaemde snijlinen 10 tr: De selve stel ick inde volghende tafel van d'eerste cromstreeck byde breeden nevens den 2 tr. der langde, als voor P Y. Daer na vergaer ick totte 6028, andermael 3014, comt 9042, daer op vinde ick t'overcommen inde voorgaende tafel 14 tr. 54 ①, de selve stel ick inde volgende tafel nevens den 3 tr. der langde, als voor *d a*; En so voort mette rest der seven Cromstreken.

MERCKT dat ick totte voorgaende langden en breeden der cromstreken, noch vervoughe haer verheden, dat sijn de langden der bogen R X, R Y, R *a*, en dierghelijcke om deur t'behulp der selve sonder eertcloot of platte caert, maer alleenelick deur ghetalen, te beantwoorden de voorstellen die vanden handel der cromstreken omgaen, en int volghende beschreven sullen sijn. Dese verheden worden aldus bekent. Om ten eersten te vinden de verheyt R X, ick segh den driehouck X Q R te hebben drie bekende palen, te weten den houck X Q R recht, den houck X R Q 78 tr. 45 ①, en de sijde Q X van 5 tr. 1 ①. Hier me ghesocht de sijde R X, wort bevonden deur het 4 voorstel der platte driehoucken van 5 tr. 6 ① 54 ②, die ick stel in d'eerste tafel by de verheden nevens 1 tr. der langde. Ten anderen om te vinden de verheyt R Y, ick segh den driehouck Y *c* X te hebben drie bekende palen, te weten den houck Y *c* X recht, Y X *c* 78 tr. 45 ①, en de sijde *c* Y 4 tr. 59 ①, als blijckt deur de tafel; want treckende P *c* 5 tr. 1 ①, als even sijnde met Q X, van P Y 10 tr. blijft voor *c* Y alsvooren 4 tr 59 ①: Met dese drie bekende palen dan, ghesocht de sijde X Y, wort bevonden deur het 4 voorstel der platte driehoucken van 5 tr. 12 ① 54 ②, die vergaert tot R X 5 tr. 6 ① 54 ②, comt voor R Y 10 tr. 19 ① 48 ②, die ick stel in d'eerste cromstreeck byde verheden nevens 2 tr. der langde. En alsoo sal ghevonden worden de verheyt van R *a*, met al d'ander.

Merckt dat wy dese verheden niet overal berekent noch ghestelt en hebben, maer alleenelick soo veel als tot ons volghende voorbeelden noodich sijn, eensdeels dat de tafelen selfgheen ghenouchsaem volcommenheyt en schijnen te hebben, ghelijck inden Anhang breeder gheseyt sal worden, als oock dat belet van ander saken ons t'selve niet toe en laet: Sulcx dat hier alleenelick de wijse ghetoont is, en open plaets ghelaten om die te meughen volmaeckt worden, by de ghene dieder lust en gheleghentheyt toe mochten hebben.

TAFEL

This preliminary of the table of the assembled secants thus having been made and in order to come now to the making of the tables of loxodromes, in the foregoing figure let *RZ* denote the first loxodrome, so that the angle *XRQ* of the triangle *XRQ* is now 78°45′. In order to find the arc *QX*, I look upon the triangle *XRQ* as a plane triangle, on account of the smallness of the sides, and say that it has three known elements, *viz.* the angle *XQR* = 90°, *XRQ* = 78°45′, the side *QR* = 1°. When herewith the side *QX* is sought, it is found by the 4th proposition of plane trigonometry [1]) to be 5°1′, which I put in the following table of the first loxodrome in the column of the latitudes, against 1° of longitude. In order to find all the subsequent latitudes of this table in a brief way, I look up in the foregoing table of assembled secants what value corresponds to the above-mentioned 5°1′ and find this to be 3,014, for the 5° makes 3,004, and 10 more is the proportional part for the 1′. This value of 3,014 serves me in general to find the values of *PY*, *DA*, and all other similar elements, which takes place as follows. When to the 3,014 we add another 3,014, we get 6,028. Corresponding to this I find in the foregoing table of assembled secants 10°. I put this in the following table of the first loxodrome in the column of the latitudes, against 2° of longitude, *viz.* for *PY*. Then to 6,028 I add once more 3,014, which gives 9,042. Corresponding to this I find in the foregoing table 14°54′. I put this in the following table, against 3° of longitude, *viz.* for *DA*, and so on with the rest of the seven loxodromes.

NOTE that I further place behind the foregoing longitudes and latitudes of the loxodromes their distances, *i.e.* the lengths of the arcs *RX*, *RY*, *RA*, and the like, in order to solve by these means, without a globe or a plane chart, but only by numbers, the propositions which concern the subject of loxodromes and which are to be described in the sequel. These distances become known in the following way. To find first the distance *RX*, I say that the triangle *XQR* has three known elements, *viz.* the angle *XQR* = 90°, the angle *XRQ* = 78°45′, and the side *QX* = 5°1′. When herewith the side *RX* is sought, it is found by the 4th proposition of plane trigonometry [1]) to be 5°6′54″, which I put in the first table in the column of the distances, against 1° of longitude. Second, to find the distance *RY*, I say that the triangle *YCX* has three known elements, *viz.* the angle *YCX* = 90°, *YXC* = 78°45′, and the side *CY* = 4°59′, as appears from the table; for when we subtract *PC* = 5°1′, as being equal to *QX*, from *PY* = 10°, the remainder is, as before, 4°59′ for *CY*. When with these three known elements the side *XY* is then sought, this is found by the 4th proposition of plane trigonometry [1]) to be 5°12′54″. When we add this to *RX* = 5°6′54″, we get for *RY* 10°19′48″, which I put in the table of the first loxodrome in the column of the distances, against 2° of longitude. And in the same way the distance *RA* and all the others have to be found.

Note that we have not calculated or noted these distances everywhere, but only as far as they are necessary for our subsequent examples, on the one hand because the tables themselves do not seem to be sufficiently accurate, as will be discussed more fully in the Appendix, but also because we are hindered by other matters from doing so, so that here the method only has been shown, and a blank has been left, to be completed by those who have a mind and an opportunity to do so.

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[1]) Stevin's *Trigonometry* (Work XI; i, 12), p. 147.

TAFELS
DER
CROMSTREKEN.

TABLES OF LOXODROMES [1]).

First Loxodrome			*Second Loxodrome*
Longitude (degr.)	Latitude (degr., min.)	Distance (degr., min.)	

[1]) We reproduce the tables for the first and for the second loxodrome, and (on p. 539) the end of the table for the seventh loxodrome.

Eerste Cromstreeck.

lang. tr.	Breede tr.	①	verhedē tr.	①	lang. tr.	Breede tr.	①	verhedē tr.	①	lang. tr.	Breede tr.	①	verhedē tr.	①
1	5	1	5.	7.	31	82	27			61	89	27		
2	10	0	10	20.	32	83	5			62	89	29		
3	14	54			33	83	39			63	89	32		
4	19	42			34	84	11			64	89	34		
5	24	22			35	84	40			65	89	36		
6	28	51			36	85	7			66	89	38		
7	33	10			37	85	32			67	89	40		
8	37	16			38	85	54			68	89	41		
9	41	9			39	86	15			69	89	43		
10	44	50			40	86	33			70	89	44		
11	48	17			41	86	51			71	89	46		
12	51	31			42	87	7			72	89	47		
13	54	32			43	87	21			73	89	48		
14	57	21			44	87	35			74	89	49		
15	59	58			45	87	47			75	89	50		
16	62	23			46	87	58			77	89	51		
17	64	38			47	88	8			79	89	52		
18	66	42			48	88	17			81	89	53		
19	68	36			49	88	26			83	89	54		
20	70	22			50	88	34			85	89	55		
21	71	59			51	88	41			87	89	56		
22	73	29			52	88	47			90	89	56		
23	74	51			53	88	53			93	89	57		
24	76	6			54	88	59			96	89	58		
25	77	16			55	89	4			99	89	58		
26	78	19			56	89	9			102	89	58		
27	79	18			57	89	13			105	89	58		
28	80	11			58	89	17			108	89	59		
29	81	0			59	89	20			111	89	59		
30	81	46			60	89	24			114	89	59		

Tweede Cromstreec.

lang. tr.	Breede tr.	①	verhedē tr.	①
1	2	24		
2	4	49		
3	7	13		
4	9	36		
5	11	58		
6	14	20		
7	16	39		
8	18	57		
9	21	13		
10	23	27		
11	25	39		
12	27	48		
13	29	55		
14	31	59		
15	34	1		
16	35	59		
17	37	55		
18	39	48		
19	41	37		
20	43	24		
21	45	8		
22	46	49		
23	48	26		
24	50	1		
25	51	32		
26	53	1		
27	54	27		
28	55	49		
29	57	9		
30	58	26		

lang. tr.	Breede tr.	①	verhedē tr.	①	lang. tr.	Breede tr.	①	verhedē tr.	①	lang. tr.	Breede tr.	①	verhedē tr.	①	lang. tr.	Breede tr.	①	verhedē tr.	①
31	59	41	5.	7.	61	81	14			91	87	31			121	89	17		
32	60	53	10.	20.	62	81	36			92	87	37			122	89	19		
33	62	2			63	81	56			93	87	43			123	89	21		
34	63	8			64	82	16			94	87	48			124	89	22		
35	64	13			65	82	35			95	87	54			125	89	24		
36	65	14			66	82	54			96	87	59			126	89	25		
37	66	14			67	83	11			97	88	4			127	89	27		
38	67	11			68	83	28			98	88	9			128	89	28		
39	68	6			69	83	44			99	88	13			129	89	29		
40	68	59			70	83	59			100	88	18			130	89	30		
41	69	50			71	84	14			101	88	22			131	89	32		
42	70	39			72	84	28			102	88	26			132	89	33		
43	71	26			73	84	42			103	88	30			133	89	34		
44	72	11			74	84	55			104	88	33			134	89	35		
45	72	55			75	85	8			105	88	37			135	89	36		
46	73	36			76	85	20			106	88	40			136	89	37		
47	74	16			77	85	31			107	88	44			137	89	38		
48	74	55			78	85	42			108	88	47			138	89	39		
49	75	32			79	85	53			109	88	50			139	89	39		
50	76	7			80	86	3			110	88	52			140	89	40		
51	76	41			81	86	13			111	88	55			141	89	41		
52	77	14			82	86	22			112	88	58			142	89	42		
53	77	45			83	86	31			113	89	0			143	89	42		
54	78	15			84	86	40			114	89	3			144	89	43		
55	78	44			85	86	48			115	89	5			145	89	44		
56	79	12			86	86	56			116	89	7			146	89	44		
57	79	38			87	87	4			117	89	9			147	89	45		
58	80	4			88	87	11			118	89	12			148	89	46		
59	80	28			89	87	18			119	89	13			149	89	46		
60	80	52			90	87	25			120	89	15			150	89	47		

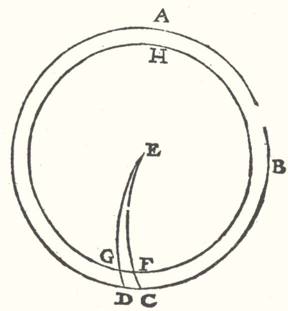
lang. tr.	Breede tr.	①	verhedē tr.	①	lang. tr.	Breede tr.	①	verhedē tr.	①	lang. tr.	Breede tr.	①	verhedē tr.	①	lang. tr.	Breede tr.	①	verhedē tr.	①
215	89	37			6	89	46			190	89	52			195	89	57		
220	89	37			12	89	46			200	89	52			210	89	57		
225	89	38			18	89	47			210	89	53			225	89	57		
230	89	38			24	89	47			220	89	53			240	89	57		
235	89	39			30	89	47			230	89	53			255	89	57		
240	89	39			36	89	47			240	89	53			270	89	58		
245	89	39			42	89	48			250	89	53			285	89	58		
250	89	40			48	89	48			260	89	54			300	89	58		
255	89	40			54	89	48			270	89	54			315	89	58		
260	89	40			60	89	48			280	89	54			330	89	58		
265	89	41			66	89	48			290	89	54			345	89	58		
270	89	41			72	89	49			300	89	54			360	89	58		
275	89	41			78	89	49			310	89	54			20	89	58		
280	89	42			84	89	49			320	89	55			40	89	58		
285	89	42			90	89	49			330	89	55			60	89	58		
290	89	42			96	89	50			340	89	55			80	89	58		
295	89	42			102	89	50			350	89	55			100	89	58		
300	89	43			108	89	50			360	89	55			120	89	58		
305	89	43			114	89	50			15	89	55			140	89	58		
310	89	43			120	89	50			30	89	56			160	89	58		
315	89	44			126	89	50			45	89	56			180	89	58		
320	89	44			132	89	51			60	89	56			200	89	59		
325	89	44			138	89	51			75	89	56			220	89	59		
330	89	44			144	89	51			90	89	56			224	89	59		
335	89	45			150	89	51			105	89	56			260	89	59		
340	89	45			156	89	51			120	89	57			280	89	59		
345	89	45			162	89	51			135	89	57			300	89	59		
350	89	45			168	89	52			150	89	57			320	89	59		
355	89	46			174	89	52			165	89	57			340	89	59		
360	89	46			180	89	52			180	89	57			360	89	59		

M 2

Tot hier toe sijn beschreven de seven tafels der seven cromstreken. Angaende de achtste die is altijt een ront evewijdich vant middelront, sulcx dattet in breede gheen verandering crijghende, soo en valter van sijn breedeschil niet te segghen, maer alleenlick van sijn verheyden, tot welcx eynde de volghende tafel dient.

VANT MAECKSEL DES *tafels der achtste cromstreeck.*

Om eerst deur een form te verclaren den sin des volghenden tafels, soo laet A B C D een cloot sijn diens middelront A B C D, en aspunt E, waer op beschreven is een cleender rondt F G H, voort sy de booch C D van 1 tr. als langdeschil tusschen C en D, en ghetrocken de twee vierendeelen ronts E D, E C, sniende het cleender rondt in F en G, soo doet G D neem ick 10 tr. als breedeschil tusschen D en G. Dit soo sijnde de booch F G doet 1 tr. der langde, en dat 10 tr. verre buyten t'middelront, welcke booch openbaerlick cleender moet sijn dan D C 1 tr. des middelronts. Nu is t'voornemen hier te vinden hoe veel 1 tr. F G langhdeschil buyten t'middelront (te weten op 10 tr. der breede D G) doet in ① en ② des middelronts: Dat is, de langde F G vervought sijnde opt middelront, van hoe veel ① en ② die daer bevonden sal worden. Dit verstaen wesende wy sullen tottet voorbeelt commen.



TGHEGHEVEN. Het sy 1 tr. langdeschil buyten t'middelrondt op 10 tr. der breede. TBEGHEERDE. Wy moeten sijn grootheyt vinden in ① en ② des middelronts.

TWERCK.

Rechthoucx houckmaet	10000000.
Gheeft schilhoucx houckmaet vande ghegheven 10 tr. doende	9848078.
Wat 1 tr. des middelronts doende.	60 ①?
Comt voor t'begheerde.	59 ① 5 ②.

Ghelijck inde volghende tafel staet, en alsoo met allen anderen.

TBEWYS.

Ghelijck de halfmiddellijn van een cloots grootste ront, als t'middelront;
Totte halfmiddellijn van haer cleender rondt, als het evewijdich rondt op de 10 tr. der breede;
Alsoo de booch eens traps des grootste of middelronts;
Totte booch eens traps des cleender ronts op de 10 tr. der breede.

Maer de halfmiddellijn eens middachronts, is even ande halfmiddellijn des middelronts, die ghenomen wort op 10000000, als rechthouckmaet: En de halfmiddellijn des rondts beschreven op de breede van 10 tr. is int middachrondt houck-

Up to this point the seven tables of the seven loxodromes have been described. As to the eighth, this is always a circle parallel to the equator, so that, since its latitude does not undergo any change, nothing can be said about its difference of latitude, but only about its distances, for which purpose the following table serves.

OF THE MAKING OF THE TABLE OF THE EIGHTH LOXODROME.

In order first to explain by means of a figure the sense of the following table, let *ABCD* be a globe, whose equator is *ABCD* and the pole *E,* on which a small circle *FGH* has been described. Further let the arc *CD* be 1°, *viz.* the difference of longitude between *C* and *D,* and when the two quarter circles *ED* and *EC* intersecting the small circle in *F* and *G* have been drawn, I assume that *GD* is 10°, *viz.* the difference of latitude between *D* and *G.* This being so, the arc *FG* is 1° of longitude, 10° outside the equator, which arc must obviously be smaller than *DC* = 1° of the equator. Now the object is to find how much 1° (*FG*) of difference of longitude outside the equator (*viz.* at 10° of latitude, *DG*) makes in minutes and seconds of arc of the equator, *i.e.* when the longitude *FG* has been placed on the equator, to how many minutes and seconds of arc this will be found to amount there. This having been understood, we shall pass on to the example.

SUPPOSITION. Let there be 1° of difference of longitude outside the equator in latitude 10°. REQUIRED. We have to find its magnitude in minutes and seconds of arc of the equator.

PROCEDURE.

Sine of a right angle	10,000,000
Gives sine of complement of the given 10°, being	9,848,078
What does 1° of the equator give, being	60′ ?
The required value is	59′5″.

As is to be found in the following table; and the same applies to all the others.

PROOF.

As the semi-diameter of a great circle on a sphere, *viz.* the equator,
To the semi-diameter of its smaller circle on said sphere, *viz.* the parallel in latitude 10°,
So the arc of one degree of the great circle or equator
To the arc of one degree of the small circle in latitude 10°.

Now the semi-diameter of a meridian is equal to the semi-diameter of the equator, which is taken to be 10,000,000, *viz.* the sine of a right angle. And the semi-diameter of the circle described in latitude 10° is in [the plane of] the meridian the sine of the complement of the arc of 10°, *i.e.* the sine of 80°, being 9,848,078.
Therefore, as 10,000,000
to 9,848,078,
so the arc of one degree of the great circle or equator
to the arc of one degree of the small circle in latitude 10°.

houckmaet des schilboochs van 10 tr. dats houckmaet van 80 tr. doende
9848078: Daerom
Ghelijck 10000000.
Tot 9848078:
Alsoo de booch eens traps des grootste of middelronts,
Totte booch eens traps des cleender ronts op de 10 tr. der breede.

Maer 10000000 ghevende 9848078, soo sal 60 ① gheven 59 ① 5 ② deur t'werck, daerom 59 ① 5 ② is t'begheerde, t'welck wy bewijsen moesten.

MERCKT.

Alsoo sijn VORSTELICKE GHENADE t'voorgaende besluyt ghelesen hadde en ghesien volcommen te wesen, seyde daer op hem ghedachtich te sijn dat hy voormael derghelijcke grootheyt eens boochs als van F tot G, ghesocht hadde deur rekening der clootsche driehoucken, om datse als van clootsche stof volcommender is dan rekening ghelijck de voorgaende: Maer overdenckende de reden van dit verschil, seyde met onderscheyt d'een wijse te dienen tot besluyt op een vraegh van cromstreken, d'ander van rechtstreken. Als by voorbeelt begeert sijnde hoe lanck de wech valien sal om te varen van F tot G op een cromstreeck, dats hier de booch eens cleender ronts, de volcommen wercking sal ghedaen worden alsvooren: Maer sulcken langde begheert wesende van F tot G op een rechtstreeck, dats de booch eens grootste ronts, de volcommen wercking bestaet dan in rekening der clootsche driehoucken. T'welck ick hier heb willen anteyckenen, eensdeels tot ghedachtnis: Ten anderen op dat de ghene dien sulcx duysterder ontmoeten mocht, hier me verlicht sy.

T'maecksel des tafels dan aldus verclaert sijnde, wy sullen de tafel beschrijven, ghetrocken uyt *Cosmographia Petri Apiani parte prima cap.* 13. als volght.

But since 10,000,000 gives 9,848,078, by the procedure 60′ will give 59′5″; therefore 59′5″ is the required value; which we had to prove.

NOTE.

When His Princely Grace had read the foregoing conclusion and had seen it to be correct, he said he remembered having formerly sought such a magnitude of an arc as that from *F* to *G*, by a calculation of spherical trigonometry, because, the matter being concerned with spheres, this is more correct than a calculation like the foregoing. But reflecting on the reason of this difference, he said that a distinction had to be made here, *viz.* that the one method served as a solution of a question concerned with loxodromes, the other of one concerned with great-circle tracks. Thus, for instance, when it is required to know the length of the route when sailing from *F* to *G* on a loxodrome, i.e. in this case the arc of a small circle, the correct procedure has to be as before. But when this length is required from *F* to *G* on a great-circle track, *i.e.* the arc of a great circle, the correct procedure then consists in a calculation of spherical trigonometry. I wanted to note this here, on the one hand to show that it has been thought of, on the other hand in order that those to whom this should appear obscure might be enlightened by it.

The making of the table thus having been explained, we shall describe the table, taken from *Cosmographia Petri Apiani parte prima cap. 13,* as follows:

EIGHTH LOXODROME, which is a table of the magnitude of 1° of difference of longitude outside the equator in minutes and seconds of arc of the equator.

In this latitude (degr., min.)	1° of differ. of longitude gives the following distance (min., sec.)

ACHTSTE CROMSTREECK, T'WELCK

is een tafel van t'ghene 1 tr. langdeschil buyten t'middelront, doet in ① en ② des middelronts.

Op dese breeden, tr.	①	Doet 1 tr. langdeschil dese verhe. ①	②		Op dese breeden. tr.	①	Doet 1 tr. langdeschil dese verhe. ①	②		Op dese breeden. tr.	①	Doet 1 tr. langdeschil dese verhe. ①	②	
0	30	59	59	0	16	0	57	40	9	31	0	51	25	16
1	0	59	59	1	16	30	57	31	9	31	30	51	9	17
1	30	59	58	1	17	0	57	22	9	32	0	50	52	17
2	0	59	57	1	17	30	57	13	10	32	30	50	36	17
2	30	59	56	1	18	0	57	3	10	33	0	50	19	17
3	0	59	55	2	18	30	56	53	10	33	30	50	2	18
3	30	59	53	2	19	0	56	43	10	34	0	49	44	18
4	0	59	51	2	19	30	56	33	11	34	30	49	6	18
4	30	59	48	2	20	0	56	22	11	35	0	49	8	18
5	0	59	46	3	20	30	56	11	11	35	30	48	50	18
5	30	59	43	3	21	0	56	0	11	36	0	48	32	18
6	0	59	40	3	21	30	55	49	12	36	30	48	14	19
6	30	59	36	3	22	0	55	37	12	37	0	47	55	19
7	0	59	33	4	22	30	55	25	12	37	30	47	36	19
7	30	59	29	4	23	0	55	13	12	38	0	47	16	19
8	0	59	24	4	23	30	55	1	13	38	30	46	57	20
8	30	59	20	5	24	0	54	48	13	39	0	46	37	20
9	0	59	15	5	24	30	54	35	13	39	30	46	17	20
9	30	59	10	5	25	0	54	22	13	40	0	45	57	20
10	0	59	5	6	25	30	54	9	14	40	30	45	37	21
10	30	58	59	6	26	0	53	55	14	41	0	45	16	21
11	0	58	53	6	26	30	53	41	14	41	30	44	56	21
11	30	58	47	6	27	0	53	27	14	42	0	44	35	21
12	0	58	41	7	27	30	53	13	15	42	30	44	14	22
12	30	58	34	7	28	0	52	58	15	43	0	43	52	22
13	0	58	27	7	28	30	52	43	15	43	30	43	31	22
13	30	58	20	7	29	0	52	28	15	44	0	43	9	22
14	0	58	13	8	29	30	52	13	16	44	30	42	47	22
14	30	58	5	8	30	0	51	57	16	45	0	42	25	22
15	0	57	57	8	30	30	51	41	16	45	30	42	3	23
15	30	57	49											

ACHTSTE CROMSTREECK, T'WELCK

is een tafel van t'ghene 1 tr. langdeschil buyten t'middelront, doet in ① en ② des middelronts.

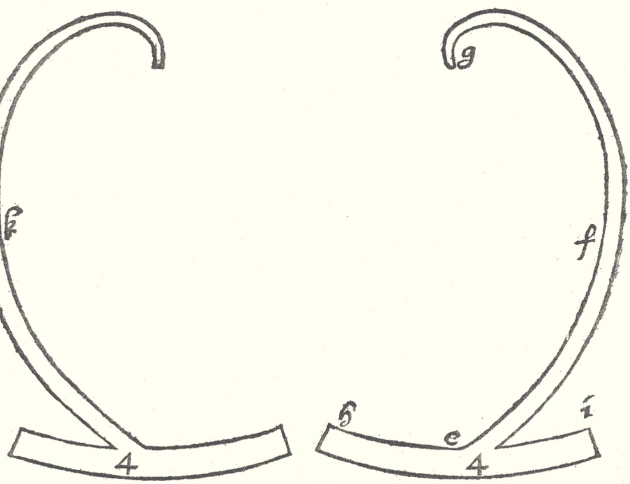
Op dese breeden,		Doet 1 tr. langdeschil dese verhe.			Op dese breeden.		Doet 1 tr. langdeschil dese verhe.			Op dese breeden,		Doet 1 tr. langdeschil dese verhe.		
tr.	①	①	②		tr.	①	①	②		tr.	①	①	②	
46	0	41	40	23	61	0	29	5	28	76	0	14	30	30
46	30	41	18	23	61	30	28	37	28	76	30	14	0	31
47	0	40	55	23	62	0	28	10	28	77	0	13	29	31
47	30	40	32	23	62	30	27	42	28	77	30	12	59	31
48	0	40	8	23	63	0	27	14	28	78	0	12	28	31
48	30	39	43	24	63	30	26	46	28	78	30	11	57	31
49	0	39	21	24	64	0	26	18	28	79	0	11	26	31
49	30	38	58	24	64	30	25	49	28	79	30	10	56	31
50	0	38	34	24	65	0	25	21	29	80	0	10	25	31
50	30	38	9	24	65	30	24	52	29	80	30	9	54	31
51	0	37	45	24	66	0	24	24	29	81	0	9	23	31
51	30	37	21	25	66	30	23	55	29	81	30	8	52	31
52	0	36	56	25	67	0	23	26	29	82	0	8	21	31
52	30	36	31	25	67	30	22	57	29	82	30	7	49	31
53	0	36	6	25	68	0	22	28	29	83	0	7	18	31
53	30	35	41	25	68	30	21	59	29	83	30	6	47	31
54	0	35	16	26	69	0	21	30	29	84	0	6	16	31
54	30	34	50	26	69	30	21	30	30	84	30	5	45	31
55	0	34	24	26	70	0	20	31	30	85	0	5	13	31
55	30	34	59	26	70	30	20	1	30	85	30	4	42	31
56	0	33	33	26	71	0	19	32	30	86	0	4	11	31
56	30	33	6	26	71	30	19	2	30	86	30	3	39	31
57	0	32	40	26	72	0	18	32	30	87	0	3	8	31
57	30	32	14	27	72	30	18	2	30	87	30	2	37	31
58	0	31	47	27	73	0	17	32	30	88	0	2	5	31
58	30	31	21	27	73	30	17	2	30	88	30	1	34	31
59	0	30	54	27	74	0	16	32	30	89	0	1	2	31
59	30	30	27	27	74	30	16	2	30	89	30	0	31	31
60	0	30	0	27	75	0	15	31	30	90	0	0	0	31
60	30	29	32	27	75	30	15	1	30					

Aldus dan inde voorgaende tafels beschreven sijnde der seven cromstreken breeden in yder middachbooch die van trap tot trap getrocken sijn, so ist openbaer hoemen daer me met groote sekerheyt op een Eertcloot de cromstreken sal meughen teyckenen, want de punten der breede ghestelt na t'behooren, en van d'een tot d'ander linikens ghetrocken, men comt tottet begheerde. Oock dienen de selve tafelen om te sien of cromstreken op Eertclooten of platte caerten wel gheteyckent sijn.

VANT MAECKSEL DER COPER CROMSTREKEN.

Maer wanttet tot verclaring der nabeschreven voorstellen bequamer soude vallen, en oock tottet ghebruyck in veel voorbeelden mijns bedunckens niet ongerievich, datmen de seven cromstreken van coper maecte, om die te leggen op yder punt des cloots daermen wil, en alsoo ten eersten t'begheerde te sien, sonder noodich te wesen de cromstreken op een Eertcloot te teyckenen: Of gheteyckent sijnde sonder te moeten doen de moeylicker wercking daer uyt volghende, soo sullen wy daer af wat breeder uytlegging doen als volght:

Op een Eertcloot gheteyckent sijnde een cromstreeck na de wijse als boven, ick neem de voorschreven noortooststreeck R X Y Z, wesende een vierde, men sal maken een dergheliicke van coper, welcke anghewesen wort mette volgende form *efg*, welverstaende dat de cant daer *efg* an comt, de eyghen lini als R X Y Z bediet, waer an noch vervought is het stick *hi*, op wiens uyterste cant *hi*, de lini *efg* sulcken houck maeckt, als R X Y Z opt middelront L N. Dese coper cromstreeck wort oock verstaen te hebben een clootsche hollicheyt, sulcx datse op den cloot gheleyt die overal gheraeckt.



Voort ghelijck hier gemaeckt is dese coper noortooststreeck, oock dienende voor zuytweststreeck, alsoo salmen maken d'ander ses, t'samen seven: En noch sulcke seven ander, van verkeerde ghestalt der voorgaende, als by voorbeelt de form

After the description in the foregoing tables of the latitudes of the seven loxodromes in each of the meridians drawn from degree to degree, it is clear how with the aid of these we shall be able to draw the loxodromes with great certainty on a globe, for when the points of latitude have been properly marked and small lines have been drawn from one to the other, the required loxodrome is obtained. The said tables also serve to ascertain whether loxodromes have been drawn correctly on globes or plane charts.

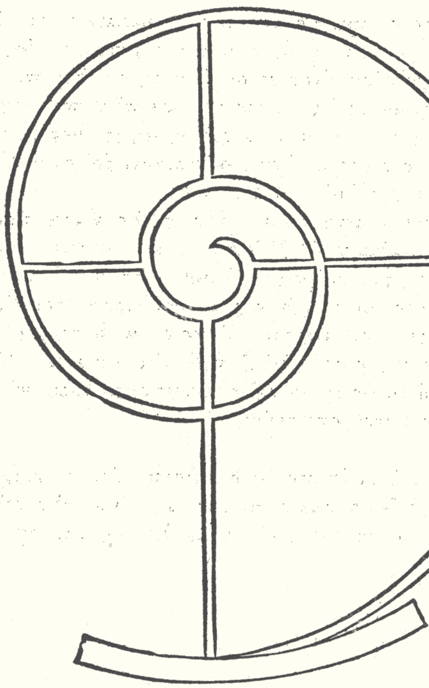
OF THE MAKING OF THE COPPER LOXODROMES.

But since it would be more suitable for the explanation of the succeeding propositions — and further in my opinion not inconvenient for use in many examples — to make the seven loxodromes of copper, so that we may put them on any desired point of the globe and thus see at once what was required, without the necessity of drawing the loxodromes on a globe, or, if they have been drawn, without having to perform the more difficult operations following therefrom, we shall explain this somewhat more fully, as follows:

After a loxodrome has been drawn on a globe in the above way — I assume the aforesaid loxodrome of northeast *RXYZ,* which is a fourth loxodrome — we should make one like it of copper, which is denoted by the figure *efg* below, it being understood that the side on which *efg* comes stands for the line *RXYZ* itself, while the piece *hi* has been attached thereto, with whose outer edge *hi* the line *efg* forms the same angle as *RXYZ* with the equator *LN*. This copper loxodrome is also understood to be spherically concave, so that, when it is placed on the globe, it touches the latter everywhere.

Further, just as this copper loxodrome of northeast, also serving for the loxodrome of southwest, has here been made, in the same way the other six should be made, seven in all. And seven more of them, having the opposite shape to form *k*, weſende van verkeerde geſtalt der noortooſtſtreeck *e f g*, dienende totte noortweſtſtreeck, en oock totte zuytooſtſtreeck. Noch machmen deſe coper cromſtreken teyckenen met ghetalen, d'eerſte met 1, de tweede met 2, en ſoo voort, om daer deur met een opſien te weten de hoemenichſte elck is.

Angaende de 7 cromſtreeck veel keeren ſoude hebben, en daerom ſlap ſijn, men ſoudeſe (met oock ander ſtreken diet noodich hadden) meughen verſtijven



ghelijck deſe form anwijſt, of anders mochtmen tot een cromſtreeck verſcheyden ſtucken maken. TBESLVYT. Wy hebben dan cromſtreken gheteyckent, na den eyſch.

5 VOORSTEL.

Weſende ghegheven langdeſchil en breede van tvvee plaetſen evener breede: Te vinden haer cromſtreeck en verheyt.

MERCKT.

Wy ſullen in elck der volghende voorſtellen drie werckinghen beſchrijven, d'eerſte mette coper cromſtreeck, de tweede mette gecromſtreeckten Eertcloot, die beyde * tuychwerckelick ſijn, de derde * wiſconſtich deur getalen, uytgenomen in dit en t'eerſtvolghende voorſtel, alwaer geen werck mette coper crom ſtreeck en valt, om datter gheen achtſte coper cromſtreeck als onnoodich ſijnde ghemaeckt en wiert. TGHEGHEVEN. Laet het langdeſchil ſijn van 30 tr. en haet

Mechanicè. *Mathematicè.*

the foregoing ones, as *e.g.* the figure *k,* having the opposite shape to the loxodrome *efg* of northeast, serving for the loxodrome of northwest, and also for the loxodrome of southeast. Further these copper loxodromes may be marked with the number of each, the first with 1, the second with 2, and so on, so that one may know at a glance which of them is meant.

Since the 7th loxodrome would have many windings and accordingly would be limp, we might stiffen it (as well as other loxodromes which might need it) as shown in the figure below, or otherwise we might combine several pieces to form one loxodrome. CONCLUSION. We have thus designed loxodromes; as required.

5th PROPOSITION.

Given the difference of longitude and the latitude of two places of equal latitude: to find their loxodrome and their distance.

NOTE.

In each of the following propositions we shall describe three operations, the first with the copper loxodrome, the second by means of the globe with the loxodromes drawn thereon, both of which are mechanical operations, the third mathematical, by means of numbers, except in the present and the next proposition, where no operation with the copper loxodrome takes place, because no eighth copper loxodrome has been made, this being unecessary.

en haer breeden 24 tr. TBEGHEERDE Wy moeten haer cromstreeck en verheyt vinden.

1 *VVerck metten ghecromstreeckten Eertcloot.*

Anghesien de twee ghegheven breeden even sijn, soo moet deur ghemeene reghel de begheerde cromstreeck de achtste wesen, dat is de oost en weststreeck, sulcx dat daer toe niet anders te doen en is.

Angaende de verheyt ick segh aldus; soo de twee ghegheven plaetsen op den Eertcloot gheteyckent stonden, en vallende op een evewijdich rondt mettet middelront, men sal den passer so nau openen, dat de rechte lini tusschen de twee voeten verdocht, gheen hinderlick verschil en hebbe vande cromme des ronts diemen meten moet, als opening van 1 tr. meer of min, na dat de cromheyt des voorghestelden ronts vereyscht, want meerder cromte hebben de bogen byden aspunt dan by t'middelront: T'welck soo sijnde, men sal sien hoe dickwils die openheyt des passers comt inde booch tusschen de twee plaetsen, daer nae de trappen t'samen vergaert, men sal in dit voorbeelt welghenouch gewrocht sijnde, vinden voor begheerde verheyt 27 tr. 24 ①.

Maer soo de twee ghegheven plaetsen op den Eertcloot niet en stonden, men sal twee uytvaghelicke punten stellen opt ghegheven langdeschil en breeden, die ghebruyckende voor ghegheven.

Maer soose niet recht en vallen op een gheteyckent evewijdich ront, men sal int meten des boochs de passer altijt doen gaen evewijdich van het naeste evewijdich ront, t'welck soot u byder ooghe niet nauwe genouch ghedaen en wierde, deur t'behulp van een ander passer gheschien can,

MERCKT.

De vraech mocht nu sijn hoe veel mijlen de boveschreven 27 tr. 24 ① maken, maer insiende de verscheydenheyt der mijlen in verscheyden landen, so en canmen daer af int ghemeen niet sekers segghen. Sulcx dat wy hier en oock int volghende, de verheyt alleenlick deur tr. en ① beschrijven, die elck verkeeren mach in sulcke mijlen alst hem belieft. Den trap wort van velen gheacht lanck te wesen ontrent 18 uyren gaens eens ghemeenen gancx: Een dier uyren wort ghenomen op 8000 stappen, oock op 1500 Rijnlantsche roeden, dat comt den stap op $2\frac{1}{4}$ Rijnlantsche voeten. Doch t'waer te wenschen dat Stierlien int ghemeen trappen en ① ghebruyckten, om malcander int ghemeen te verstaen.

2 *VVerck door ghetalen.*

Anghesien de twee ghegheven breede even sijn, soo moet deur ghemeene regel, als oock int 1 werck ghesevt is, de begeerde cromstreeck de achtste wesen.

Om nu de verheyt te vinden, ick souc inde tafel des 8 cromstreecx int 4 voorstel de ghegheven breede van 24 tr. en daer nevens wat verheyt tot die plaets overcomt op 1 tr. langdeschil, bevinde 54 ① 48 ②: Hier me segh ick, 1 tr. langdeschil, doet tot dese plaets 54 ① 48 ② verheyt, wat de ghegheven 30 tr. langdeschil? Comt voor begeerde verheyt 27 tr. 24 ①: waer af t'bewijs deur t'werck openbaer is. TBESLVYT. Wesende dan ghegheven langdeschil en breede van twee plaetsen evener breede, wy hebben ghevonden haer cromstreeck en verheyt, na den eysch.

MERCKT.

SUPPOSITION. Let the difference of longitude between the places be 30° and their latitude 24°. REQUIRED. We have to find their loxodrome and their distance.

1st Procedure, by means of the Globe with the Loxodromes Drawn thereon.

Since the two given latitudes are equal, by the general rule the required loxodrome must be the eighth, *i.e.* the east and west loxodrome, so that nothing else has to be done about this.

As to the distance, I say as follows: if the two given places are marked on the globe and fall on a circle parallel to the equator, one must open the compasses so little that the straight line imagined between the two points does not differ appreciably from the arc of the circle that has to be measured, *viz.* an opening of about one degree, as the curvature of the circle in question requires, for arcs near the pole have a greater curvature than arcs near the equator. This being so, one must ascertain how often this opening of the compasses is included in the arc between the two places; the degrees afterwards being added up, in the present example — if the procedure has been exact enough — one will find 27°24′ for the required distance.

But if the two given places are not marked on the globe, one must mark two erasable points having the difference of longitude and the latitude given, using them as if they were given.

But if they do not fall exactly on a drawn parallel, in measuring the arc one must always keep the compasses parallel to the nearest parallel, which, if it should not be done accurately enough at sight, can be effected with the aid of another pair of compasses.

NOTE.

The question might now be asked to how many miles the above-mentioned 27°24′ corresponds, but considering the inequality between miles in different countries, in general nothing can be said with certainty about this. So that here as well as in the sequel we shall only describe the distance in degrees and minutes, which everyone may reduce to such miles as he pleases. A degree is considered by many people to be equivalent to about 18 hours' walk at an ordinary pace. One such hour is taken to be 8,000 paces, also 1,500 Rhineland roods, so that one pace is 2 1/4 Rhineland feet. But it were to be wished that navigators generally used degrees and minutes, so as to understand each other generally.

2nd Procedure, by Numbers.

Since the two given latitudes are equal, by the general rule, as has also been said in the 1st procedure, the required loxodrome must be the eighth.

Now in order to find the distance, I look up in the table of the 8th loxodrome in the 4th proposition the given latitude of 24°, and against it I see what distance corresponds in that place to 1° of difference of longitude, for which I find 54′48″. I now say: 1° of difference of longitude gives in this place a distance of 54′48″; what does the given difference of longitude of 30° give? We get 27°24′ for the required distance; the proof of which is evident from the procedure. CONCLUSION. Given the difference of longitude and the latitude of two places of equal latitude, we have thus found their loxodrome and their distance; as required.

MERCKT.

Soo de ghegheven ghetalen niet gantſchelick en overquamen mette ghetalen inde tafel beſchreven, men ſoude (ſoo hier als inde volghende voorſtellen) om t'begheerde ſeer na te crijghen, vinden wat het voorgeſtelde ghetal eygentlick toecomt, ghelijck in ander tafels ghebruyckt wort, en ſoo daer voorbeelt af is int 11 voorſtel des houckmaetmaeckſels.

6 VOORSTEL.

Weſende ghegeven verheyt en breede van tvvee plaetſen evener breede: Te vinden haer cromſtreeck en langdeſchil.

TGHEGHEVEN. Laet tweer plaetſen verheyt ſijn van 27 tr. 24 ①, en haer breeden 24 tr. TBEGHEERDE. Wy moeten haer cromſtreeck en langdeſchil vinden.

1 VVerck metten ghecromſtreeckten Eertcloot.

Anghesien de twee ghegheven breeden even ſijn, ſoo moet deur ghemeene reghel als int 1 werck des 5 voorſtels gheſeyt is, de begheerde cromſtreeck de achtſte ſijn.

Angaende het langdeſchil, ick ſegh aldus: Soo de twee ghegheven plaetſen op den Eertcloot gheteyckent ſtonden op een evewijdich ront, men ſal op den pasſer nemen een deel des middelronts als 1 tr. meer of min en meten daer me de ghegheven 27 tr. 24 ①, int voorſchreven rondt dat deur de twee plaetſen ſtreckt, brenghende voorts onder het middachront t'eynde der ſelve, daer nae t'begin, en ſal alſdan de booch des middelronts, begrepen tuſſchen ſoodanige twee middachronden, t'begheerde langdeſchil ſijn, en bevonden worden van 30 tr.

Maer ſoo de twee ghegheven plaetſen op den Eertcloot niet en ſtonden, men ſal daer me doen alſoo van derghelijcke gheſeyt is int 5 voorſtels eerſte werck.

2 VVerck door ghetalen.

Anghesien de twee ghegheven breeden even ſijn, ſoo moet deur ghemeene reghel als int 1 werck des 5 voorſtels gheſeyt is, de begheerde cromſtreeck de achtſte ſijn.

Om nu het langdeſchil te vinden, ick ſouck inde tafel des 8 cromſtreecx int 4 voorſtel de ghegheven breede van 24 tr. en daer nevens wat verheyt tot die plaets overcomt op 1 tr. langdeſchil, bevinde 54 ① 48 ②: Hier me ſegh ick 54 ① 48 ②, gheven tot deſe plaets 1 tr. langdeſchil, wat de ghegheven verheyt 27 tr. 24 ①? Comt voor begeert langdeſchil 30 tr. waer af t'bewijs deur t'werck openbaer is. TBESLVYT. Weſende dan ghegheven verheyt en breede van twee plaetſen evener breede, wy hebben ghevonden haer cromſtreeck en langdeſchil, na den eyſch.

7 VOOR-

NOTE.

If the given numerical values do not correspond altogether to the numerical values given in the table, one must (both here and in the subsequent propositions), in order to get the required value very accurately, find what the value in question properly corresponds to, as is commonly done in other tables and in the way of the example given thereof in the 11th proposition of the work on the making of tables of sines [1]).

6th PROPOSITION.

Given the distance and the latitude of two places of equal latitude: to find their loxodrome and their difference of longitude.

SUPPOSITION. Let the distance between two places be 27°24′ and their latitude 24°. REQUIRED. We have to find their loxodrome and their difference of longitude.

1st Procedure, by means of the Globe with the Loxodromes Drawn thereon.

Since the two given latitudes are equal, by the general rule, as said in the 1st procedure of the 5th proposition, the required loxodrome must be the eighth.

As to the difference of longitude, I say as follows. If the two given places are marked on the globe on a parallel, one must take between the compasses a part of the equator, *viz.* about one degree, and measure therewith the given 27°24′ in the aforesaid circle passing through the two places, bringing first one end thereof and then the other under the meridian; then the arc of the equator contained between these two meridians will be the required difference of longitude and will be found to be 30°.

But if the two given places are not marked on the globe, one must proceed in the same way as has been said of a similar case in the first procedure of the 5th proposition.

2nd Procedure, by means of Numbers.

Since the two given latitudes are equal, by the general rule, as said in the 1st procedure of the 5th proposition, the required loxodrome must be the eighth.

Now in order to find the difference of longitude, I look up in the table of the 8th loxodrome in the 4th proposition the given latitude of 24°, and against it I see what distance corresponds in that place to 1° of difference of longitude, which I find to be 54′48″. I now say: 54′48″ gives in this place 1° of difference of longitude; what does the given distance of 27° 24′ give? We get 30° for the required difference of longitude, the proof of which is evident from the procedure. CONCLUSION. Given the distance and the latitude of two places of equal latitude, we have thus found their loxodrome and their difference of longitude; as required.

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[1]) Stevin's *Trigonometry* (Work XI; i, 11), p. 58, where he explains how one is to find the sine of a given angle by means of the tables.

7 VOORSTEL.

Weſende ghegheven tvveer plaetſen cromſtreeck en breeden: Te vinden (midts dat de ghegheven breeden niet even en ſijn) haer langdeſchil en verheyt.

Tis kennelick ſoo de twee ghegheven breeden even waren, dat de cromſtreeck ſoude de 8 vallen, of waer de ghegheven cromſtreeck de achtſte, dat de breeden ſouden moeten even ſijn: Maer want langdeſchil en verheyt gevonden worden deur breedeſchil, en dat hier gheen en is, ſo en canmen deur ſulck ghegheven het langdeſchil en de verheyt niet vinden, en daerom iſt dat int voorſtel deſe uytneming ſtaet, namelick, *midts dat de ghegeven breeden niet even en ſijn*; waer uyt kennelick is, dat wanneer in voorbeelden deſes voorſtels de ghegeven cromſtreeck ſeer nae de achtſte ſtaet, datmen inde daet t'beſluyt niet ſeer ſeker en heeft.

MERCKT.

T'ghebeurt wel dat een Sierman ſeylende van d'een plaets tot d'ander op een ſelve ſtreeck, deur dadelicke ervaring ghenomen heeft die twee plaetſen breede, en dat hy hier uyt wil vinden de verheyt, om te ſien hoe ſijn giſſing die hy int ſeylen daer op ghenomen mach hebben, overcomt met deſe reghelen: Voort hoe daer me overcomt het langdeſchil dat op Eertclooten en in tafels daer af ghestelt mach ſijn: Oft anders ſoo de voornoemde giſſing en langdeſchil onbekent waren, hoemen die bekent ſal maken, en tot ſulcken eynde dient dit voorſtel. TGHEGHEVEN. Laet der twee plaetſen cromſtreeck ſijn de 4, en de weſtelicker breede 5 tr. 59 ①, d'ander breede 28 tr. 42 ①.

TBEGHEERDE. Wy moeten haer langdeſchil en verheyt vinden.

1 *VVerck mette coper cromſtreeck.*

Ick neem een Eertcloot welcke A B C D ſy, diens aſpunt A, en middelront D B, teycken daer op een verborghen of uytvagelick punt E, ſulcx dat ſijn breede F E doe 5 tr. 59 ① der ghegheven weſtlicker plaets; neem voort na t'inhout van t'ghegeven de vierde coper cromſtreeck, welcke ſy G H I K, vervough haer gront G H opt middelront D B, die daer langs henen ſchuyvende tot dat de wijſlijn I K comt op E; keer daer na den Eertcloot weſtwaert (om dat d'ander plaets ooſtlicker is) tot dat de cromſtreeck het middachront deurſnijt in d'ander ghegheven breede van 28 tr. 42 ①, t'welck valt neem ick an L, en de gemeene ſne des middachronts en middelronts ſy alſdan M. Dit ſoo weſende F M is der twee plaetſen langdeſchil, t'welck op den Eertcloot in dit voorbeelt bevonden moet worden voor t'begheerde van 24 tr.

Voort is des cromſtreecx lini E L de verheyt, welcke gemeten als int 5 voorſtels eerſte werck, moet in dit voorbeelt bevonden worden voor t'begheerde van 32 tr. 8 ①.

2 *VVerck metten ghecromſtreeckten Eertcloot.*

Ick verkies op den Eertcloot eenighe cromſtreeck als de ghegeven, dats de 4, en breng die onder het middachront, ſnyende t'ſelve inden 5 tr. 59 ① der ghegheven weſtelicker breede; Ick keer daer na den Eertcloot weſtwaert (om dat

d'ander

7th PROPOSITION.

Given the loxodrome and the latitudes of two places: to find (provided the given latitudes are not equal) their difference of longitude and their distance.

It is obvious that if the two latitudes were equal, the loxodrome would be the eighth, or if the given loxodrome were the eighth, the latitudes would have to be equal. Now because the difference of longitude and the distance are found by means of the difference of latitude and there is none in this case, it is not possible to find the difference of longitude and the distance by means of this datum, and that is why the proposition contains this condition, *viz. provided the given latitudes are not equal,* from which it is obvious that if in examples of this proposition the given loxodrome is very near to the eighth, in practice the solution is not very accurate.

NOTE.

It sometimes happens that a navigator, sailing from one place to another on the same course, has found the latitudes of those two places by practical observation and that from these he wants to find the distance, in order to see how far his conjecture, which he may have made during the voyage, agrees with these rules. Further, how far the difference of longitude that may be given thereof on globes and in tables agrees with it. Or else, if the aforesaid conjecture and difference of longitude were unknown, how they have to become known; and it is for this purpose that the present proposition serves.

SUPPOSITION. Let the loxodrome of the two places be the fourth, and the latitude of the more westerly place 5°59′, the latitude of the other 28°42′.

REQUIRED. We have to find their difference of longitude and their distance.

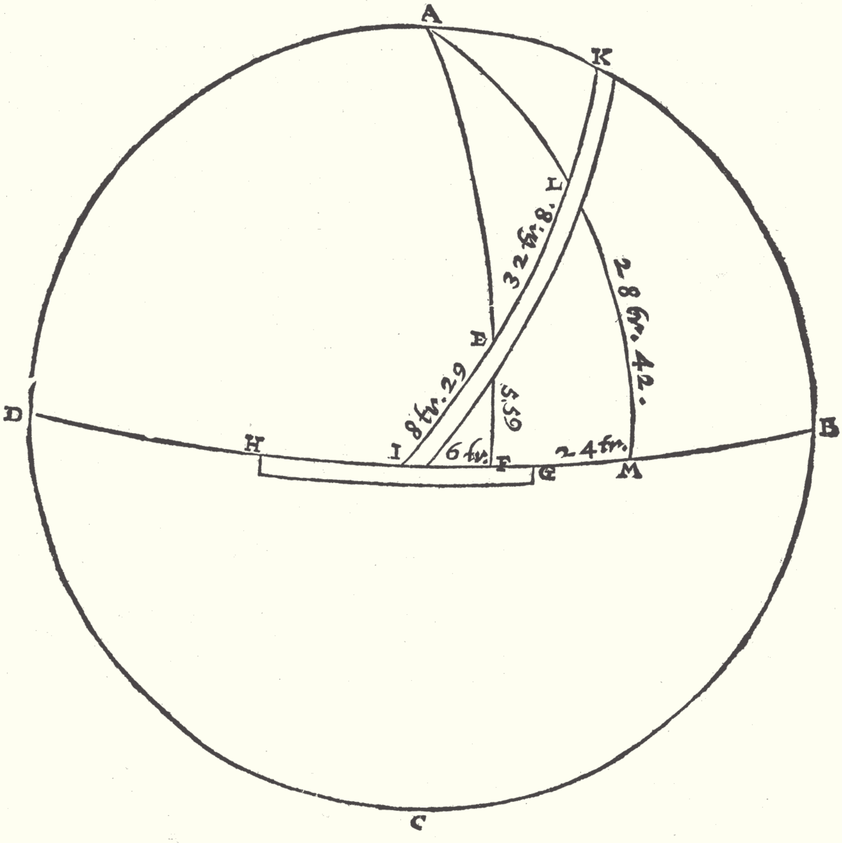
1st Procedure, with the Copper Loxodrome.

I take a globe, which shall be *ABCD,* whose pole is *A* and the equator *DB,* and mark on it an erasable point *E* so that its latitude *FE* is the 5°59′ of the more westerly of the given places. Further, in accordance with the supposition, I take the fourth copper loxodrome, which shall be *GHIK,* place its base *GH* on the equator *DB,* moving it along the latter until the index line *IK* falls through *E,* then turn the globe to the west (because the other place is further to the east) until the loxodrome intersects the meridian in the other given latitude of 28°42′, which is, I assume, at *L*; and let the point of intersection of the meridian and the equator then be *M*. This being so, *FM* is the difference of longitude between the two places, which on the globe in this example, to satisfy the requirement, must be found to be 24°.

Further the part *EL* of the loxodrome is the distance, which, when measured as in the first procedure of the 5th proposition, in this example, to satisfy the requirement, must be found to be 32°8′.

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

I choose on the globe a loxodrome for the one given, which is the 4th, and bring it under the meridian, so that it intersects the latter in the 5°59′ of the



d'andet plaets ooſtlicker is) tot dat de cromſtreeck het middachront ſnijt inden 28 tr. 42 ① van d'ander ghegheven breede: T'welck ſoo ſijnde, het middelront verloopen vande eerſte ſtandt totte tweede, is het langdeſchil, dat in dit voorbeelt bevonden moet worden van 24 tr.

Voort is de cromſtreecx lini tuſſchen die twee ſtanden de verheyt, welcke gemeten als int 5 voorſtels 1 werck, moet in dit voorbeelt bevonden worden voor t'begheerde van 32 tr. 8 ①.

3 VVerck door ghetalen.

Ick ſouck inde tafel des 4 voorſtels de ghegheven 4 cromſtreeck, en ſie wat langden en verheden overcommen op de twee ghegeven breeden, bevinde op de cleenſte breede 5 tr. 59 ①, t'overcommen langde 6 tr. verh. 8 tr. 29 ①.
En op de grootſte breede van 28 tr. 42 ①, bevinde ick t'overcommen langde 30. verh. 40 tr. 37 ①.
Daer af ghetrocken langde en verheyt eerſte in d'oirden, blijft voor t'begheerde langdeſchil 24 tr. verh. 32 tr. 8 ①.
Waer af t'bewijs deur t'werck openbaer is. TBESLVYT. Weſende dan

N ghegiven more westerly latitude. I then turn the globe to the west (because the other place is further to the east) until the loxodrome intersects the meridian in the 28°42′ of the other given latitude. This being so, the part of the equator between the first and the second position is the difference of longitude, which in this example must be found to be 24°.

Further the part of the loxodrome between those two positions is the distance, which, when measured as in the 1st procedure of the 5th proposition, in this example, to satisfy the requirement, must be found to be 32°8′.

3rd Procedure, by means of Numbers.

I seek in the table of the 4th proposition the given 4th loxodrome and look up what longitudes and distances correspond to the two given latitudes. I find that the values corresponding to the
smaller latitude 5°59′ are .... the longitude 6° and the distance 8°29′,
and those corresponding to the greater
latitude 28°42′ I find to be .... the longitude 30° and the distance 40°37′.
When from this we subtract the
longitude and distance first mentioned,
we get for the required .... difference of longitude 24°, for the distance 32°8′;
the proof of which is evident from the procedure. CONCLUSION. Given the

ghegheven tweer plaetsen cromstreeck en breeden : Wy hebben ghevonden (midts dat de ghegheven breeden niet even en waren) haer langdeschil en verheyt, na den eysch.

8 VOORSTEL.

Wesende ghegheven tvveer plaetsen breeden en langdeschil: Te vinden haer cromstreeck en verheyt.

TGHEGHEVEN. Laet de breede der westlicker plaets sijn van 5 tr. 59 ①, d'ander plaetsens breede 28 tr. 42 ①, en haer langdeschil 24 tr.

TBEGHEERDE. Wy moeten de cromstreeck en verheyt vinden.

1 VVerck mette coper cromstreeck.

Soo de ghegheven plaetsen op den Eertcloot niet gheteyckent en waren, men salder uyt vaghelicke punten setten, volghende t'ghegheven, die ick inde form des 7 voorstels neem te wesen de twee punten E en L : Daer na salmen uyt de seven coperen cromstreken een verkiesen die ons uyter oogh dunckt t'begheerde ten naesten te commen, legghende haer gront G H mette wijslijn opt middelront B D, die daer langs henen schuyvende tot dat des cromstreecx wijslijn I K, comme op d'een der twee ghegheven plaetsen, ick neem op E: Soo d'ander plaets L alsdan comt te gheraken de selve wijslijn I K, so is die coper streeck de begheerde; maer sulcx niet ghebeurende, men sal een ander nemen daer op passende, of die nemen welcke ten naesten comt, de selve moet in dit voorbeelt bevonden worden voor t'begheerde de vierde.

Voort is des cromstreecx lini E L de verheyt, welcke ghemeten als int 5 voorstels eerste werck, moet in dit voorbeelt bevonden worden voor t'begheerde van 32 tr. 8 ①.

2 VVerck metten ghecromstreeckten Eertcloot.

Soo de twee plaetsen op den Eertcloot gheteyckent stonden, en datse by ghevalle op een selve gheteyckende cromstreeck lagen, tis kennelick dat die cromstreeck de begheerde soude sijn: Maer want dat selden ghebeurt, so sullen wy de saeck hier by voorbeelt nemen datse beyde in gheen selve gheteyckende cromstreeck en vallen, waer toe den voortganck dusdanich is: Keert den cloot tot dat eenige cromstreeck het middachront deursnijt op de breede van d'een plaets, ick neem de westelicker : Daer nae anghesien d'ander plaets oostelicker is, soo keert den cloot westlicker, tot datter een booch des middelronts verloopen is van 24 tr. te weten het ghegheven langdeschil: Siet dan of de ghenomen cromstreeck het middachront deursnijt, op de ghegheven grootste breede der 28 tr. 42 ① van d'ander plaets, want dat soo ghebeurende, die cromstreeck is de begheerde: Dies niet , neemt een ander cromstreeck, en doet daer me alsvooren en dergheliicke soo dickwils tot dat ghy de begheerde cromstreeck crijcht, of de begheerde ten naesten, welcke in dit voorbeelt bevonden sal worden de vierde.

Om daer na de verheyt te hebben, men sal tusschen de twee boveschreven plaetsen inde vierde cromstreeck ghevonden, meten de langde des selven cromstreecx met een passer, als int 5 voorstels 1 werck, en moet in dit voorbeelt bevonden worden voor t'begheerde van 32 tr. 8 ①.

loxodrome and the latitudes of two places, we have thus found (provided the given latitudes are not equal) their difference of longitude and their distance; as required.

8th PROPOSITION.

Given the latitudes and the difference of longitude of two places: to find their loxodrome and their distance.

SUPPOSITION. Let the latitude of the more westerly place be 5°59′, the latitude of the other place 28°42′, and their difference of longitude 24°.
REQUIRED. We have to find the loxodrome and the distance.

1st Procedure, with the Copper Loxodrome.

If the given places are not marked on the globe, one must mark thereon erasable points, in accordance with the supposition, which in the figure of the 7th proposition I assume to be the two points *E* and *L*. Thereafter one must choose from the seven copper loxodromes one which at sight appears to approximate the required loxodrome most, placing its base *GH* with the index line on the equator *BD*, moving it along the latter until the index line *IK* of the loxodrome falls through one of the two given places, I assume through *E*. If the other place *L* then falls on the said index line *IK*, this copper loxodrome is the one required; but if this is not the case, one must take another, which does fit on it, or take the one that approximates it most; in this example, to satisfy the requirement, this must be found to be the fourth.
Further the part *EL* of the loxodrome is the distance, which, when measured as in the first procedure of the 5th proposition, in this example, to satisfy the requirement, must be found to be 32°8′.

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

If the two places were marked on the globe and happened to lie on the same drawn loxodrome, it is obvious that this loxodrome would be the one required. But because this rarely happens, we shall here give an example where the two do not fall on the same drawn loxodrome, for which the procedure is as follows. Turn the globe until a certain loxodrome intersects the meridian in the latitude of one of the places, I assume the more westerly one. Then, since the other place lies further to the east, turn the globe further to the west until an arc of the equator of 24° has been traversed, *viz.* the given difference of longitude. Then see whether the chosen loxodrome intersects the meridian, in the given greater latitude of the 28°42′ of the other place, for if this is the case, that loxodrome is the one required. If not, take another loxodrome and proceed therewith as before, and similarly until you get the required loxodrome, or approximately the one required, which in this example will be found to be the fourth.
In order to find next the distance, one must measure, between the two above-mentioned places found in the fourth loxodrome, the length of this loxodrome by means of a pair of compasses, as in the 1st procedure of the 5th proposition; in this example, to satisfy the requirement, this must be found to be 32°8′.

3 VVerck door ghetalen.

Ick ſouck inde tafel des 4 voorſtels de cleenſte gegeven breede van 5 tr. 59 ①, in eenighe der 7 cromſtreken, ick neem inde vierde, en ſie wat langde
daer me overcomt, bevinde 6 tr.
Daer na ſouck ick inde ſelve vierde cromſtreec wat langde datter overcomt op d'ander ghegheven breede 28 tr. 42 ①, bevinde 30 tr.
Daer af ghetrocken 6 tr. eerſte in d'oirden, blijft langdeſchil. 24 tr.

By aldien nu t'ſelve langdeſchil niet even en waer, oft immers niet na ghenouch even mettet ghegheven langdeſchil, ſo en ſoude die ghenomen tafel der 4 cromſtreeck de begheerde niet ſijn, en daerom ſoudemen dierghelijcke moeten verſoucken op een ander cromſtreeck, en dat ſoo dickwils tot datmen het derde des oirdens even vonde, of ten naeſten even mettet ghegheven langdeſchil: Maer t'valt in dit voorbeelt even, daerom de begheerde cromſtreeck is de vierde.

Om nu de verheyt te vinden, ick ſouc inde boveſchreven vierde cromſtreeck d'een ghegheven breede 5 tr. 59 ①, vinde daer op te overcommen verheyt van 8 tr. 29 ①,
En inde ſelve vierde cromſtreeck ſouck ick d'ander ghegheven breede 28 tr. 42 ①, vinde daer op te overcommen verheyt van 40 tr. 37 ①.
Diens verſchil vande verheyt 8 tr. 29 ① vijfde in d'oirden, is voor de begheerde verheyt. 32 tr. 8 ①.

Waer af t'bewijs deur t'werck openbaer is. TBESLVYT. Weſende dan ghegheven tweer plaetſen breeden en langdeſchil, wy hebben ghevonden haer cromſtreeck en verheyt, na den eyſch.

9 VOORSTEL.

Weſende ghegeven tvveer plaetſen breede en verheyt: Te vinden de cromſtreeck en langdeſchil.

MERCKT.

T'ghebeurt dat ymant bekent ſijn tweer plaetſen breeden, en verheyt van d'een tot d'ander, deur giſſing die int ſeylen daer op mach hebben ghenomen gheweeſt, en dat hy begheert te weten de cromſtreeck en langdeſchil, om ſijn toecommende ſeyling daer na te rechten, of om ſulcke twee plaetſen op den Eertcloot recht te teyckenen, of daer op gheteyckent ſijnde, om te ſien hoe ſijn rekening daer me overcomt, en tot ſulcken eynde can dit voorſtel dienen.

TGHEGHEVEN. Laet de weſtlicker plaetſens breede ſijn van 5 tr. 59 ①, van d'ander 28 tr. 42 ①, en haer verheyt 32 tr. 8 ①. TBEGHEERDE. Wy moeten haer cromſtreeck en langdeſchil vinden.

1 VVerck mette coper cromſtreeck.

Om eerſt de cromſtreeck te vinden, ick neem een Eertcloot welcke beteyckent ſy mette form des 7 voorſtels, ſet daer op d'een ghegheven plaets, latet ſijn de weſtlicker, diens breede 5 tr. 59 ① als t'punt E: Ick neem daer na een der ſeven coper cromſtreken die my uyter oogh dunckt t'begheerde ten naeſten te commen, legghende haer gronts wijſlijn opt middelront B D, die daer langs henen ſchuyvende tot dat des cromſtreecx wijſlijn I K comme opt punt E: Ick

 meet

3rd Procedure, by means of Numbers.

I look up in the table of the 4th proposition the smaller of the given latitudes 5°59′, under one of the 7 loxodromes, I assume under the fourth, and see what longitude corresponds thereto, which I find to be 6°

Thereafter I look up in the said table of the fourth loxodrome what longitude corresponds to the other given latitude 28°42′, which I find to be 30°

When from this we subtract 6°, the first in the present list, the difference of longitude is 24°

Now if the said difference of longitude were not equal, or at least almost equal, to the given difference of longitude, the chosen table of the 4th loxodrome would not be the one required, and consequently we should have to try similarly with another loxodrome, until the third figure in the present list were found to be equal, or approximately equal, to the given difference of longitude. But in this example it is equal, consequently the required loxodrome is the fourth.

Now in order to find the distance, I look up in the above-mentioned table of the fourth loxodrome the one of the given latitudes 5°59′, and find as the distance corresponding thereto 8°29′

And in the said table of the fourth loxodrome I look up the other given latitude 28°42′, and find as the distance corresponding thereto 40°37′

The difference of the latter from the distance 8°29′, the fifth in the present list, is, for the required distance, 32° 8′

The proof of which is evident from the procedure. CONCLUSION. Given the latitudes and the difference of longitude of two places, we have thus found their loxodrome and their distance; as required.

9th PROPOSITION.

Given the latitudes of and the distance between two places: to find the loxodrome and the difference of longitude.

NOTE.

It sometimes happens that a man knows the latitudes of two places and the distance from one to the other, by a conjecture which he may have made in practice, and that he wants to know the loxodrome and the difference of longitude, in order to direct his future sailing thereby, or in order to mark these two places correctly on the globe, or, if they are marked thereon, to see how his reckoning agrees therewith, and for this purpose the present proposition may serve.

SUPPOSITION. Let the latitude of the more westerly of the places be 5°59′, that of the other 28°42′, and their distance 32°8′. REQUIRED. We have to find their loxodrome and their difference of longitude.

1st Procedure, with the Copper Loxodrome.

In order to find first the loxodrome, I take a globe, on which the figure of the 7th proposition shall have been drawn. I mark thereon the one given place, let it be the more westerly one, whose latitude is 5°59′, *viz.* the point *E*. I then take one of the seven copper loxodromes which at sight appears to me to approximate the required loxodrome most, putting its base on the equator *BD*, moving it along the latter until the index line *IK* of the loxodrome falls through the point *E*.

meet voort met een passer na de wijse als int 5 voorstel van E langs de wijslijn der crom streeck, tot dat ick heb de ghegeven verheyt 32 tr. 8 ①, die comt neem ick an L, welck punt soo vallende, dat sijn breede de ghegheven is van 28 tr. 42 ①, die ghenomen coper cromstreeck is de begheerde: Maer sulcx niet ghebeurende, men sal een ander nemen, en daer me dergelijcke doen, en dat so dickwils tot datmen een crijcht waer in sulcke breede alsoo even valt, of ten naesten comt, t'welck in dit voorbeelt moet sijn voor t'begeerde de vierde cromstreeck.

Om daer na het langdeschil te hebben, men salt soucken vande twee punten E, L, t'welck sy F M, en moet in dit voorbeelt bevonden worden van 24 tr.

2 *vverck metten ghecromstreeckten Eertcloot.*

Om eerst de cromstreeck te vinden, ick verkies op een ghecromstreeckten Eertcloot eenighe cromstreeck die my uyter oogh dunckt t'begheerde ten naesten te commen: Latet inde form des 7 voorstels sijn de cromstreeck I K, teycken daer op d'een ghegheven plaets, latet sijn de westelicker als t'punt E, diens breede F E 5 tr. 59 ①: Ick meet voort met een passer na de wijse als int 5 voorstel, van E langs de cromstreeck tot dat ick heb de ghegheven verheyt van 32 tr. 8 ①, die comt neem ick an t'punt L, welck punt soo vallende, dat sijn breede L M de ghegheven is van 28 tr. 42 ①, de ghenomen cromstreeck is de begeerde. Maer sulcx niet ghebeurende, men sal een ander nemen, en daer me dergelijcke doen, en dat so dickwils tot datmen een crijcht waer in sulcke breede also even valt, of ten naesten comt, t'welck in dit voorbeelt moet sijn voor t'begheerde de vierde cromstreeck.

Om daer na het langdeschil te hebben, men salt soucken vande twee punten E, L, t'welck sy F M, en moet in dit voorbeelt bevonden worden voor t'begeerde van 24 tr.

3 *vverck door ghetalen.*

Om eerst de cromstreeck te vinden, ick souck inde tafels des 4 voorstels d'een ghegheven breede in een der cromstreken, latet sijn de cleenste van 5 tr. 59 ①, voor t'eerste inde 4 cromstreeck, en sie wat verheyt daer me overcomt, bevinde — 8 tr. 29 ①.

Daer toe ghedaen de ghegheven verheyt (ick segh daer toe ghedaen om dat d'eerste breede de cleenste was, want waerse de grootste gheweest men soude moeten aftrecken) doende — 32 tr. 8 ①.

Comt verheyt — 40 tr. 37 ①.

Daer na sien ick inde selve vierde cromstreeck, wat breede datter overcomt op de verheyt van 40 tr. 37 ① derde in d'oirden, bevinde — 28 tr. 42 ①.

Dese breede even sijnde mette ghegeven breede der tweede plaets, soo is dese ghenomen vierde cromstreeck de begheerde: Waerse daer me oneven gheweest, men soude derghelijcke moeten met een ander versoucken, tot datmen sulcx bevonde te overcommen, of ten naesten.

Om nu het langdeschil te vinden, ick souck eerst de cromstreeck alsvooren, en sie wat langde datter overcomt op d'een ghegeven breede van 28 tr. 42 ①, bevinde — 30 tr.

S'ghe-

I next measure by means of a pair of compasses, by the method described in the 5th proposition, from *E* along the index line of the loxodrome until I have got the given distance 32°8′, which I assume to be in *L;* and if this point falls in such a way that its latitude is the given latitude 28°42′, the chosen copper loxodrome is the one required. But if this is not the case, one must take another and proceed similarly therewith, until one gets a loxodrome in which this latitude is equal or nearly so, which in this example, to satisfy the requirement, must be the fourth loxodrome.

In order to get next the difference of longitude, one must seek it for the two points *E* and *L,* which shall be *FM,* and this in the present example must be found to be 24°.

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

In order to find first the loxodrome, I choose on a globe with the loxodromes drawn thereon a loxodrome which at sight appears to me to approximate the required one most. Let it be, in the figure of the 7th proposition, the loxodrome *IK.* I mark thereon the one given place, let it be the more westerly one, *viz.* the point *E,* whose latitude *FE* is 5°59′. I next measure by means of a pair of compasses, by the method described in the 5th proposition, from *E* along the loxodrome until I have got the given distance of 32°8′, which I assume to be in the point *L;* and if this point falls in such a way that its latitude *LM* is the given latitude 28°42′, the chosen loxodrome is the one required. But if this is not the case, one must take another and proceed similarly therewith, until one gets a loxodrome in which this latitude is equal or nearly so, which in this example, to satisfy the requirement, must be the fourth loxodrome.

In order to get next the difference of longitude, one must seek it for the two points *E* and *L,* which shall be *FM,* and this in the present example, to satisfy the requirement, must be 24°.

3rd Procedure, by means of Numbers.

In order to find first the loxodrome, I look up in the table of the 4th proposition the one given latitude — let it be the smaller one of 5°59′ — under one of the loxodromes, to begin with under the 4th loxodrome, and I see what distance corresponds thereto, which I find to be 8°29′

When we add thereto the given distance (I say: add, because the first latitude was the smaller one, for if it had been the greater one, we should have to subtract), being 32° 8′

The distance becomes 40°37′

Thereafter I see in the said table of the fourth loxodrome what latitude corresponds to the distance of 40°37′, the third in the present list, which I find to be 28°42′

Since this latitude is equal to the given latitude of the second place, this chosen fourth loxodrome is the one required. If it had not been equal to it, we should have to try similarly with another, until we found it to correspond, or nearly so.

In order to find now the difference of longitude, I first seek the loxodrome, as before, and see what longitude corresponds to the one given latitude 28°42′, which I find to be 30°

S'ghelijcx wat langde datter overcomt op d'ander ghegeven breede de van 5 tr. 59 Ⓘ, bevinde 6 tr. — 6 tr.

Die ghetrocken vande 30 tr. vijfde in d'oirden, blijft voor begheert langdeſchil — 24 tr.

Waer af t'bewijs deur t'werck openbaer is. TBESLVYT. Weſende dan ghegheven tweer plaetſen breede en verheyt, wy hebben ghevonden de cromſtreeck en langdeſchil, na den eyſch.

10 VOORSTEL.

Weſende ghegeven tvveer plaetſen cromſtreeck, langdeſchil, en d'een plaetſens breede: Te vinden d'ander plaetſens breede en verheyt.

TGHEGHEVEN. Laet de cromſtreeck ſijn de vierde, langdeſchil 24 tr. en d'een plaetſens breede weſende de weſtelickſte en cleenſte 5 tr. 59 Ⓘ.

TBEGHEERDE. Wy moeten vinden d'ander plaetſens breede en verheyt.

1 VVerck mette coper cromſtreeck.

Om ten eerſten te vinden d'ander plaetſens breede, ick teycken op eenighen Eertcloot als die des 7 voorſtels een punt ghelijck E, op de ghegeven breede van 5 tr. 59 Ⓘ: Daer na, want de ghegeven cromſtreeck de vierde is, ſo vervough ick de vierde coper cromſtreeck alſoo, dat haer gronts wijſlijn G H paſſe opt middelront D B, en dat de wijſlijn I K gerake t'punt E: T'ſelve gebrocht onder het middachront, ick teycken des middachrondts ſne F int middelrondt D B, keer daer na den Eertcloot weſtwaert tot datter int middelront van F af deurloopen ſijn 24 tr. des ghegheven langdeſchils, t'welck valt neem ick van F tot M, en teycken alſdan des middelronts ſne inde cromſtreeck, als ter plaets van L: Dit ſoo ſijnde, L M is d'ander plaetſens begheerde breede, die in dit voorbeelt bevonden moet worden van 28 tr. 42 Ⓘ. En E L is de begheerde verheyt, welcke ghemeten met een paſſer na de wijſe des 5 voorſtels, moet in dit voorbeelt bevonden worden van 32 tr. 8 Ⓘ.

2 VVerck metten ghecromſtreeckten Eeertcloot.

Om eerſt te vinden d'ander plaetſens breede, angheſien de vierde cromſtreeck de ghegheven is, ick teycken op den ghecromſtreeckten Eertcloot in eenighe 4 cromſtreeck welcke I K ſy, een punt als E, op de ghegheven breede van 5 tr. 59 Ⓘ: T'ſelve punt E ghebrocht onder het middachront, ick teycken des middachronts ſne F int middelront D B: keer daer na den Eertcloot weſtwaert, tot datter int middelront van F af deurloopen ſijn 24 tr. des ghegheven langdeſchils, t'welck valt neem ick van F tot M, en teycken alſdan des middelronts ſne inde cromſtreeck, als ter plaets van L. Dit ſoo ſijnde L M is d'ander plaetſens begeerde breede, die in dit voorbeelt bevonden moet worden van 28 tr. 42 Ⓘ: En E L de begheerde verheyt, welcke ghemeten na de wijſe des 5 voorſtels moet in dit voorbeelt bevonden worden van 32 tr. 8 Ⓘ.

Likewise what longitude corresponds to the other given latitude 5°59′, which I find to be 6°

When we subtract the latter from the 30°, the fifth in the present list, the remainder is, for the required difference of longitude, 24°

The proof of which is evident from the procedure. CONCLUSION. Given the latitudes of and the distance between two places, we have thus found the loxodrome and the difference of longitude; as required.

10th PROPOSITION.

Given the loxodrome and the difference of longitude of two places, and the latitude of one place: to find the latitude of the other place and the distance.

SUPPOSITION. Let the loxodrome be the fourth, the difference of longitude 24°, and the latitude of one place, being the more westerly and smaller one, 5°59′.

REQUIRED. We have to find the latitude of the other place and the distance.

1st Procedure, with the Copper Loxodrome.

In order to find first the latitude of the other place, I mark on a globe, *viz.* the one of the 7th proposition, a point, *viz. E,* in the given latitude of 5°59′. Thereafter, because the given loxodrome is the fourth, I place the fourth copper loxodrome in such a way that the base *GH* fits on the equator *DB* and the index line *IK* passes through the point *E.* The latter having been brought under the meridian, I mark the point of intersection *F* of the meridian with the equator *DB,* then turn the globe to the west until the 24° of the given difference of longitude have been traversed in the equator from *F,* which I assume to be from *F* to *M,* and then I mark the point of intersection of the equator with the loxodrome, *viz. L.* This being so, *LM* is the required latitude of the other place, which in this example must be found to be 28°42′. And *EL* is the required distance, which, when measured with a pair of compasses by the method described in the 5th proposition, in this example must be found to be 32°8′.

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

In order to find first the latitude of the other place, since the fourth loxodrome is the given loxodrome, I mark on the globe with the loxodromes drawn thereon, on a 4th loxodrome, which shall be *IK,* a point, *viz. E,* in the given latitude of 5°59′. The said point *E* having been brought under the meridian, I mark the point of intersection *F* of the meridian with the equator *DB,* then turn the globe to the west until the 24° of the given difference of longitude have been traversed in the equator from *F,* which I assume to be from *F* to *M,* and then mark the point of intersection of the equator with the loxodrome, *viz. L.* This being so, *LM* is the required latitude of the other place, which in this example must be found to be 28°42′. And *EL* must be the required distance, which, when measured by the method described in the 5th proposition, in this example must be found to be 32°8′.

3 VVerck met ghetalen.

Ick souck de ghegheven breede 5 tr. 59 ① inde tafel des 4 voorstels inde ghegheven vierde cromstreeck, en sie wat verheyt en langde daer op overcomt, bevinde verheyt 8 tr. 29.
En langde 6 tr.
Daer toe vergaert het ghegheven langdeschil (ick segh vergaert om dat de ghegheven breede de cleenste is, waerse de grootste men sou moeten aftrecken) doende 24 tr.
Comt langde 30 tr
Daer op vinde ick te overcommen inde boveschreven 4 streeck, de breede der tweede plaets voor t'begheerde 28 tr. 42 ①.
En vinde oock daer nevens de verheyt van 40 tr. 37.
Diens verschil vande verheyt 8 tr. 29 ①, eerste in d'oirden, doet voor begheerde verheyt 32 tr. 8.
Waeraf t'bewijs deur t'werck openbaer is. TBESLVYT. Wesende dan gegeven tweer plaetsen cromstreeck, langdeschil, en d'een plaetsens breede; Wy hebben ghevonden d'ander plaetsens breede en verheyt, na den eysch.

11 VOORSTEL.

Wesende ghegheven tvveer plaetsen cromstreeck, d'een plaetsens breede en verheyt: Te vinden d'ander plaetsens breede en het langdeschil.

MERCKT.

Genomen an ymant kennelick te sijn op wat cromstreeck datmen van d'een plaets tot d'ander seylt, voort d'een plaetsens breede en de verheyt, die deur gissing int seylen mach ghevonden sijn, waer me hy begheert te vinden d'ander plaetsens breede, en het langdeschil, om die op een Eertcloot te teyckenen, of daer op wesende te meughen sien hoe sijn rekening daer me overcomt, en tot sulcken eynde can dit voorstel dienen. TGHEGHEVEN. Laet de cromstreeck sijn de 4, de breede der westelicker plaets de cleenste wesende van 5 tr. 59 ①, en de verheyt 32 tr. 8 ①. TBEGHEERDE. Wy moeten vinden d'ander plaetsens breede, en het langdeschil.

1 VVerck mette coper cromstreeck.

Ick teycken op den Eertcloot des 7 voorstels eenich punt als E, op de ghegeven breede van 5 tr. 59 ①, vervough daer op de vierde coper cromstreeck, te weten alsoo dat den gront G H passe opt middelront D B, en de wijslijn I K op E: Ick meet voorts met een passer langs de selve wijslijn I K, van E oostwaert (om dat E deur t'gegeven westelicker is) na K, de ghegheven verheyt van 32 tr. 8 ①, na de wijse des 5 voorstels, welcke comt neem ick tot L. Dit soo sijnde, ick souck de breede van L, welcke is L M, die in dit voorbeelt bevonden moet worden voor t'begheerde van 28 tr. 42 ①.

Daer na souck ick der twee plaetsen langdeschil F M, t'welckmen in dit voorbeelt bevinden moet voor t'begheerde van 24 tr.

2 Werck

3rd Procedure, by means of Numbers.

I look up the given latitude of 5°59′ in the table of the 4th proposition under the given fourth loxodrome, and see what distance and longitude correspond thereto, and I find the distance to be 8°29′
And the longitude 6°
When we add thereto the given difference of longitude (I say: add, because the given latitude is the smaller one; if it were the greater, we should have to subtract), being 24°
The longitude becomes 30°
Corresponding thereto in the table of the above-mentioned 4th loxodrome I find the required latitude of the second place 28°42′
And I also find against it the distance 40°37′
The difference between the latter and the distance 8°29′, the first in the present list, is the required distance 32° 8′
The proof of which is evident from the procedure. CONCLUSION. Given the loxodrome and the difference of longitude of two places and the latitude of one place, we have thus found the latitude of the other place and the distance; as required.

11th PROPOSITION.

Given the loxodrome of two places, the latitude of one place, and the distance: to find the latitude of the other place and the difference of longitude.

NOTE.

Assuming that a man knows on what loxodrome one sails from one place to the other, further the latitude of one place and the distance, which may have been found by conjecture in practice, from which he wants to find the latitude of the other place and the difference of longitude, in order to mark them on a globe or, if they are marked thereon, to see how his reckoning agrees therewith, for this purpose the present proposition may serve. SUPPOSITION. Let the loxodrome be the 4th, the latitude of the more westerly place, being the smaller one, 5°59′, and the distance 32°8′. REQUIRED. We have to find the latitude of the other place and the difference of longitude.

1st Procedure, with the Copper Loxodrome.

I mark on the globe of the 7th proposition a point, *viz.* *E,* in the given latitude of 5°59′, place thereon the fourth copper loxodrome, *viz.* in such a way that the base *GH* fits on the equator *DB* and the index line *IK* passes through *E.* I next measure with a pair of compasses along the said index line *IK* from *E* eastward (because *E* by the supposition is more westerly) to *K* the given distance of 32°8′, by the method of the 5th proposition, which I assume to come as far as *L.* This being so, I seek the latitude of *L,* which is *LM,* which in this example, to satisfy the requirement, must be found to be 28°42′.

I next seek the difference of longitude *FM* of the two places, which in this example, to satisfy the requirement, must be found to be 24°.

2 *VVerck metten ghecromstreeckten Eertcloot.*

Ick verkies eenighe gheteyckende vierde cromstreeck daer ick de ghegheven breede in teyckenen can: Latet inden Eertcloot des 7 voorstels sijn de cromstreeck I K, waer in gheteyckent is t'punt E, soo dat sijn breede F E doe de ghegheven 5 tr. 59 ①: Ick meet voort met een passer langs de wijslijn I K van E oostwaert (om dat E deur t'ghegheven de westlicker plaets is) na K, de ghegheven verheyt van 32 tr. 8 ①, na de wijse des 5 voorstels, welcke comt neem ick tot L. Dit soo sijnde ick souck de breede van L, welck is L M, die in dit voorbeelt bevonden moet worden voor t'begheerde van 28 tr. 42 ①.

Daer na souck ick der twee plaetsen langdeschil F M, t'welckmen in dit voorbeelt bevinden moet voor t'begheerde van 24 tr.

3 *VVerck door ghetalen.*

Ick souck de ghegheven breede 5 tr. 59 ① inde ghegeven vierde cromstreeck der tafels des 4 voorstels, en sie wat langde en verheyt daer op overcomt, bevinde langde 6 tr.

En verheyt 8 tr. 29 ①.

Daer toe vergaert de ghegheven verheyt (ick segh vergaert om dat de ghegeven breede de cleenste is, waerse de grootste men soude moeten aftrecken) doende 32 tr. 8 ①.

Comt verheyt 40 tr. 37. ①.

Die ghesocht inde boveschreven vierde cromstreeck, ick vinde daer nevens te overcommen voor begheerde breede der tweede plaets 28 tr. 42 ①.

En vinde oock daer nevens de langde van 30 tr.

Diens verschil vande langde 6 tr. eerste in d'oirden, doet voor begheert langdeschil. 24 tr.

Waer af t'bewijs deur t'werck openbaer is. TBESLVYT. Wesende dan ghegheven tweer plaetsen cromstreeck, d'een plaetsens breede en verheyt: Wy hebben ghevonden d'ander plaetsens breede, en het langdeschil, na den eysch.

MERCKT.

Wy hebben hier vooren ghezeyt vant seylen op rechte en cromme seylstreken, maer wantter in groote zeevaerden, noch ghebruyckt wort daert de gheleghentheyt toelaet een wijse van seyling ghemengt van beyden, te weten een achtste streeck met een middachront, soo sullen wy daer af hier wat vermaen doen. Laet inde volghende form A B den Eertcloot beteyckenen, diens middelront A B, en noortschen aspunt C, voort sijn D, E, twee plaetsen van verscheyden langden en breeden, deur welcke ghetrocken sijn de twee evewijdige ronden D F en E G. Om nu van E tot D te seylen, niet op een rechte noch cromme streeck alsvooren, maer op een middachront met een achtste streeck: Men seylt eerst van E af recht noortwaert an, opt middachront E F, tot datmen sich vint op de breede diemen weet D te hebben, t'welck sy neem ick tot F: daer na keertmen westwaert, blijvende gheduerlick op de selve breede, dat is geduerlick varende op de achtste cromstreeck tot datmen ter begheerde plaets D comt.

Merckt noch datmen soude meughen eerst beginnen te seylen van E recht

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

I choose a certain drawn fourth loxodrome on which I can mark the given latitude. Let it be, on the globe of the 7th proposition, the loxodrome *IK*, on which the point *E* has been marked, so that its latitude *FE* is the given 5°59′. I next measure with a pair of compasses along the index line *IK* from *E* eastward (because *E* by the supposition is the more westerly place) to *K* the given distance of 32°8′, by the method of the 5th proposition, which I assume to come as far as *L*. This being so, I seek the latitude of *L*, which is *LM*, which in this example, to satisfy the requirement, must be found to be 28°42′.

I next seek the difference of longitude *FM* of the two places, which in this example, to satisfy the requirement, must be found to be 24°.

3rd Procedure, by means of Numbers.

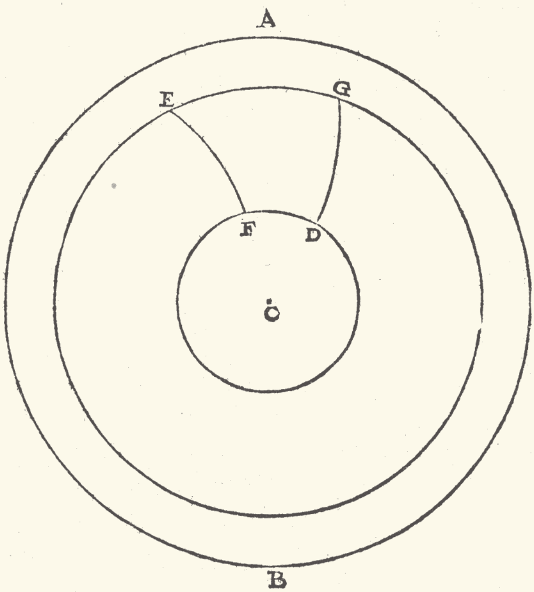
I look up the given latitude of 5°59′ under the given fourth loxodrome of the table of the 4th proposition, and see what longitude and distance correspond thereto, and I find the longitude to be	6°
And the distance	8°29′
When we add thereto the given distance (I say: add, because the given latitude is the smaller one; if it were the greater, we should have to subtract), being	32° 8′
The distance becomes	40°37′
This being looked up under the above-mentioned fourth loxodrome, I find against it, as the value corresponding thereto, the required latitude of the second place	28°42′
And I also find against it the longitude	30°
The difference between the latter and the longitude 6°, the first in the present list, is the required difference of longitude	24°

The proof of which is evident from the procedure. CONCLUSION. Given the loxodrome of two places, the latitude of one place, and the distance, we have thus found the latitude of the other place and the difference of longitude; as required.

NOTE.

We have spoken above of sailing on great-circle tracks and loxodromes, but because on long voyages use is also made, as occasion permits, of a method of sailing that is a combination of the two, *viz.* an eighth loxodrome and a meridian, we shall say something about this here. In the figure overleaf let *AB* designate the earth, whose equator is *AB* and the north pole *C;* further *D* and *E* are two places of different longitudes and latitudes, through which have been drawn the two parallels *DF* and *EG*. Now in order to sail from *E* to *D*, neither on a great-circle track nor on a loxodrome, as above, but on a meridian and an eighth loxodrome, one first sails from *E* straight to the north, on the meridian *EF*, until one is in the latitude which one knows *D* to have; let this be, I assume, at *F*. Thereafter one turns to the west, remaining constantly in the same latitude, *i.e.* constantly sailing on the eighth loxodrome until one arrives at the desired place *D*.

Note further that one might first begin to sail from *E* straight to the west, westwaert an, dats gheduerlick op de achtste cromstreeck, tot datmen hadde de langde van D, als tot G, en daer na van G recht noortwaert an, tot datmen op D comt: Doch alst niet en waer uyt oirsaeck van winden of stroomen, diemen van te vooren wist dat hinderlick souden sijn, so waert beter eerst te seylen van



E op F, en van F op D, dan eerst van E op G, en van G op D, uyt oirsaeck datmen met meerder sekerheyt van F tot D op een selve breede can blijven, dan van G tot D op een selve langde, inder voughen dat soo veel men deur afleydende stroomen of qua rekening, in langde misgiste, soo verre soudemen te oostlick of westlick van D commen: Ia sulcx, datmen somwijlen als D een cleen Eylant waer, niet weten en soude (ghelijckt metter daet dickwils ghebeurt (ofmen daer af oost of west waer, niet teghenstaende men de breede versekert hadde: Maer van E eerst noortwaert an varende, t'sy datmen wat oostlicker of westlicker dan op F ter begheerde breede comt, ten doet daer sulcken hinder niet, want blijvende int seylen nae D altijt op de behoirlicke breede, men moet D ontmoeten.

Tot dese wijse van seyling op een achtste cromstreeck met een middachront, en behouftmen gheen rekeninghen als de voorgaende van rechte en cromme seylstreken, maer ten is soo corten wech niet, gelijckmen inde form openbaerlick siet.

ANHANG

i.e. constantly on the eighth loxodrome, until one is in the longitude of *D*, *viz.* at *G*, and then from *G* straight to the north, until one arrives at *D*. But if it were not because of winds or currents which one knew beforehand would be troublesome, it would be better first to sail from *E* to *F* and then from *F* to *D*, rather than first from *E* to *G* and then from *G* to *D*, because one can remain with greater certainty in the same latitude from *F* to *D* than in the same longitude from *G* to *D*, so that as much as the conjecture concerning the longitude was wrong owing to diverting currents or miscalculation, so much one would arrive too far to the east or west of *D*, even to the extent that sometimes, if *D* were a small island, one would not know (as often happens in actual fact) whether one was to the east or west of it, in spite of the fact that one had made sure of the latitude. But if one first sails from *E* to the north, it does not matter so much if one arrives in the required latitude somewhat further east or west than *F*, for if in sailing to *D* one always remains in the proper latitude, one is bound to come to *D*.

For this method of sailing on an eighth loxodrome and a meridian one requires no calculations like the foregoing, of great-circle tracks and loxodromes, but it is not so short a route, as is plainly seen in the figure.

ANHANG DER CROMSTREKEN.

1 HOOFTSTICK.

Verhael op de oirden der cromſtreken.

SOmmige als *Robert Hues*, nemen de cromſtreken te beginnen vant middachront af na t'middelront toe, noemende die van noort ten ooſten d'eerſte, noort noortooſt de tweede, en ſoo voorts. Ander als *Edwart Wright*, tellen van t'middelront af na t'middachront, noemende de cromſtreeck van ooſt ten noorden d'eerſte, ooſt noordooſt de tweede, en ſoo voorts. Maer wanttet om twijffeling te weeren, goet waer dat alle die van deſe ſtof handelen, int noemen der cromſtreken een ſelve oirden volghden, ſoo heeft my oirboir ghedocht mijn ghevoelen te verclaren, waerom ick meyn datter meerder reden is te beginnen van het middachront, ghelijck wy hier vooren oock ghedaen hebben, dan van t'middelront, als volght:

Voor al wanneermen ſeght d'eerſte cromſtreeck die te weſen, welcke eerſt na t'middelront volght, ſoo en is dat niet eyghentlick geſproken den * Doender buyten t'middelront ſijnde, dadelick of deur t'gheſtelde, wantmen in ſulck anſien dan niet en ſoude moeten ſegghen d'eerſte cromſtreeck vant middelront af, dan d'eerſte cromſtreeck van een evewijdich rondt mettet middelront: Maer dat ſoo ghenomen, t'ſelve evewijdich ront en ſal in d'oirden der cromſtreken niet vallen, t'welck nochtans gheen rechte ſtreeck ſijnde een cromſtreeck is, volghende haer bepaling. Dit anghemerckt, de natuerlicke oirden ſchijnt te vereyſſchen datmen t'middachront neme voor begin, want van daer af tellende, ſoo valt de ooſtenweſtſtreeck me onder de cromſtreken, weſende altijt de achtſte na t'behooren, en daerom hebben wy hier vooren die wijſe ghevolght.

* *Efficiens.*

2 HOOFTSTICK.

Van *Petrus Nonius* feyl, angaende de ghetalen der cromſtreken.

Nadien de Portuguijſen en Spaengnaerden eerſt ernſtelick de groote zeevaerden anghevanghen hadden, ſoo viel by hun anmercking op de gedaente en eyghenſchappen der cromſtreken: Waer af den vermaerden Wiſconſtnaer *Petrus Nonius* handelende, heeft gheſchreven vande ghetalen dienende tottet formen der ſelve, maer ſy en wierden by hem niet recht ghenouch getroffen, t'welck ick niet en ſegh tot ſijn verachting, want den gront daer hy op boude, hadde uyterlick ſoo vaſten anſien, dat d'alder ervarenſte voor t'eerſte lichtelick ſouden ghemeent hebben de ſaeck ſoo te weſen, en by aldien hem ſulcke oirſaeck van proef ontmoet hadde, als anderen na hem wel bejeghende, hy ſoude ſoo wel als anderen t'ghebreck bemerckt hebben.

Tis dan te weten dat hy int 23 Hoofdſtick ſijns 2 boucx *de Reg. & inſtr.* beſluyt de houckmaten der boghen vanden aſpunt totte cromſtreeck in

*ghe-

APPENDIX

OF THE LOXODROMES.

1st CHAPTER.

Account about the order of the loxodromes.

Some people, like *Robert Hues* [1]), regard the loxodromes as starting from the meridian and proceeding to the equator, calling that of north by east the first, north northeast the second, and so on. Others, like *Edward Wright* [2]), count from the equator to the meridian, calling the loxodrome of east by north the first, east northeast the second, and so on. But since, to avoid doubt, it would be advisable for all those dealing with this subject to follow the same order in naming the loxodromes, it seemed expedient to me to set forth my view why I think there is more reason to start from the meridian, as we have done in the foregoing, than from the equator, as follows:

Especially if we say that the first loxodrome is the first, reckoning from the equator, this is not properly expressed, when the observer is outside the equator, in actual fact or by the supposition, because in this respect we ought not to say the first loxodrome from the equator, but the first loxodrome from a parallel to the equator. But even if this is done, this parallel will not fall under the order of the loxodromes, and yet, not being a great-circle track, it is a loxodrome according to the definition. Considering this, the natural order of things seems to demand that the meridian be taken as the starting-point, for, reckoning thence, the east and west track also falls under the loxodromes, being always the eighth, as it should be, and that is why we have followed this method in the foregoing.

2nd CHAPTER.

Of the error of *Petrus Nonius* [3]), concerning the numerical values of the loxodromes.

When the Portuguese and the Spaniards had seriously started on long sea-voyages, their attention was caught by the appearance and properties of the loxodromes. The famous mathematician *Petrus Nonius,* dealing with this, wrote about the numerical values serving to form them, but he did not fix these values accurately enough, which I do not say to disparage him, for the foundation on which he built seemed outwardly so secure that even the most experienced people would at first easily have thought that the matter was like this. And if he had had the same opportunity of testing it as others after him found, he would have perceived the error just as well as others.

Thus it is to be noted that in the 23rd Chapter of his 2nd book *de Reg. & instr.* [4]) he concludes that the sines of the arcs from the pole to the loxodrome

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[1]) *Cf. Introduction,* p. 488.
[2]) *Cf. Introduction,* p. 482, 483, 494.
[3]) *Cf. Introduction,* p. 491-492.
[4]) *Cf. Introduction,* p. 491.

Continua proportione.

*gheduerighe everedenheyt te wesen; Als inde voorgaende form des 3 voorstels, (alwaer wy nemen R Z de vierde te sijn) dat ghelijck houckmaet van M R, tot houckmaet van M X, also houckmaet van M X, tot houckmaet van M Y, en van M Y tot die van M *a*, en van M *a* tot die van M *b*; en van M *b*, tot die van M Z, en daerom oock ghelijck houckmaet van M R, tot houckmaet van M X, alsoo houckmaet van M *b*, tot houckmaet van M Z: Doch dat gemist te wesen blijckt aldus: De driehouck M R X heeft drie bekende palen, te weten de sijde M R 90 tr. den houck M R X 45 tr. en den houck R M X 1 tr. Hier me ghesocht de sijde M X, wort bevonden van 89 tr. diens houckmaet doet 9998: Sulcx dat de houckmaet van M R, in soodanighen reden is totte houckmaet van M X, als 10000 tot 9998. Laet nu de cromstreeck van R tot *b*, soo na den * aspunt ghecommen wesen, dat M *b* doe 10 tr. diens houckmaet is 1736. Dit soo ghenomen, de houckmaet van M Z soude moeten doen 1736, want seggende houckmaet van M R 10000, geeft houcmaet M X 9998, wat houckmaet van M *b* 1736? Comt houckmaet die sijn soude voor M Z alsvooren 1736: Maer de selve soo niet te wesen wort aldus bethoont: De driehouck M *b* Z heeft drie bekende palen, te weten de sijde M *b* 10 tr. den houck M *b* Z 45 tr. en den houck *b* M Z 1 tr. Hier me ghesocht de sijde M Z, wort bevonden deur het 42 voorstel der clootsche driehoucken van 8 tr. 33 ①, diens houckmaet 1487, groot verschil heeft vande voorschreven 1736, t'welck sy soude moeten doen om met d'ander everedenich te sijn. Sulcx dat de booch M Z, die volghende t'ghestelde maer en doet 8 tr. 33 ①, wort na d'ander reghel bevonden van 10 tr. t'welck 1 tr. 27 ① te veel is. Maer sulcken feyl vallende op alleenelick een everedenheyts wercking, soo machmen dencken wat grooter feyl datter op M Z boven dat noch moet commen, deur al de voorgaende versaemde feylen der werckinghen eermen totten boveschreven 10 tr. vanden aspunt gheraeckt.

Polum.

3 HOOFTSTICK.

Vant feyl inde tafels der cromstreken deur *Edvvart VVright*.

Na de Portuguysen en Spaengnaerden sijn in groote zeevaerden de Enghelschen ghevolght, welcke op dese ghedaente der cromstreken oock acht nemende, hebben t'feyl van *Nonius* bemerckt, en tot verbetering van dien soo sijnder onlancx uytghegheven tafelen der cromstreken deur *Edwart Wright*, als die des 4 voorstels van desen, welcke de saeck naerder commen: De proef daer ick sulcke meerder naerheyt deur vermoede, was dat ick na d'eerste wijse des maecksels der tafels vande cromstreken int boveschreven 4 voorstel, socht de breeden des vierden cromstreecx, (in welcke t'werck licht valt, deur gheduerighe vergaring sonder menichvulding of deeling, om dat raecklijn en rechthouckmaet van 45 tr. daer even vallen) en dat tot op den 78 tr. der langde, waer op ick bevant t'overcommen 61 tr. 26 ①: Maer *Wrights* tafels hebben tot sulcken plaets 61 tr. 14 ①, die op soo grooten langde maer 12 ① en schillen: Boven dien was my noch bekent dattet ware ghetal, volghende sulck ghestelde, minder moest sijn dan die 61 tr. 26 ①: Sulcx dat my dit, als gheseyt is, vermoen gaf van die tafels de sake na te commen: (Maer hoe de ghetalen der tafels van ander cromstreken met sulcke rekeningen overcommen, en heb ick deur ander belet niet versocht) Doch en isser de rechte gront noch niet volcommelick ghetroffen. Om hier af verclaring te doen, soo schrijf ick eerst t'navolghende

VER-form a continued proportion. Thus, in the foregoing figure of the 3rd proposition (where we assume *RZ* to be the fourth loxodrome), as the sine of *MR* is to the sine of *MX,* so is the sine of *MX* to the sine of *MY,* and that of *MY* to that of *MA,* and that of *MA* to that of *MB,* and that of *MB* to that of *MZ;* and consequently also: as the sine of *MR* is to the sine of *MX,* so is the sine of *MB* to the sine of *MZ*. Now that this is wrong, appears as follows: The triangle *MRX* has three known elements, *viz.* the side *MR* = 90°, the angle *MRX* = 45°, and the angle *RMX* = 1°. When herewith the side *MX* is sought, it is found to be 89°, the sine of which is 9,998, so that the sine of *MR* is to the sine of *MX* in the ratio of 10,000 to 9,998. Now let the loxodrome from *R* to *B* have come so near to the pole that *MB* is 10°, the sine of which is 1,736. Assuming this, the sine of *MZ* would have to be 1,736, for when we say: sine of *MR* = 10,000 gives sine of *MX* = 9,998, what does the sine of *MB* = 1,736 give? The sine, which would be that of *MZ* — as before — is 1,736. Now that this is not true, is proved as follows. The triangle *MBZ* has three known elements, *viz.* the side *MB* = 10°, the angle *MBZ* = 45°, and the angle *BMZ* = 1°. When herewith the side *MZ* is sought, it is found by the 42nd proposition of spherical trigonometry [1]) to be 8°33′, the sine of which = 1,487 differs a good deal from the aforesaid 1,736, which it would have to be in order to be proportional to the other. So that the arc *MZ,* which according to the supposition is only 8°33′, according to the other rule is found to be 10°, which is 1°27′ too much. Now if such an error occurs in only one operation of proportion, one may consider how much greater will be the error that must occur in *MZ* in addition, from all the previous errors accumulated of the operations carried out before one reaches the above 10° from the pole.

3rd CHAPTER.

Of the error in the tables of the loxodromes by *Edward Wright.*

The Portuguese and the Spaniards were succeeded in long sea-voyages by the English, who, also paying attention to this appearance of the loxodromes, noticed the error of *Nonius;* and to correct this error, tables of the loxodromes were recently published by *Edward Wright, viz.* those of the 4th proposition of the present work, which are more accurate. The test on the ground of which I expected this greater accuracy was that I sought the latitudes of the fourth loxodrome according to the first method of making the tables of the loxodromes in the above-mentioned 4th proposition (in which case the operations are easy, owing to constant addition, without multiplication or division, because the tangent of an angle of 45° is equal to the sine of a right angle [2]), up to 78° of longitude, with which I found 61°26′ to correspond. But *Wright*'s tables have for this place 61°14′, which differs only 12′ on so great a longitude. Moreover it was known to me that the correct value, according to this supposition, must be less than this 61°26′, so that — as I have said — this made me expect that those tables were rather accurate (But since I was prevented by other business, I have not examined how the values of the tables of other loxodromes agree with these

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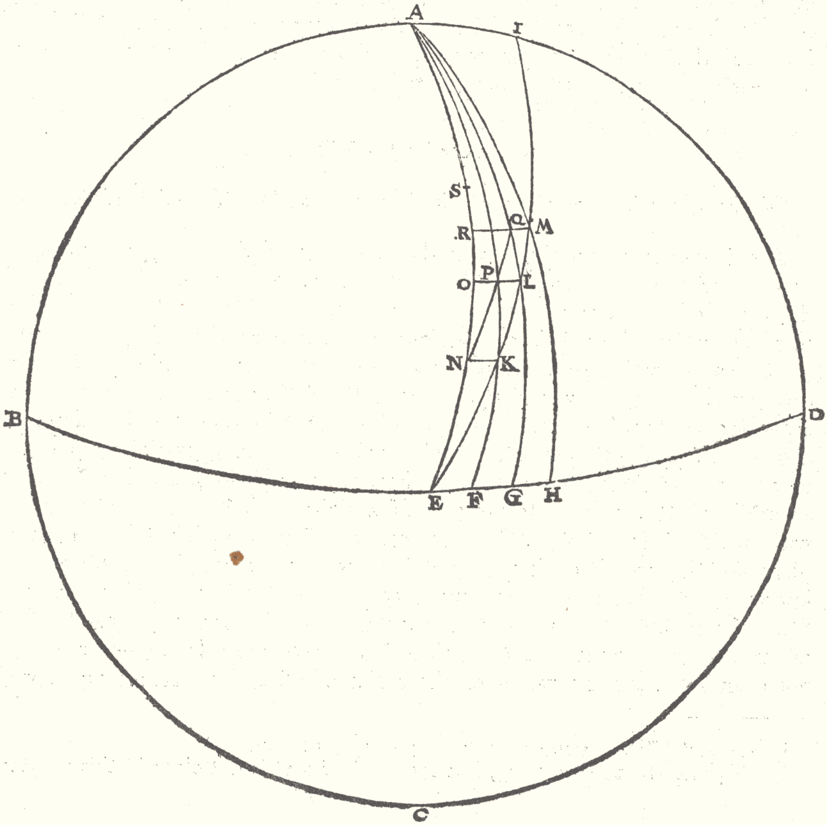
[1]) Stevins's *Trigonometry* (Work XI; i, 13), p. 302.
[2]) This must be the meaning, but the original text is not clear in this respect.

VERTOOCH.

Ghelijck des cromstreecx afvvijcking vant middelront op haer voortganck eens langdetraps, tot haer afvvijcking op haer voortganck van noch een langdetrap daer an volghende: Alsoo seer na snylijn door t'begin van die laetste voortganck, tot snylijn door t'begin des eersten voortgancx.

TGHEGHEVEN. Laet A B C D een Eertcloot sijn diens aspunt A, middelront B E D, waer in gheteyckent sijn de vier punten E,F,G,H, sulcx dat E F, F G, G H, elck doen 1 tr. Op dese vier punten E,F,G,H, sijn ghetrocken de vier middachboghen A E, A F, A G, A H; voort sy E I een cromstreeck, ick neem d'eerste, snyende de vier middachboghen in K,L,M,I; Daer na sy ghetrocken de booch K N evewijdich met F E, en L O met G E, snyende A F in P, sghelijcx sy ghetrocken de booch M Q R evewijdige met H E en snyende A G in Q: Daer



na sy

calculations). But the correct basis has not yet been perfectly laid. To explain this, I first give the following

THEOREM.

As the deviation of the loxodrome from the equator during its advance through one degree of longitude is to its deviation during its advance through the next degree of longitude, so very nearly is the secant through the beginning of the latter advance to the secant through the beginning of the former advance.

SUPPOSITION. Let *ABCD* be a globe, whose pole is *A*, the equator *BED*, in which are marked the four points *E*, *F*, *G*, and *H*, so that *EF*, *FG*, and *GH* are each 1°. On these four points *E*, *F*, *G*, and *H* are erected the four meridians *AE*, *AF*, *AG*, and *AH*. Further let *EI* be a loxodrome, I assume the first, intersecting the four meridians in *K*, *L*, *M*, and *I*. Thereafter let the arc *KN* be drawn parallel to *FE*, and *LO* parallel to *GE*, intersecting *AF* in *P*; similarly let the arc *MQR* be drawn parallel to *HE* and intersecting *AG* in *Q*. Thereafter let the first loxodrome be drawn from *N* *via* *P* to *Q*, which must be equal and similar to the part *KLM*.

This being so, *FK* is the deviation of the loxodrome from the equator *FED* during its advance through one degree of longitude; *i.e.* the loxodrome having advanced from *E* to *K*, where its change of longitude was one degree, *EF*, during this advance through one degree of longitude its deviation has become *FK*, or *EN*, as being equal thereto, because *KN* is parallel to *FE* by the supposition. And in the same way it is to be understood that *NO* is the deviation of the loxodrome during its advance through another degree of longitude, coming from *K* to *L*, where it has received as much deviation as from *N* to *O*, because *LO* is parallel to *GE* by the supposition. And for the same reason *OR* is the deviation of the loxodrome during its advance through yet another degree of longitude, na sy ghetrocken d'eerste cromstreeck van N deur P tot Q, die even en gelijck moet sijn mettet deel K L M.

Dit soo wesende F K is des cromstreecx afwijcking vant middelront F E D op haer voortganck eens langdetraps: Dat is, de cromstreeck voortghegaen hebbende van E tot K, daerse in langde verandering ghecreghen heeft een trap E F; soo is op die voortganck eens langdetraps, haer afwijcking gheworden F K, of E N als daer me even sijnde, om dat K N evewijdich is met F E deur t'gegeven: En alsoo salmen verstaen dat N O is des cromstreecx afwijcking op haer voortganck van een langdetrap meer, met te commen van K tot L, waer op sy afwijcking ghecreghen heeft soo veel als van N tot O, om dat L O evewijdich is met G E deur t'ghegheven: En om derghelijcke redenen is O R des cromstreecx afwijcking op haer voortganck van noch een langdetrap meer, met te commen van L tot M. Sulcx dat N O is des cromstreecx afwijcking op haer voortganck eens langdetraps, en O R afwijcking op haer voortganck van noch een trap daer an volghende. TBEGHEERDE. Wy moeten bewijsen dat ghelijck N O tot O R, alsoo seer na de snylijn van des cloots middelpunt deur L begin des laetsten voortgancx L M, tot snylijn deur K begin des eersten voortgancx K L: Maer de snylijn deur N, is even ande snylijn deur K, om dat K N evewijdighe is met F E: En sghelijcx is de snylijn deur O, even ande snylijn deur L, om dat L O evewijdighe is met G E, en daerom moeten wy bewijsen dat ghelijck N O tot O R, alsoo snylijn deur O, tot snylijn deur N.

Maer om hier te segghen tot wat eynde dit streckt, en de somme des voornemens int cort te verclaren, soo is te weten dat wy bethoonen sullen, de ghetalen inde boveschreven tafel ghevonden, niet te overcommen met dese evereden-heyt, en daerom niet heel recht te wesen. Voort hoemen deur sulcken gront ghewisse tafels can maken, hoe wel deur een moeylicker wercking.

TBEWYS.

Laet ons nemen de form in d'eerste bepaling des houckmaetmaecksels, waer in te ansien valt dat den driehouck A B I, ghelijck is metten driehouck A F C, diens * lijckstandighe sijden everedenich sijn, te weten

Homologa.

Ghelijck A I tot A B, alsoo A C tot A F,

Maer A C is even met A B, en G C met A F, daerom

Ghelijck A I tot A B, alsoo A B tot G C.

Maer A I is snylijn, A B rechthouckmaet, en G C houckmaet van C E, te weten des schilboochs van B C, daerom segh ick by ghemeene reghel, dat

Ghelijck eens boochs snylijn tot rechthouckmaet,
alsoo rechthouckmaet, tot schilboochs houckmaet.

Dit soo sijnde, wy commen nu totte form deses vertoochs, waer me ick om dese bewesen everedenheyt aldus segh:

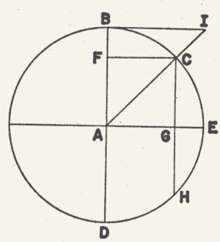
Ghelijck int vierendeelronts A E, snylijn deur N, dats snylijn des boochs N E, tot rechthouckmaet, also rechthoucmaet, tot schilboochs houckmaet van N E wesende houckmaet van A N: Dats andermael int cort gheseyt: Ghelijck snylijn deur N, tot rechthouckmaet, alsoo rechthouckmaet, tot houckmaet van A N. En weerom

Ghelijck snylijn deur O, tot rechthouckmaet,
alsoo rechthouckmaet, tot houckmaet van A O.

Maer

coming from *L* to *M*. So that *NO* is the deviation of the loxodrome during its advance through one degree of longitude, and *OR* the deviation during its advance through yet another subsequent degree. REQUIRED. We have to prove that as *NO* is to *OR*, so very nearly is the secant from the centre of the sphere through *L*, the beginning of the latter advance *LM*, to the secant through *K*, the beginning of the former advance *KL*. Now the secant through *N* is equal to the secant through *K*, because *KN* is parallel to *FE*. And similarly the secant through *O* is equal to the secant through *L*, because *LO* is parallel to *GE*, and consequently we have to prove that as *NO* is to *OR*, so is the secant through *O* to the secant through *N*.

Now in order to state here for what purpose this serves and to set forth briefly the sum of what we propose to say, it is to be noted that we are going to stress the fact that the numerical values found in the above-mentioned table do not agree with this proportion, and consequently are not quite exact. Furthermore, how it is possible to make accurate tables on this basis, though by more difficult operations.



PROOF.

Let us take the figure in the first definition of the work on the making of tables of sines [1]), where it can be seen that the triangle *ABI* is similar to the triangle *AFC*, whose homologous sides are proportional to each other, *viz.:*

As *AI* to *AB*, so *AC* to *AF;*

Now *AC* is equal to *AB*, and *GC* to *AF*, therefore

As *AI* to *AB*, so *AB* to *GC*.

Now *AI* is the secant, *AB* the semi-diameter, and *GC* the sine of *CE*, *viz.* of the complement of *BC;* I therefore say by the general rule that

As the secant of an arc is to the semi-diameter,
So is the semi-diameter to the sine of the complement.

This being so, we now come to the figure of the present theorem, about which on account of this proved proportion I say as follows:

As in the quarter circle *AE* the secant through *N*, *i.e.* the secant of the arc *NE*, is to the semi-diameter, so is the semi-diameter to the sine of the complement of *NE*, being the sine of *AN*. That is, stated briefly once again: As the secant through *N* is to the semi-diameter, so is the semi-diameter to the sine of *AN*. And again:

As the secant through *O* is to the semi-diameter, so is the semi-diameter to the sine of *AO*.

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[1]) Stevin's *Trigonometry* (Work XI; i, 11), Fol. 1.

Maer rechthouckmaet aldus inde twee voorgaende everedenheden middel-everedenighe sijnde, soo volght daer uyt dat

Ghelijck snylijn deur N, tot snylijn deur O,
alsoo houckmaet van A O, tot houckmaet van A N.

Maer ghelijck houckmaet van A O, tot houckmaet van A N, alsoo het rondt diens halfmiddellijn de houckmaet is van A O, dats t'rondt daer L P deel af is, tottet rondt diens halfmiddellijn de houckmaet is van A N, dats t'ront daer K N deel af is: Daerom

Ghelijck snylijn deur N, tot snylijn deur O, alsoo het rondt daer L P deel af is, tottet rondt daer K N deel af is.

Maer ghelijck het rondt daer L P deel af is, tottet rondt daer K N deel af is, alsoo het deel L P, tottet deel K N, want elck is 1 tr. sijns rondts deur t'ghegheven: Daerom

Ghelijck snylijn deur N, tot snylijn deur O, alsoo L P, tot K N.

Maer L P en K N sijn lijckstandighe sijden in twee driehoucken Q L P, P K N, die na ghenouch ghelijck sijn, (ick segh na ghenouch, uyt oirsaeck datse so na malcander staen, en so cleene sijden hebben, waer deur oock int vertooch gheseyt is *Alsoo seer na*) daerom sy sijn met d'ander lijckstandighe Q L, P K everedenich, dat is

Ghelijck L P tot K N, alsoo Q L tot P K: En vervolghens
Ghelijck snylijn deur N, tot snylijn deur O, alsoo Q L tot P K.

Maer R O is even met Q L, en O N met P K: Daerom

Ghelijck snylijn deur N, tot snylijn deur O, alsoo R O tot O N.

En deur verkeerde reden,

Ghelijck O N tot O R, alsoo snylijn deur O, tot snylijn deur N.

TBESLVYT. Ghelijck dan des cromstreecx afwijcking vant middelront, op haer voortganck eens langdetraps, tot haer afwijcking op haer voortganck van noch een langdetrap daer na volghende: Alsoo seer na snylijn door t'begin van die laetste voortganck, tot snylijn door t'begin des eersten voortgancx, t'welck wy bewijsen moesten.

Dese everedenheyt aldus vast en bekent sijnde, wy sullen tot ons voorghenomen verclaring van t'feyl der tafels commen.

Laet E I inde form deses vertoochs, beteyckenen d'eerste cromstreeck, daer in wy ons voorstellen te moeten vinden de twee breeden F K, G L. Om dan eerst te vinden F K, ick segh den driehouck K F E (die ick ten eersten voor plat ansie, om dat sulcke driehoucken inde tafels voor plat ghenomen wierden) te hebben drie bekende palen, te weten den houck K E F 78 tr. 45 ①, K F E recht, en F E 1 tr. Hier me ghesocht de sijde K F, en alles berekent met rechthouckmaet van 10000000, tot op ② toe, wort bevonden deur het 4 voorstel der platte driehoucken van 5 tr. 1 ① 38 ②. Maer E N is even an F K, daerom E N doet oock 5 tr. 1 ① 38 ②. Nu dan N E bekent sijnde, en om na de regel des voorgaenden vertoochs te vinden N O, ick seg aldus: Snylijn deur N, dats van N E 5 tr. 1 ① 38 ② doende 10038616, gheeft snylijn deur E doende 10000000, (die wy hier snylijn heeten, hoe wel het eyghentlick gheen en is, maer om de naem na de reghel te ghebruycken) wat de booch N E 5 tr. 1 ① 38 ②? Comt voor de booch N O 5 tr. 0 ① 28 ②: Die vergaert tot N E 5 tr. 1 ① 38 ②, comt voor E O, dats oock voor de begheerde G L 10 tr. 2 ① 6 ②: En soo veel wort oock volcommelick bevonden volghende d'eerste wijse des maecksels vande tafels der cromstreken int 4 voorstel, want N K als gront des rechthouckighen driehoucx P K N, doet 59 ① 46 ②, waer me de reghel gevolght, deur rekening der platte driehoucken, Now since thus the semi-diameter in the two foregoing proportions is the mean proportional, it follows from this that

As the secant through *N* is to the secant through *O*,
So is the sine of *AO* to the sine of *AN*.

Now as the sine of *AO* is to the sine of *AN*, so is the circle whose semi-diameter is the sine of *AO*, *i.e.* the circle of which *LP* is a part, to the circle whose semi-diameter is the sine of *AN*, *i.e.* the circle of which *KN* is a part. Therefore:

As the secant through *N* is to the secant through *O*, so is the circle of which *LP* is a part to the circle of which *KN* is a part.

Now as the circle of which *LP* is a part is to the circle of which *KN* is a part, so is the part *LP* to the part *KN*, for each is 1° of its circle by the supposition. Therefore:

As the secant through *N* is to the secant through *O*, so is *LP* to *KN*.

Now *LP* and *KN* are homologous sides in two triangles *QLP* and *PKN*, which are nearly similar (I say: nearly, because they are so close to each other and have such small sides, on which account the theorem also spoke of *so very nearly*), therefore they are proportional to the other homologous sides *QL* and *PK*, *i.e.*:

As *LP* is to *KN*, so is *QL* to *PK*. And consequently:

As the secant through *N* is to the secant through *O*, so is *QL* to *PK*.

Now *RO* is equal to *QL*, and *ON* to *PK*. Therefore:

As the secant through *N* is to the secant through *O*, so is *RO* to *ON*.

And by the inverse proportion:

As *ON* is to *OR*, so is the secant through *O* to the secant through *N*.

CONCLUSION. As therefore the deviation of the loxodrome from the equator during its advance through one degree of longitude is to its deviation during its advance through the next degree of longitude, so very nearly is the secant through the beginning of the latter advance to the secant through the beginning of the former advance; as we had to prove.

This proportion thus being established and known, we shall come to our proposed exposition of the error of the tables [1]).

Let *EI* in the figure of the present theorem designate the first loxodrome, on which we propose we have to find the two latitudes *FK* and *GL*. In order to find first *FK*, I say that the triangle *KFE* (which to begin with I regard as plane, because such triangles were taken as plane in the tables) has three known elements, *viz.* the angle *KEF* = 78°45′, *KFE* = 90°, and *FE* = 1°. When herewith the side *KF* is sought and everything is calculated with a semi-diameter = 10,000,000, to the nearest second of arc, it is found by the 4th proposition of plane trigonometry [2]) to be 5°1′38″. But *EN* is equal to *FK*, therefore *EN* is also 5°1′38″. Now *NE* thus being known, in order to find *NO* according to the rule of the foregoing theorem, I say as follows: Secant through *N*, *i.e.* *NE* = 5°1′38″, being 10,038,616, gives secant through *E*, being 10,000,000 (which we here call secant, although properly speaking it is no secant, but we do so in order to use the name that accords with the rule); what does the arc *NE* = 5°1′38″ give? The arc *NO* becomes 5°0′28″. When we add this to *NE* = 5°1′38″, *EO*, *i.e.* also the required *GL*, becomes 10°2′6″. And exactly the same value is also found according to the first method of the making of tables of the loxodromes in

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[1]) *Cf. Introduction*, p. 488-490.
[2]) Stevin's *Trigonometry* (Work XI; i, 12), p. 147.

men bevint K P, dats oock voor N O van 5 tr. 0 ① 28 ②, en vervolghens E O inde boveſchreven volcommenheyt. Maer niet alſoo deur de tweede wijſe, want alles met ② ghewrocht, daer comt voor G L 10 tr. 1 ①. Maer om de ſelve tweede wercking hier breeder te verclaren ick ſegh aldus: Gheſocht inde tafel der verſaemde ſnylijnen watter overcomt op 5 tr. 1 ① 38 ② van E N, wort bevonden 3020, daer toe vergaert noch eens 3020 comt 6040, waer op inde ſelve tafel overcommen voor G L of E O 10 tr. 1 ①, die 1 ① 6 ② verſchillen van d'ander 10 tr. 2 ① 6 ②, en daerom en is dit niet heel volcommen, want hier me voortgaende men gheraeckt allenx tot noch wat meerder verſchil.

Merckt noch wijder de reden te vereyſſchen, datmen int ſoucken van K F des driehoucx K F E, den ſelven driehouck niet en behoort te nemen voor plat, gemerckt de twee ſijden daermen rekening me maeckt als E F, F K boghen ſijn, ſulcx dat F K geſocht deur rekening der clootſche driehoucken, wort bevonden van 5 tr. 0 ① 51 ②, die van d'ander 5 tr. 1 ① 38 ② verſchillen 47 ②: Want hoewel die in haer ſelven voor t'eerſte cleen ſijn, int vervolgh wordet feyl grooter.

4 HOOFTSTICK.

Hoe t'maeckſel van ghevviſſe tafels der cromſtreken ſoude meughen gheſchien na t'ghevoelen des Schrijvers.

Ghelijck ghevonden is N O, deur een wercking getrocken uyt het vertooch int 3 Hoofſtick deſes Anhangs, alſoo ſal ghevonden worden O R; ſegghende ſnylijn van C E, gheeft ſnylijn van N E, wat de booch N O? t'gene daer uyt comt is voor de booch O R, welcke vergaert tot O E, men crijcht E R, dats oock H M breede des 3 tr. der langde. En om daer nae te vinden de breede van noch een langdetrap voorder, die ick neem te weſen R S, ick ſegh: Snylijn van R E, gheeft ſnylijn van O E, wat de booch O R? t'ghene daer uyt comt is voor de booch R S, en ſoo voort met d'ander.

MERCKT nu noch dat deur t'nemen van cleender boghen der langde, ſekerder werck valt dan deur grooter, om bekende oirſaken: Maer om verſekert te ſijn datmen de boghen cleen ghenouch neemt, dat ſal gheſchien deur een dobbel wercking, aldus toegaende: Benevens t'vinden der breeden deur t'nemen der langdeboghen van trap tot trap als hier vooren, ſoo ſalmen noch doen een ander wercking van halve trap tot halve trap, ſoo langhe alſſe gheen hinderlick verſchil en hebben: Maer beginnende hinderlick verſchil te crijghen, men ſal d'eerſte wercking van trap tot trap verlaten, blijvende by die van halve trap tot halve trap, en daerbenevens noch vougende een wercking van vierendeel traps tot vierendeel traps, dats van 15 ① tot 15 ①, welcke voorder commende, ſooſe oock hinderlick verſchil creghen, men ſal die van halve trap tot halve trap verlaten, en voughen by d'ander een wercking van noch cleender boghen als van 10 ① tot 10 ①. En daer me canmen, hoe wel de wercking moeylick valt, tot ſekerheyt commen, want gheen hinderlick verſchil vallende tuſſchen twee werckinghen, d'eene deur t'nemen van heele langdetrappen, d'ander van halve, ten is niet noodich ſoo verre met noch cleender boghen te wercken dan met halve, want t'ghene men daer deur naerder vonde, ſoude door meerder moeyte gheſchien ſonder tot voordeel te ſtrecken.

Merckt noch dat ſoomen deſe wercking wilde doen na d'eerſte wijſe des maeckſels vande tafels der cromſtreken int 4 voorſtel, t'ſoude oirboir ſijn eerſt te maken een tafel van t'ghene 1 tr. langdeſchil buyten t'middelront, doet

in

the 4th proposition, for *NK*, being the base of the right-angled triangle *PKN*, is 59′46″, and when the rule is followed, by a calculation of plane trigonometry, *KP*, *i.e.* also *NO*, is found to be 5°0′28″, and consequently *EO* is found as exactly as above. But it is not like this by the second method, for when everything is calculated to the nearest second, *GL* becomes 10°1′. Now in order to set forth this second operation more fully, I say as follows: When one looks up in the table of the assembled secants what value corresponds to the 5°1′38″ of *EN*, this is found to be 3,020. When one adds thereto another 3,020, this becomes 6,040, to which there corresponds in the said table for *GL* or *EO* 10°1′, which differs 1′6″ from the other value of 10°2′6″, and consequently this is not quite exact, for if one continues in this way, the difference gradually increases.

Note further that reason demands that when seeking *KF* of the triangle *KFE*, one ought not to regard this triangle as plane, since the two sides by means of which the calculation is made, *viz.* *EF* and *FK*, are arcs, so that *FK*, when sought by a calculation of spherical trigonometry, is found to be 5°0′51″, which differs 47″ from the other, 5°1′38″. For although at first it is small in itself, the error will become greater in the sequel.

4th CHAPTER.

How the making of accurate tables of the loxodromes might be effected in the opinion of the author.

As *NO* has been found, by an operation derived from the theorem in the 3rd Chapter of this Appendix, so *OR* must be found, saying: secant of *CE* gives secant of *NE*, what does the arc *NO* give? The result of this is the value of the arc *OR*, and when we add this to *OE*, we get *ER*, *i.e.* also *HM*, the latitude at 3° of longitude. And in order to find then the latitude at yet one degree of longitude further, which I assume to be *RS*, I say: Secant of *RE* gives secant of *OE*, what does the arc *OR* give? The result of this is the value of the arc *RS*, and so on with the others.

NOTE further that when taking smaller arcs of longitude, the operations are more accurate than when taking larger ones, for known reasons. Now to ensure that the arcs are taken small enough, a double operation has to be performed, as follows. Besides the finding of the latitudes by taking the arcs of longitude from degree to degree, as hereinbefore, another operation must be performed from one half degree to the next half degree, as long as they have no appreciable difference. Now when an appreciable difference begins to appear, one must stop the first operation from degree to degree, keeping to that from one half degree to the next, and adding thereto an operation from one fourth degree to the next one fourth degree, *i.e.* from 15′ to 15′, and when this has gone on for some time, if again an appreciable difference arises, one must stop the operation from one half degree to the next, and add to the other an operation with even smaller arcs, *viz.* from 10′ to 10′. And thus, though the operation is difficult, one can attain to accuracy, for if there is no appreciable difference between two operations, the one by taking whole degrees of longitude and the other by taking half degrees, it is not necessary to operate further with arcs even smaller than half arcs, for what one might thus gain in greater exactness would be obtained by greater trouble without yielding any advantage.

Note further that if one wanted to perform this operation according to the in ①,② en ③ des middelronts, als de tafel des achtsten cromstreecx, en dat niet van 30 tot 30 ① der breede, noch berekent tot op ② als die, maer van ① tot ① der breede, en berekent tot op ③.

De moeyte des maeckfels van sulcken tafel ghedaen wesende, tis kennelick dat de voorder wercking dan gheschien soude alleen deur menichvulding sonder deeling, daermen na de voorgaende ander wijse met snylijnen, benevens de menichvulding een deeling moet doen, doch en isser dan sulcken tafel niet te maken.

Dit is de sekerste wech die my nu te vooren comt, en hoe wel de wercking moeylick soude vallen, doch eens wel ghedaen wesende, men soude hem daer op meughen betrouwen. Maer sooder middeler tijt deur ymant ander lichter wercking met ghenouchsaem bewesen sekerheyt ghevonden wierde, t'waer billich die te aenvaerden.

5 HOOFTSTICK.

Hoemen scherper opt Zeecompas soude connen seylen dan na t'ghemeen ghebruyck.

Het zeecompas placht eertijts ghedeelt te sijn in 8 streken, t'welck daer nae doen de wijder zeevaerden nauwer toesicht vereyschten, ghecommen is tot 32. En hoewel eenige die halven, voorder commende tot 64 streken, doch houden ander dese laetste deeling voor onnoodich, achtende s'menschen gesicht in een varende schip op soo cleene ghedeelten des ronts gheen seker oirdeel te connen hebben. Maer sijn VORSTELICKE GHENADE desen handel der scherpseyling grondelick overdenckende, heeft daer af middel voorgewent, om smenschen oirdeel seker te connen sijn niet alleen op 64 streken, maer op streken van trap tot trap der 360 daermen de ronden in deelt, ja tot op ghedeelten eens traps, sulcx dat verscheyden menschen op een varende schip t'samen in het zeecompas siende, sullen al ghelijckelick uyt eenen mont noemen een selve trap, ja helft of vierendeel van dien, na dat den tuych groot en sorchvuldelick mocht ghemaeckt sijn. Ende want dit stof is mette voorgaende ghemeenschap hebbende, soo vervough ick t'selve in desen Anhang, daer af een corte verclaring doende met twee voorbeelden,

1 Voorbeelt deur een zeecompas mette lely op stijf papier.

Laet A B C D E F G een zeecompas beduyen, waer in een rondt stijf papier sy, niet met 32 streken na de ghemeene wijse, maer met 360 tr. te weten elck vierendeelronts in 90 tr. gheteyckent opt uyterste des cants van t'selve papier, en beginnende de telling in yder vierendeel vant middachrondt af: En daer onder noch een ander rondt stijf papier, waer in het yser gestreken met zeylsteen vast light, om alsoo na t'ghemeen ghebruyck de afwijcking des ysers vande lely soo groot te stellen, als de seylnaeldens afwijcking vant noorden op de tegenwoordighe plaets vereyscht.

Opden buytecant der casse van sulcken compas, sy ghetrocken een lini recht over eynde als C E, daer na het liniken C H, van C na t'middelpunt des compas streckende, en noch een ander van H recht neerwaert evewijdich commende met C E: Voort sy het punt A soo, dat de verdochte lini van C tot A, strecke first method for the making of the tables of the loxodromes in the 4th proposition, it would be expedient first to make a table of what 1° of difference of longitude outside the equator is equivalent to in minutes, seconds, and sixtieths of seconds of the equator, like the table of the eighth loxodrome, not from 30′ to 30′ of latitude, nor calculated to the nearest second, like the latter, but from minute to minute of latitude and calculated to the nearest sixtieth of a second.

The trouble having been taken to make this table, it is evident that the further operations would then have to be performed only by multiplication, without division, whereas according to the foregoing other method by means of secants it is necessary besides the multiplication to perform a division, but in that case no such table has to be made.

This is the most accurate method that I can think of now, and even though the operations should be difficult, once they had been performed satisfactorily, one might rely thereon. But if at any time an easier operation of sufficiently tested accuracy were found by someone else, it would be reasonable to adopt it.

5th CHAPTER.

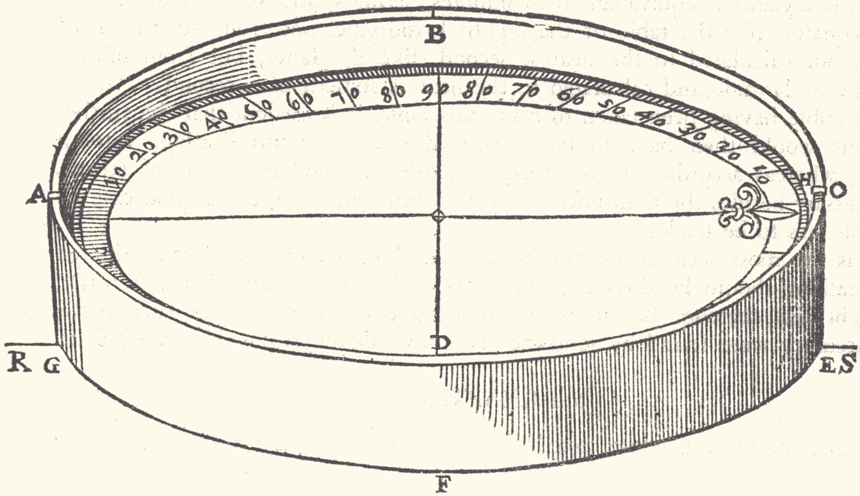
How one might steer more exactly by the mariner's compass than is usually done.

The mariner's compass was formerly divided into 8 points, which number was afterwards, when the longer voyages called for greater accuracy, increased to 32. And although some people halve these, thus arriving at 64 points, others consider this latter division unnecessary, being of the opinion that man's eyesight cannot judge with accuracy in a moving ship about such small parts of a circle. But His Princely Grace, thoroughly reflecting on this subject of accurate steering, put forward a means to ensure the accuracy of a man's judgment not only with 64 points, but with points from degree to degree of the 360 into which a circle is divided, and even to parts of a degree, so that several people in a moving ship, when looking together at the compass, will all with one accord and unanimously name the same degree, or even one half or one fourth thereof, according as the instrument has been constructed large and accurate. And because this is a subject that is connected with the foregoing, I include it in this Appendix, giving a short explanation thereof with two examples.

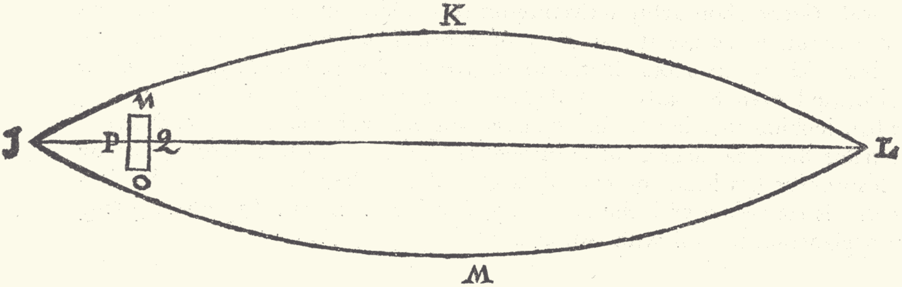
1st Example, by means of a Mariner's Compass with the Fleur-de-lys on Cardboard.

Let ABCDEFG denote a mariner's compass, containing a circular piece of cardboard, not with 32 points, as usual, but with 360 degrees, *viz.* each quarter of the circle marked in 90 degrees on the circumference of the said piece of cardboard, the counting commencing in each quarter from the meridian. And underneath it there is yet another circular piece of cardboard, on which the needle touched with loadstone is fixed, in order to adjust by the general custom the deviation of the needle from the fleur-de-lys to the value required by the deviation of the magnetic needle from the north at the place in question.

On the outside of the box of this compass let there be drawn a vertical line, *viz. CE,* then the small line *CH* extending from *C* to the centre of the compass, and yet another from *H* vertically downward, coming parallel to *CE*. Further let



recht over t'middelpunt vant compas: Daer na sy van A noch een lini recht neertwaert ghetrocken opden buytecant tot G, als A G, sulcx dat de verdochte lini van E tot G, strecken moet recht onder t'middelpunt vant compas. Dit soo sijnde, Laet nu I K L M de grontteyckening van een schip bedien, waer in N O is de plaets daermen al seylende t'compas op stelt: Op t'plat van t'selve sal ghe-



trocken worden een lini P Q. evewijdich streckende mettet kiel des schips, (die ick daerom kiellini noem) te weten in een lini of gespannen draet als I L, commende uyt het middel vant achterste des schips, totter middelste vant voorste. Maer want P Q in dese teyckening seer cort valt, soo sal ick om int volghende bequamelicker verclaring te meughen doen, andermael trecken een langher lini R S, die ick nu houde als voor kiellini in een schip ghetrocken te sijn, na de meyning alsvooren. Op dese kiellini R S salmen het compas stellen, soo dat de twee punten E en G effen commen daer op te passen.

Om nu hier deur met sekerheyt te seylen tot op trappen of cleender gedeelte, als by voorbeelt te moeten gheseylt worden opden 17 tr. van westen na zuyden, men sal t'schip soo stieren, dat den selven 17 tr. altijt passe op de lini die geseyt is te strecken van H neerwaert evewijdeghe met C E, want soo lang die overcommen, machmen seggen t'voorghenomen oirdeel van nauwer sekerheyt gevonden te wesen, overmits dat so haest dien selven 17 tr. een trap of een halve wijct vande voorschreven lini onder H, al de ghene diet sien, sullen, al of sy uyt eenen mont spraken, terstont t'samen connen seggen hoe veel het buyten den wech is.

2 *Voor-*

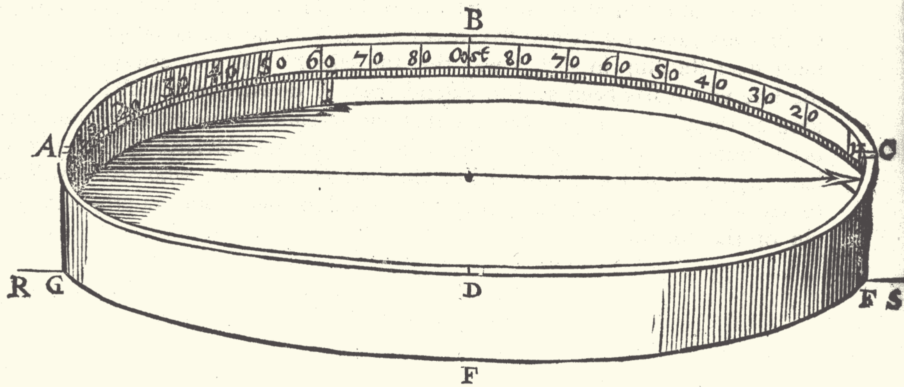
the point *A* be such that the imaginary line from *C* to *A* extends straight through the centre of the compass. Thereafter let there be drawn from *A* a line vertically downward on the outside to *G*, *viz.* *AG*, so that the imaginary line from *E* to *G* must pass straight below the centre of the compass. This being so, let *IKLM* denote the plan of a ship, in which *NO* is the place where during the voyage the compass is mounted. In the plane of this let a line *PQ* be drawn parallel to the keel of the ship (which I therefore call "keel-line" [1]), *viz.* in a line or stretched thread, *viz.* *IL*, extending from the middle of the rear part of the ship to the middle of the front part. But because *PG* in this drawing is very short, in order to make possible a more convenient explanation in the following, I will draw once more a longer line *RS*, which I now consider as having been drawn for the keel-line in a ship, as meant before. One must place the compass in this keel-line *RS*, so that the two points *E* and *G* coincide exactly therewith.

Now in order to steer an exact course by this means to the nearest degree or smaller divisions, *e.g.* if one has to sail on the 17th degree from west to south, one must steer the ship in such a way that the said 17th degree always coincides with the line which has been said to extend from *H* downward parallel to *CE*, for as long as these coincide, we can say that the intended more accurate judgment has been found, since, as soon as the said 17th degree deviates one degree or one half degree from the aforesaid line through *H*, all those who see it will be able to say with one accord, as if they were speaking with one mouth, how far it is out of the course.

[1]) Fore-and-aft line.

2 Voorbeelt deur een zeecompas met een seylnaelde.

Maer want de punt van een ſeylnaelde alleen draeyende ſonder ſtijf papier, ſcherper en ſekerder verthoont de naeldens natuerlicke wijſing, dan twee cromme beſtreken yſerkens diemen ghemeenelick ruytwijs teghen t'papier plackt, ſoo can ſulcx deur ſoodanighen bloote ſeylnaelde met noch meerder ſekerheyt gedaen worden, aldus: Deſe A B C D E F G H R S van beteyckening ſijnde als int eerſte voorbeelt, men ſal inden binnecant der caſſe daer de lini van H neerwaert incomt, teyckenen de 360 tr. te weten viermael 90 tr. beginnende t'een vierendeel van H na B, t'ander van H na D, ſgelijcx van A na B, en van A na D.



Voort ſalmen an t'punt H ſchrijven noort, an A zuyt: Maer an B, t'welck na de ghemeene wijſe weſt ſoude ſijn, ſalmen ooſt ſtellen, en an D weſt. Daer na ſoo veel de ſeylnaeldens afwijcking ter teghenwoordighe plaets is, op ſulcken verheyt ſalmen t'punt E vande kiellini R S ſtellen. Als by voorbeelt weſende de afwijcking 10 tr. van noort na ooſt, men ſal den 10 trap van C na B doen overcommen op de kiellini R S, want alſdan moet t'punt E oock 10 tr. vande kiellini verſchillen. Dit ſoo ſijnde, en datmen by voorbeelt gheſeyt wilde varen 17 tr. van weſt na zuyt, men ſal t'ſchip ſoo ſtieren, dat de punt vande ſeylnaelde paſſe opden 17 trap vant weſten af na zuyt dat inde caſſe ſtaet, en is openbaer datmen dan t'begheerde ſal doen, met meerder ſekerheyt dan na d'eerſte wijſe.

Maer ſoomen vreeſde dat deſe verkeerde ſtelling van ooſt en weſt, om d'onghewoonte wille dwaling mocht veroirſaken, voor de bootſgheſellen die te roer ſtaen: De Stierman ſoude in plaets vande namen der vier winden, meugen ghebruycken de vier letters A, B, C, D, en in plaets van hun te bevelen datſe ſeylen ſouden 17 tr. van weſt na zuyt, mach bevelen te ſeylen 17 tr. van D na A.

Voort om met meerder bequaemheyt het noort der caſſe of t'punt E, ſo verre vande kiellini te ſtellen, als de naeldens afwijcking vereyſcht, men ſoude van de kiellini af over beyde ſijden 40 of 50 tr. meughen teyckenen, om de caſſens punt E te ſtellen op ſulcken trap als de afwijcking mebrengt, makende int middelpunt van ſulcken ront een pinneken, om de caſſe met haer bodems middelpunt (een putken daer in gheboort ſijnde) op te draeyen.

DER SEYLSTREKEN

EYNDE.

2nd Example, by means of a Mariner's Compass with one Magnetic Needle.

Now because the point of a magnetic needle turning by itself, without a piece of cardboard, shows the natural pointing of the needle more sharply and accurately than two bent magnetized irons such as are usually stuck in the form of a rhomb against the paper, this can be done with even greater accuracy by means of a simple magnetic needle, as follows: The figure *ABCDEFGHRS* having the same letters as in the first example, in the inside of the box, where the line descends from *H* downward, the 360° must be marked, *viz.* four times 90°, starting one quarter from *H* to *B,* the other from *H* to *D,* similarly from *A* to *B* and from *A* to *D.* Further one must write north at the point *H,* south at *A.* But at *B,* which ordinarily would be west, one must place east, and at *D* west. Thereafter, as much as is the deviation of the magnetic needle in the place where the ship is at the moment, at such a distance [1]) from the keel-line *RS* one must place the point *E.* Thus, for instance, if the deviation is 10° from north to east, one must cause the 10th degree from *C* to *B* to coincide with the keel-line *RS,* for then the point *E* must also deviate 10° from the keel-line. This being so, and if one wanted, for instance, to sail 17° from west to south, one must steer the ship in such a way that the point of the magnetic needle coincides with the 17th degree from west to south marked in the box; it is obvious that one will then perform the required operation with greater accuracy than by the first method.

But if one should be afraid that this reversed placing of east and west might cause errors, on account of its unusual nature, for the sailors who are at the helm, the navigator might use instead of the names of the four points of the compass the four letters *A, B, C,* and *D,* and instead of ordering them to steer 17° from west to south may order them to steer 17° from *D* to *A.*

Furthermore, in order that one may place the north of the box or the point *E* more conveniently as far from the keel-line as the deviation of the needle requires, one might mark 40° or 50° from the keel-line to both sides, in order to place the point *E* of the box on such a degree as the deviation calls for, fitting in the centre of the circle a small pin for the box to pivot on at the centre of its bottom (in which a small pit has been drilled).

END OF THE SAILINGS.

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[1]) Stevin means a rotation through so many degrees.

DE WYSENTYT

THE AGE OF THE SAGES

From the *Wisconstighe Ghedachtenissen* (Work XI, i, 21)

This chapter forms the introduction to Stevin's Physical Geography. He gives an exhaustive exposition of his views about the "Age of the Sages", which was already discussed by us in the introduction to Vol. I (p. 44; *cf.* also the *Discourse,* pp. 59 ff.). In connection with the promise there made we now reproduce two fragments of this chapter, while a summary is given of the rest. This treatise gives a curious insight into Stevin's personality and into his opinions about language, the Renaissance, and the study of science generally.

6 BEPALING.

WYSENTYT noemen vvy die, vvaer in by de menschen een seltsaem vvetenschap ghevveest heeft, t'vvelck vvy deur seker teyckens ghevvisselick mercken, doch sonder te vveten by vvie, vvaer, of vvanneer.

Anghesien het int volghende dickwils te pas sal commen, t'onghewoonlick woort *Wysentyt* te noemen, en dat mijn voornemen is t'sijnder plaets te weten inde navolgende vernieuwing des *Wysentyts*, verclaring te doen, hoemen mijns bedunckens de saeck an soude meughen legghen, om weerom te gheraken tot alsulcken groote wetenschappen soo in Hemelloop als ander stoffen, ghelijck wy mercken by de menschen eertijts gheweest te hebben, so heeft my noodich ghedocht den selven tijt te bepalen alsvooren.

Maer om hier af breeder verclaring te doen ick segh aldus: Tis int ghebruyck datmen den tijt van over ontrent neghen of thien hondert jaren, tot over ontrent 150 jaren, noemt *Barbarum sæculum*, soo veel te segghen als Leecketijt, om dat de menschen seven of acht hondert jaren lanck waren als leecke, sonder oeffening der letters of vrye consten: T'welck sijn oirspronck nam doen de Christenen d'overhant creghen boven de Heydenen: Van welcke sy te vooren veel gheleden hebbende, en daer benevens de Heydensche Religie seer hatende, verbranden en vernielden niet alleenelick alle boucken der Religie, mette ghene daer eenich vermaen van hare Goden in stont, maer oock der vrye consten d'een metten anderen, waer syse crijghen conden. Ten laetsten heeft dit een eynde ghenomen, sulcx datmen heel verkeert de verborghen overbleven Heydensche boucken, weerom in allen houcken ghesocht heeft, int licht ghebrocht, en met groote neersticheyt en cost doen drucken, niet alleen van vrye consten, maer oock hun Goden aengaende, sulcx dattet nu yder Christen vry staet die in sijn ghedichten te aenroepen; Ia ghedichten der Christelicke Religie te vermenghen met veersen vande rammeling der Heydensche Goden, en die daer in seer ervaren sijn, worden daerom oock seer gheleert ghenoemt. Nu alsoo benevens eenighe ydelheden, oock ernstighe dinghen voort quamen, en dat de letters en vrye consten weerom op de beenen gherochten, men heeft die voorschreven tijt van seven oft acht hondert jaren, tot onderscheyt des tijts diet nu is, en ontrent duysent jaer daer te vooren was, ghenoemt als gheseyt is Leecketijt, by welcke d'ander verleken, *Wysetyt* soude meughen heeten: Doch tot sulcke *Wysetyt* en streckt ons meyning in de boveschreven bepaling niet, want die met d'ander seven of acht hondert jaren, al t'samen niet dan leecke tijt en sijn, verleken by den onbekenden tijt die wy deur teyckens sekerlick mercken gheweest te hebben. Maer om vande selve teyckens breeder verclaring te doen ick segh:

TEN EERSTEN datter byde menschen een groote ervaring en kennis des Hemelloops gheweest heeft, welcke ten tijde van *Hypparchus* en *Ptolemeus* bycans te niet en vergaen was, sulcx dat al t'ghene sy daer af beschreven hebben, maer voor overblijffels te houden en sijn van t'ghene datter gheweest hadde, want den eersten grondt waer deur het inhoudt dier overblijffels ghevonden wiert, te weten de ervarings dachtafels, sijn verloren, en men heeft sedert gheen ander ghemaeckt.

Angaende het ongheregelt roersel der Dwaelders dat *Ptolemeus* de * tweede oneven- *Secundam in æqualitatem.*

6th DEFINITION.

AGE OF THE SAGES we call that time in which exceptional learning was to be found among men, a fact which we perceive with certainty from certain signs, but without knowing among whom, where or when.

Since in the sequel the unusual term *Age of the Sages* will often be mentioned, while it is my intention to set forth in its place, to wit in the subsequent treatise on the restoration of the *Age of the Sages,* how in my view we might set out to attain once more to such great learning, both in astronomy and in other subjects, as we perceive was formerly found among men, it appeared necessary to me to define this time as above.

Now to set this forth more fully, I say as follows. It is customary to call the time from about nine hundred or one thousand years ago to about 150 years ago *Barbarum saeculum,* that is to say *Age of the Ignorant,* because for seven or eight hundred years men were ignorant, not practising letters or the liberal arts; which [condition] originated in the days when the Christians prevailed over the pagans. Having previously suffered a great deal from the latter and moreover deeply hating the pagan religion, they burned and destroyed not only all religious books, along with those which contained any information whatever about their gods, but also those on the liberal arts, one along with the other, wherever they could lay hands on them. Finally this came to an end, so that on the contrary the hidden pagan books that were left were again sought for in every corner, brought to light, and printed with great zeal and at great expense, not only those on the liberal arts, but also those concerning their gods, so that now every Christian is free to invoke them in his poems, nay, to introduce in Christian religious poetry verses concerned with the whole lot of the pagan gods, and those who are greatly proficient in it are called very learned on this account. Now since, besides some trivialities, also serious things emerged, and letters and the liberal arts were thus restored again, the aforesaid period of seven or eight hundred years was named — to distinguish it from the present time and that of about one thousand years ago — as has been said: Age of the Ignorant, in comparison with which the other period might be called *Age of the Sages*. But it is not this *Age of the Sages* which we mean in the above definition, for this, along with the other seven or eight hundred years, is nothing but an age of the ignorant in comparison with the unknown period which, judging from certain signs, we perceive with certainty to have existed. Now to set forth these signs in greater detail, I say as follows:

Firstly, that there existed among men great experience and knowledge of astronomy, which at the time of *Hipparchus* and *Ptolemy* had almost disappeared, so that all they have written about it is only to be regarded as remnants of what had existed, for the first foundation on which the content of those remnants was based, to wit the empirical ephemerides, have been lost, and no other such tables have since been made.

As to the irregular motion of the planets, which *Ptolemy* calls the second inequality, about which he says in the 4th chapter of his 5th book, and even more clearly in the 2nd chapter of his 9th book, that he thinks he was the first to have observed it and that it had not been noted by his predecessors, frequently accusing them of carelessness in observing the positions and motions of the heavenly bodies — to this it is replied that his predecessors must have seen the said second onevenheyt noemt, t'welck hy int 4 Hoofstick sijns 5 boucx, en noch meer int 2 Hoofstick sijns 9 boucx, meent selfs eerst gagheslaghen te hebben, en by sijn voorganghers niet bemerckt gheweest te sijn, beschuldighende dickmael hun onnachtsaemheyt int gaslaen vande plaetsen en loopen der Hemelsche lichten: Daer wort op gheantwoort, dat de voorgangers de selve tweede onevenheden ghesien hebben, eer hun meughelick was d'eerste onevenheyt, of Dwaelders middelloop soo aerdich te beschrijven ghelijckse *Ptolemeus* daer nae gecreghen heeft, en datse om tot soo grooten vondt te gheraken niet onachtsaem en hadden gheweest, maer neerstigher danmen van *Hypparchus* tijt af, en voor hem hoe langhe en weet ick niet, tot nu toe gheweest heeft, of meughelick is gheweest te sijne, om datmen na den Wijsentijt, de saeck op sulcken voet niet aenghetast en heeft ghelijckmen doen dede, soo wy terstont t'sijnder plaets daer af breeder verclaring sullen doen.

Het voorgaende wort bevesticht deur dienmē nae *Ptolemeus* tijt in ettelicke Arabische schriften vernomen heeft, dat voor hem by verscheyden * gheslachten van volck int ghebruyck is gheweest, de vaste sterren op de Hemelclooten in ander formen te teyckenen dan de ghemeene Egypsche, van welcke *Ptolemeus* noch *Hypparchus* nerghens eenich ghewach en doen: Soodanighe heeft my ghetoont den Edelen Hoochgheleerden Heer *Iosephus Scaliger*, in boucken met verclaring der teyckens ghedaen in Arabische spraeck, en dat niet op een wijse, maer wel tot drie clooten toe, elck verscheyden vanden anderen. De formen van een dier clooten wierden gheseyt Hemelteyckens der Indianen, van welcke namen, doch sonder schilderie, my oock ghedenckt ghelesen te hebben in een Latijns bouck van seer ouden druck, maer des schrijvers naem is my vergeten, ick en weet oock niet waer t'bouck bleven is. Een deel ander teyckens heb ick geschildert gesien tegen de mueren van een camer op s'Conincx hof in Polen tot Craco, wesende monsters, diens leden gemengt waren uyt verscheyden afcomsten van ghedierten, en stont daer by geschreven SIGNA HERMETIS, dats teyckens van Hermes. Niet een van al de boveschreven Hemelteyckens en vinde ick als gheseyt is by *Ptolemeus* vermaent te sijn: Waer uyt schijnt te meughen besloten worden, die t'sijnder handt niet ghecommen te wesen, veel min de Hemelloopsche leeringhen die elck * gheslacht na sijn wijse daer op ghegront hadde. Voort hebben de Hemelmeters eertijts wel gheweten dat den Eertcloot om de Son draeyde, sonder dat de eyghentlicke voorstellen van dien *Ptolemeus*, soot schijnt, ter hant ghecommen sijn, want had hyse ghesien, tis daer voor te houden dat hy (soomen oirdeelen mach uyt sijn verstandt en redelickheyt inde rest) soude toeghelaten hebben het natuerlick roersel byde ervaren Hemelmeters beschreven, en dat der eerste leerlinghen verlaten, of alleenlick leerings halven by * stelling gebruyckt. Van dit roersel des Eertcloots hebben vernomen *Philolaus Pytagoricus* als *Plutarthus* seght, voort *Aristarchus Samius* so *Archimedes* int bouck des * santtals betuycht; Maer dat sulcx inden wijsentijt niet en ghebeurde, schijnt ghenouch uyt *Archimedes* woorden te meughen besloten worden, inhoudende dat *Aristarchus* schreef teghen de Hemelmeters die de weerelt seyden een cloot te sijn, wiens middelpunt des Eertcloots middelpunt is, en halfmiddellijn even ande lini tusschen t'middelpunt der Son, en t'middelpunt des Eertcloots, welcke lini (benevens ander slechticheden) d'een tijt langher wesende als d'ander, soo en schijnter niet veel bescheyts in. Voort soomen insiet de form der * everedenheydt by *Aristarchus* aldus ghestelt:

Diversis gentes.

Natio.

Propositionem.

Arenæ numero.

Proportionis.

inequalities before they were able to describe the first inequality, or the planets' mean motion, as nicely as *Ptolemy* afterwards succeeded in doing, and that, to make such a great discovery, they cannot have been careless, but more diligent than people were from the time of *Hipparchus* and I do not know how long before him up to the present, or may have been, because after the Age of the Sages the matter was not tackled on the same scale as it had been, as we shall presently set forth more fully in its place.

The foregoing is confirmed by the fact that after the time of *Ptolemy* it became known from some Arabic writings that before him it was customary among various nations to mark the fixed stars on celestial globes in figures different from the common Egyptian ones, of which neither *Ptolemy* nor *Hipparchus* makes mention anywhere. Such figures were shown to me by the Noble Very Learned Mr. *Josephus Scaliger* [1]), in books with an explanation of the figures given in the Arabic language, not in one way, but on as many as three globes, each different from the others. The figures on one of these globes were said to be constellations of the inhabitants of India, about which names, but without a representation, I also remember having read in a Latin book of a very old edition, but I have forgotten the author's name, nor do I know what has become of the book. Other signs I have seen painted on the walls of a room in the royal palace at Cracow in Poland [2]); these were monsters whose members were a combination of those of different species of animals, with an inscription: *Signa Hermetis, i.e.* signs of Hermes. As has been said, I do not find any of the above-mentioned signs mentioned in *Ptolemy,* from which it may apparently be concluded that they had not been transmitted to him, and even less so the astronomical doctrines which every nation in its own way had based thereon. Further, astronomers formerly knew quite well that the earth revolved round the sun, but the actual propositions concerning this apparently had not come down to *Ptolemy,* for if he had seen them, it is to be assumed that (if we may judge by his intelligence and reasonability in the rest) he would have admitted the natural motion described by experienced astronomers and would have abandoned that according to the earlier doctrines [3]) or used it merely for instructive purposes, by way of hypothesis. The following authors learned of this motion of the earth: *Philolaus Pythagoricus,* as *Plutarch* says [4]), further *Aristarchus Samius,* as *Archimedes* states in the book of the number of the grains of sand [5]). Now that this did not happen in the Age of the Sages apparently can be concluded sufficiently from *Archimedes'* words, which are to the effect that *Aristarchus* wrote against the astronomers who said that the world is a sphere whose centre is the earth's centre, while its radius is equal to the line joining the centre of the sun and the centre of the earth, this line (among other imperfections) being longer at one time than at another, and so it does

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[1]) Joseph Julius Scaliger (Agen 1540–Leiden 1609), Italian humanist, philologist and chronologist, professor at Ghent and at Leiden. He published excellent editions of Latin authors and wrote the monumental works *De emendatione temporum* (1583), *Thesaurus Temporum* (1606). *Cf.* note 4 on p. 325.

[2]) This interesting passage shows that Stevin visited Poland. *Cf.* Biographical introduction, Vol. I, p. 5. Such monsters belong to the world of Accadian and Sumerian mythology and astronomy.

[3]) The word *leerlinghen* in the Dutch text is evidently a printer's error for *leeringhen.*

[4]) *De placitis philosophorum* III, *cap.* 13.

[5]) Commonly called: *the Sand Reckoner (= Arenarius). Opera*, ed. Heiberg (Leipzig 1913) II, 218.

Ghelijck cloots middelpunt,
Tot clootvlack,
Alsoo Eertclootwech,
Tot verheyt der vaste sterren.

Alwaer tegen Wisconstenaers reghel verlijcking van * verscheenslachtighe sijnde, te weten punt met vlack, soo schijnt et selve als vooren ghseyt is daer uyt te moghen besloten worden. Angaende *Archimedes* de boveschreven everedenheyt uytleght, en seght aldus behooren verstaen te worden: *Heterogeneia*

Ghelijck Eertcloot,
Tottet ghene men de Weerelt noemt,
Alsoo Eertclootwechs cloot,
Tot vaste sterrens cloot.

T'mach daer me sijn hoet wil, dan wisconstighe redenen vereysschen gewisse woorden: In somme daer en is gheen teycken van wijsentijt.

Ander getuychnis van een groote oeffening dieder voor *Ptolemeus* tijt inden Hemelloop geweest heeft, hebben wy door de verscheyden manieren van leeringen der clootsche driehoucken op verscheyden gronden gesticht, naderhant te voorschijne ghecommen in Arabische spraeck, en daer uyt int Latijn gherocht. Want nadien de menschen eertijts saghen hoe noodich goede manier van rekening der clootsche driehoucken was, om na volcommen kennis des hemelloops te trachten, soo heeft hun verstant in die stof wonderlick gearbeyt. De manier an *Ptolemeus* ter hant ghecommen, en door hem beschreven, is cort en aerdich, bestaende in vergaring en aftrecking van redens der linien, die verdocht worden in seker plat datmen * clootsne noemt, maer t'gebruyc is moeyelick, want den Doender ant werck commende, en vint in dien beschreven driehouckhandel tot alle ontmoetende voorbeelden gheen gesormde * werckstucken, om die met lichticheyt te volgen, maer moet geduerlick becommert sijn, met t'overdencken wat manier van vergaring of aftrecking der redens hy tot sijn voorbeelt uyt de clootsne verkiesen sal. Een ander manier ist diemen oock ghebruyckt met bedencking der gemeene sneen van platten der ronden op den cloot gheteyckent. En noch een ander diemen by *Regiomontanus* bevint. *Sectiosphaerica.* *Problemata.*

Angaende sommighe achten de vonden des Hemelloops niet soo seer oudt te wesen, maer dat de meeste besonderheden van dien deur *Hypparchus* tot smenschen kennis souden ghecommen sijn. En dat *Timochares* gheleeft hebbende 30 jaren na den Grooten *Alexander*, onder de sterflicke den eersten was, die t'vinden en opschrijven der plaetsen vande vaste sterren beneerstichde, mijn gevoelen is daer af anders: Wel wil ick toestaē, dat so *Hypparchus* uyt de boucken van sijn voorgangers niet beschreven en hadde het bouck daer na *Ptolemeus* ter handt ghecommen, en van hem tot ons gherocht, dat wy nu vanden loop der Dwaelders weynich kennis souden hebben: Maer dat hy van die seltsame voorstellen een vinder soude gheweest sijn, t'en schijnt niet om onder anderen dese reden: *Ptolemeus* segt int 11 Hoofstick sijns 4 boucx, dat *Hypparchus* een swaricheyt ontmoete, om dat hy deur * stelling des Maenloops in een * inrondt, tot ander besluyt quam dan deur stelling in een * uytmiddelpuntichront: Bewijst voort dat sulck verschil niet en quam deur de verscheydenheyt die *Hypparchus* inde twee stellinghen vermoet, dan deur sijn misrekening, of ander ongheval: Dit soo sijnde, het schijnt te meughen besloten worden, dat *Hypparchus* onder ander vonden diemen hem toeschrijft, voor al gheen vinder en was des * vertoochs, deur t'welck bewesen wort d'een en d'ander stelling een selve besluyt te gheven, maer veel eer dat hy sulck vertooch (t'welck wy niet en segghen tot sijn *Positionem.* *Epicyclo.* *Eccentrico circulo.* *Theorematis.*

ver-not seem to make us much wiser. Further, if we understand the form of the proportion, given as follows in *Aristarchus:*

As the centre of the sphere is to the surface of the sphere,
so is the earth's orbit to the distance of the fixed stars,

where, contrary to the rule of mathematicians, heterogeneous things are compared, to wit a point and a surface, it seems that the aforesaid statement may be inferred therefrom. As to the fact that *Archimedes* explains the above-mentioned proportion and says it ought to be understood as follows:

As the earth is to that which is called the world,
so is the sphere of the earth's orbit to the sphere of the fixed stars,

however this may be, mathematical arguments call for exact words. To sum up, there is no sign of the Age of the Sages there.

Other evidence of the fact that people were greatly versed in astronomy before *Ptolemy*'s time we have through the various doctrines about spherical triangles, established on different foundations, which afterwards appeared in Arabic and thus got into Latin. For since people formerly saw the necessity of a good method for calculations on spherical triangles in striving to attain perfect knowledge of the heavenly motions, their minds worked marvellously at this subject. The method which came into *Ptolemy*'s hands and was described by him [1]) is short and elegant, consisting in addition and subtraction [2]) of the ratios of the lines which are imagined in a given plane, which is called "section of the sphere", but the practical application of it is difficult, for when the practician sets to work, in this treatise on trigonometry he does not find for all the examples encountered by him worked-out problems, which can easily be followed, but he has to be continually concerned to think which manner of addition or subtraction of the ratios he has to choose for his example from the section of the sphere. Another method is that also used in studying the common intersections of the planes of the circles drawn on the sphere. And yet another method is that found in *Regiomontanus*.

As to the fact that some people think the astronomical discoveries are not so very ancient, but that most of the particulars about the subject have come to men's knowledge through *Hipparchus*, and that *Timocharis* [3]), who lived 30 years after *Alexander* the Great, was the first among mortals to have diligently practised the finding and writing down of the positions of the fixed stars, I am of a different opinion about this. I am prepared to admit indeed that, if *Hipparchus* had not written — on the basis of his predecessors' books — the book which afterwards came into *Ptolemy*'s hands, and through him has reached us, we should now have little knowledge of the motion of the planets. But that he was one of the discoverers of these extraordinary propositions does not seem to be true, among other things for the following reason. *Ptolemy* says in the 11th chapter of his 4th book that *Hipparchus* encountered a difficulty because by assuming the moon to move on an epicycle he reached another result than by assuming it to move on an eccentric circle. He further proves that this difference was not due to the difference between the two hypotheses as suspected by *Hipparchus*, but

[1]) *Syntaxis* I, *cap.* 13.

[2]) By "addition and subtraction of ratios" Stevin means what *we* should call „multiplication and division of ratios".

[3]) In the Dutch original the orthography *Timochares* is used. This Greek astronomer lived at Alexandria around 230 B.C.

vermindering maer om t'voornemen te bewijsen) niet begrepen en heeft, want anders hét waer so veel al oft ymant wel verstaende * t'wisconstich bewijs vant laetste voorstel des eersten boucx van *Euclides*, nochtans daer na totte dadelicke ervaring commende, twijfelde of de twee viercanten der twee cortste sijden eens rechthouckigen driehoucx, even sijn ant viercant der langste sijde, om dat hyse dadelick bevint te verschillen, sonder te weten dat sulcx niet en comt deur ghebreck des voorstels, maer deur feyling der handen, ooghen, deur misrekening, of ander ongeval. Angaende dat *Ptolemeus* int 2 Hoofdstick sijns 3 boucx, de manier van *Hypparchus* prijst voor die der ouden, nopende het vinden der Sonnens plaets tusschen de vaste sterren, om daer uyt de lanckheyt des jaers te berekenen, welcke plaets sy meynen de ouden ghesocht te hebben deur de Sonnens evenaer breede doense was in haer * Noortstant, welcke ondersoucking *Hypparchus* met meerder sekerheyt dede doense inde lentsne was. Hier op segtmen, dat sy die door soodanige langduerighe seltsaem oeffeninghen en steeroogينghen, gherocht waren ter kennis van der Dwaelders middelloopen, niet en souden bemerckt hebben soodanighe ervaringhen gantsch onbequamelick in Sonstant ghedaen te worden, ick en siender gheen teycken af: Maer by aldiense haer ervaringhen in Sonstant ghedaen hebben, soo *Hipparchus* seght, tis te vermoeden dat sulcx niet en ghebeurde deur ondersoucking vande Sonnens evenaerbreede, maer veel eer met meerder sekerheyt deur t'nemen der schijnbaer duysteraerlangde tusschen haer en eenige der vaste sterren. Wantse beneven de Son, alsmen vlietelick ghenouch steerooght, ghesien connen worden: T'welck *Ptolemeus* onbekent wesende, so en schijnt sijn verheffing van *Hipparchus* vont, boven die van sijn oude voorganghers in gheen ghenouchsaem reden gegront. Soo veel *Timochares* angaet, dat hy als ghesteyt is onder de sterflicke den eersten soude gheweest hebben, die t'vinden en opschrijven der plaetsen vande vaste sterren beneerstichde: Anghesien hemlien die dat segghen, en *Timochares* me, onbekent schijnen gheweest de beschrijvinghen der Hemelclooten, op welcke haer oude voorganghers van verscheyden gheslachten, de sterren in ander formen vervinghen als vooren gheseyt is, so en schijnt *Timochares* sulcken eersten gaslagher der vaste sterren niet gheweest te hebben. Te meer dat der Dwaelders middelloopen *Hypparchus* ter hant ghecommen, ghevonden schijnen deur sekerder beschrijving der vaste sterren dan van *Timochares*, die 10 ⦶ voor cleenste maet ghebruyckte, soo in *Ptolemeus* tafelen blijckt.

Mathematicam demonstrationem.

Solstitio æstivale.

HET TWEEDE TEYCKEN is de wonderlicke ervarentheyt die wy sien eertijts by de menschen inde Telconst gheweest te hebben, waer afmen een van de vreemde seltsaemheden houden mach de * Stelreghel, die over weynich jaren deur Arabische boucken weer te voorschijne gecommē is, daer afmen deur naghelaten schriften niet en merckt gheweten te hebben Caldeen, Hebreen, Griecken (want *Diophantus* is jonck) of Romeynen, al welcke gheen * Telders diemen deur weerdicheyt Telders noemt, gheweest en sijn. Oock insiende dat hemlien daer toe noodighe reetschap ghebrack, namelick talletters, met soodanighe thiende voortganck als de uytghesproken getalen hebben, so wast hemlien onmeughelick. Maer mochten sulcke telders sijn, als die nu met penninghen legghen, of met crijtschreefkens rekenen, of dierghelijcke. Dese talteyckens van thiende voortganck sijn inde Arabische spraeck weerom voortghecommen, in sulcker voughen datmen daer me anvanghende, mach leeren wat t'begin of punt des ghetals is, t'welck de leeckentijt (ick meen vanden anvang der vermaerde Griecken tot nu toe) qualick verstaende, gheseyt heeft de * eenheyt te wesen. Want den Edelen Hoochgeleerden Heer *Iosephus Scaliger*, heeft my ge-

Algebram.

Arithmetici.

Vnitatem.

to his faulty calculation or some other accident. This being so, it may apparently be concluded that *Hipparchus,* though many other discoveries were attributed to him, was not the discoverer in particular of the theorem by means of which it is proved that the two hypotheses give the same result, but much rather that he did not understand this theorem (which we do not say to belittle him, but to prove our intention), for otherwise it would be as if a man, though properly understanding the mathematical proof of the last proposition [1]) of the first book of *Euclid,* yet, when afterwards it comes to practical work, should doubt whether the two squares on the two shortest sides of a right-angled triangle are [together] equal to the square on the longest side, because he finds them to differ in actual practice, without knowing that this is not due to a defect in the proposition, but to errors of the hands or eyes or to faulty calculation or some other accident. As to the fact that *Ptolemy* in the 2nd chapter of his 3rd book praises the method of *Hipparchus* above that of the ancients, concerning the finding of the sun's position among the fixed stars, in order to calculate therefrom the length of the year, which position they thought the ancients had sought by means of the sun's equatorial latitude when it was at its summer solstice, which research *Hipparchus* carried out with greater accuracy when it was at the vernal equinox, — to this it is said: I see no evidence that those who through such prolonged and exceptional practice and watching had arrived at knowledge of the planets' mean motions should not have noticed that such observations are made quite improperly at the solstice. Now if they made their observations at the solstice, as *Hipparchus* says, it may be suspected that this was not done by seeking the sun's equatorial latitude, but rather with greater accuracy by taking the apparent ecliptic longitude between the sun and some of the fixed stars. For if we watch diligently enough, they can be seen by the side of the sun; and since this was not known to *Ptolemy,* his exaltation of *Hipparchus'* discovery above those of his ancient predecessors seems to be founded on insufficient grounds. As regards *Timocharis, viz.* that — as has been said — he should have been the first among mortals to have diligently practised the finding and writing down of the positions of the fixed stars: since those who say this, and *Timocharis* as well, seem to have been unacquainted with the descriptions of the celestial globes on which their ancient predecessors of different nations marked the stars in other figures, as has been said above, *Timocharis* does not seem to have been this first observer of the fixed stars. The more so because the planets' mean motions which were handed down to *Hipparchus* seem to have been found by a more accurate description of the fixed stars than that of *Timocharis,* who used 10′ for the smallest measure, as appears from *Ptolemy*'s tables.

The second sign is the wonderful experience in arithmetic which it has been found man formerly possessed, one of the curious peculiarities of which may be considered to be *Algebra,* which came to light again a few years ago from Arabic books, which subject, from the writings they left, is seen not to have been known to the Chaldeans, the Hebrews, the Greeks (for *Diophantus* is no ancient writer [2])) or the Romans, all of whom were no arithmeticians worth the name. Considering also that they lacked the necessary instruments, *viz.* numerals, with the same tenth progression as the numbers have when pronounced, it was impossible for

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[1]) Actually this is the penultimate proposition of the first book.

[2]) Diophantus lived at Alexandria about A.D. 250. Stevin seems to have a mistaken view of the epoch when this mathematician flourished. His *Arithmetica* is actually a textbook of algebra.

my getoont, dat de Arabiers daer voor teyckendē een punt, aldus . t'selve oock punt noemende, en wierden die punten onder de talletters ghebruyckt in placts daer wy o stellen, overcommende mettet ghene wy over eenighe jaren in onse Fransche Arithmetique onder de 2 bepaling daer af seyden. D'oirsaeck waerom in plaets van dat punt, byde Europianen nu een o gestelt wort, acht ick dese, dat wy ghewoon sijn punten te ghebruycken int eynde en onderscheyding der geschreven redens, welcke punten oock dickwils achter ghetalen volghen, maer soomen daer punten stelde, sy souden twijfeling maken oft een punt waer des ghetals, dat onbehoirlick vermeerderende, oft een punt als onderscheyt des redens, en om soodanighe twijffeling te voorcommen, heeftmen het punt verandert, en daer voor een o gheschreven. Nu dan o inden Wysentijt punt geheeten hebbende, wy sullent om die te volghen nu voortaen oock dien naem gheven, en dat tot onderscheyt vant meetconstich punt, Talpunt noemen, verlatende d eerste naem *Begin*, die wy daer toe dus langhe gebruyckt hebben. Angaende dat sommighe niet ghenouch deur natuerlicke reden connen oirdeelen, hun ghedraghen totte * loofweerdicheyt der ghene die daer af ghehandelt hebben, ick acht die op een goeden wech te wesen, midts datse de loofweerdichste loofweerdicheyt volghen, dat sijn de ervaren Telders des Wysentijts, en verlaten de Griecken die gheen telders en waren, noch volcommelick sijn en conden deur ghebreck van rechte talteyckens als vooren ghesseyt is: want hoe wel *Euclides* schoone Telconstighe * vertooghen beschrijft, die uyt den Wysentijt t'sijnder hant getocht waren, daer en sijn * werckstucken noch * Teldaet by, welcke als ghesseyt is onlancx te voorschijn commen sijn in Arabische spraeck. Sulcx dat *Euclides* vertooghen ghetuyghenis gheven des Wysentijts die te voren geweest hadde, en doen niet en was. De reden waerom wy hier so ernstelick van dit punt seggen, is dat bepaling der eenheyt voor punt des getals, onder anderen getuychnis geeft des Wysentijts diet was doenment punt teyckende en daer voor hielt: En oock des Leeckentijts diet sedert geweest heeft, so onvolcommen * Telders makende, als bepaling des deels der grootheyt voor punt der grootheyt, onvolcommen Meters soude mebrenghen. Noch canmen mercken, dat inden selven Wysentijt veel Telconstige werckingen met besonder lichticheyt afgeveerdicht wierden deur rekening op thiende voortganck ghegront. Om van t'welck breeder reden te verclaren, het is te weten dat alsoo ick over eenige jaren de thiende beschreef, en my inbeelde met groote lichticheyt te meugen gebruyct worden in deyling der houckmaten en bogen met thiende voortganck; En alsó ick daer na de selve manier eyghentlick beschreef, in sulcker vougen als van dies inden volgenden Hemelloop t'sijnder plaets een hooftstuck ghemaect sal worden met sulcke cortheyt als blijcken sal: So heb ick daer na bemerct dat sgelijcx voor my al gedaen had geweest, oft immers gedaē scheen geweest te sijne in ouden tijt, die ick meyne dat den Wysentijt was, om dese redenē: De tafel der houckmaten van *Regiomontanus*, diens halfmiddellijn gedeelt is in 10000000, begreep in haer volcommelick den thienden voortganck die ick socht: Want nemende de halfmiddellijn gedeelt te sijn in 100 even deelen, in plaets van 60 der Egyptenaers, En daer nae my selven voorstellende de deeling te wesen van thiende voortganck, in plaets der t'sestichde vande Egiptenaers, ick bevant met die tafel al ghedaen werck, soo veel de houckmaten angaet: En meende doen *Regiomontanus* daer af een eerste vinder geweest te sijne, te meer dat hy int begin van sijn houckmaetmaecksel, seght dat die voor hem geweest sijn, de middellijn in weynich stucken deelden, als *Ptolemeus* in 120, *Arzahel* in 300, deelende elck van dien in 60 ①, en elcke 1 ① in 60 ②. Maer naderhant is my yet te vooren ghecommen, waer uyt ick nu anders vermoede, t'welck aldus toeginck: Alsoo ick

Autoritatem.

Theoremata.

Problemata neque praxis Arithmeticæ.

Arithmeticos.

Geometricos.

them; but they may have been arithmeticians of the kind who now count by laying out pennies or reckon with chalk marks, or the like. These numerals of the tenth progression have come to light again from Arabic books, so that, beginning with this, we can learn what is the origin or point of number, which the Age of the Ignorant (I mean from the beginning of the famous Greeks up to the present), misunderstanding it, stated to be unity. For the Noble and Very Learned Mr. *Josephus Scaliger* has shown me that the Arabs drew a point for it, as follows: ., also calling it point, and these points were marked underneath the numerals, where we put 0, which corresponds to what we said about it some years ago in our French Arithmétique, in the 2nd definition [1]). I consider that the cause why instead of this point a 0 is now used by Europeans is that we are accustomed to use points at the end and the demarcation of written sentences, which points also often succeed numbers; if there points were used, they would raise doubt as to whether is was a point belonging to the number, increasing the latter unduly, or a point destined to demarcate the sentence, and in order to prevent such doubt, the point was changed and replaced by a 0. Now then, since in the Age of the Sages 0 was called point, in order to imitate them we shall henceforth also give it this name and call it Numerical Point, so as to distinguish it from the geometrical point, abandoning the previous name of *Commencement,* which we hitherto used for it. As to the fact that some people, not being able to judge sufficiently by means of natural reason, act on the authority of those who have dealt with the subject, I consider that they are on the right road, provided they follow the most authoritative authority, *i.e.* the skilled arithmeticians of the Age of the Sages, and do not follow the Greeks, who were no arithmeticians, nor could perfectly be so, through lack of the proper numerals, as has been said above. For though *Euclid* writes elegant arithmetical theorems, which had come down to him from the Age of the Sages, they include neither problems nor practice of arithmetic, which, as has been said, have recently come to light from Arabic books, so that *Euclid*'s theorems bear witness to the Age of the Sages which had previously existed and did not then exist. The reason why we here discuss this point so seriously is that the definition of unity as the point of number is evidence, among other things, of the Age of the Sages which existed when the point was drawn and looked upon as such; and also of the Age of the Ignorant which has existed since then, producing as imperfect arithmeticians as the definition of a part of a magnitude as a point of the magnitude would testify to imperfect geometers. It may also be noted that in this Age of the Sages many arithmetical operations were performed with extraordinary ease by means of computation based on the tenth progression. In order to explain this more fully, it is to be noted that while some years ago I described the Tenth and imagined that it might be used with great ease in the division of sines and arcs according to the tenth progression; and while thereafter I described this method properly, in such a way as in its place a chapter is to be made about it in the subsequent book on astronomy, with the succintness that will appear there, I perceived afterwards that this had already been done before me, or at least seemed to have been done in ancient times, which I think was the Age of the Sages, for the following reasons. The table of sines of *Regiomontanus,* whose radius is divided into 10,000,000, completely comprised the tenth progression which I sought. For assuming the radius to be divided into 100 equal parts instead

[1]) Vol. II B, pp. 495 *et seq.*

eens wilde ondersoucken ofmen deur de houckmaettafel van *Regiomontanus*, de reden van des ronts middellijn totten omtreck, so na soude vinden dat de palen bleven tusschen de palen van *Archimedes* reden, of niet: Ick sach tot desen eynde hoe veel de houckmaet van 1 Ⓘ dede, bevantse van 2909, in sulcke deelen alsser de halfmiddellijn 10000000 doet, Maer die houckmaet is bycans even an haer booch, om des boochs cleenheyt wille, daerom de 5400 Ⓘ makende t'vierendeelronts, sijn bycans even an 5400 mael 2909, dats evē met 15708600, waer deur het vierendeelronts by na in sulcken reden is totte halfmiddellijn, als 15708600 tot 10000000: En vervolgens het heel ront doende viermael so veel, dats 62834400, is by na in sulcken reden totte heele middellijn, als 62834400 tot 20000000. Dese reden bevant ick binnen de voorschreven palen der reden van *Archimedes*, te weten cleender als van 22 tot 7, en grooter dan van 227 tot 71. Nu alsulcken inval als ick hier hadde, van te willen ondersoucken de reden der middellijn totten omtreck deur de houckmaettafels, dergelijcke inval schijnt eertijts byde ouden oock geweest te sijn, om dese redenen: Ter hant van *Georgius Peurbachius* gecommen sijnde seker schriften (soo hy seght in sijn handel opt houckmaetmaecksel) inhoudende t'ghevoelen van verscheyden geslachten, als Indianen, Egiptenaers, en Arabiers, angaende de reden van des ronts middellijn totten omtreck, datter oock eenige geweest sijn diese stelden op van 20000 tot 62832, ghenouchsaem de selve letters alsvooren, doch de ware reden noch wat naerder. Maer die dat deden, en sulcken houcmaettafel gebruycten, de selve tafel hadde de middellijn van 2 met ettelicke talpunten, welcke in menichte niet alleen seven en waren, gelijck in *Regiomontanus* tafel, maer moesten soot schijnt van een talpunt meer sijn, uyt oirsaeck dat dit ghetal 62832, vast gaet tot op de vijfde letter, daer het onse maer vast en gaet tot op de vierde: T'welc blijct alsmen de palē des redens veel nauwer neemt, gelijc gedaē heeft dē vermaerden * Telder M. *Ludolf vā Ceulen*, te wetē dat alsmen de middellijn neemt op 200000000000000000000

Arithmeticus.

Soo is den omtreck corter dan 628318530717958647694
Maer langher als 628318530717958647690

Inder vougen dattet schijnt te meughen besloten worden, datmen voor *Regiomontanus* tijt de halfmiddellijn inde houckmaettafel gedeelt heeft in 10000000, of in een ghetal met een talpunt meer, t'welckmen met reden vermoeden mach den onbekenden wysentijt gheweest te hebben, ghemerckt datter geen teycken en is sulcx in bekende tijt gheschiet te sijn.

Tot hier toe is gheseyt van der Ouden deeling der houckmaten met thiende voortganck, maer datse boven dien oock alsoo den booch des vierendeelronts meugen gedeelt hebben, om daer deur te becommē de lichticheyt die indē Anhang des Hemelloops verclaert sal worden, dat machmē vermoen uyt des ronts deeling in 1600, daermen voormael de * Wisconsttuygen in deelde, na t'seggen van *Ptolemeus* int 2 Hoofstick sijns 3 boucx, waer uyt volghde dat ghelijck het vierendeel ronts na de Egipsche wijse in 90 tr. ghedeelt wort, en in de wisconsttuygen elcke trap dickwils in vieren, niet tegenstaende men in rekening de tsestichde voortganck volghde: Dat hier alsoo elck vierendeelronts in 100 tr. ghedeelt wiert, en inde wisconsttuyghen elcke trap dickwils in vieren, niet teghenstaende in rekeningen de thiende voortganck ghevolgt wiert. Want dat sy stelders der talletters met thiende voortganck, en der reghels van rekeninghen die deur opsicht der thiende voortganck ghewrocht worden, ghelijck wy daer af int bouck der ghemengde stoffen eyghentlicker segghen sullen, niet en souden bemerckt hebben het groot voordeel der thiende voortganck, inde deyling des ronts soo dickwils in rekeninghen der Hemelloopen te vooren commende, ten schijnt niet.

Instrumenta Mathematica.

HET

of the 60 of the Egyptians, and subsequently imagining the division to be in tenths instead of in sixtieths of the Egyptians, I found that in this table the work had already been accomplished as far as the sines are concerned. And I then thought that *Regiomontanus* had been the first discoverer of it, the more so as at the beginning of his trigonometry he says that those who preceded him divided the diameter into a small number of parts, *e.g. Ptolemy* into 120, *Arzahel* [1]) into 300, subdividing each of them into 60′ and each minute into 60″. But later on I hit upon something on account of which I now have different surmises; this happened as follows. When one day I wanted to investigate whether by means of the table of sines of *Regiomontanus* the ratio between the diameter of a circle and its circumference would be found so closely that the limits remained within the limits of *Archimedes'* ratio or not, I looked up to this end how much was the sine of 1′ and found it to be 2,909, in the same parts as the radius has 10,000,000. But this sine is nearly equal to its arc, because of the smallness of the arc; consequently the 5,400′ constituting a quarter circle are nearly equal to 5,400 times 2,909, *i.e.* equal to 15,708,600, so that the quarter circle is to the radius nearly in the ratio of 15,708,600 to 10,000,000. And consequently the whole circle, which is four times as great, *i.e.* 62,834,400, is to the whole diameter nearly in the ratio of 62,834,400 to 20,000,000. I found this ratio to fall within the aforesaid limits of the ratio of Archimedes, to wit less than 22 to 7 and greater than 223 to 71 [2]). Now the same idea which struck me, *viz.* that of wishing to seek the ratio of the diameter to the circumference by means of the tables of sines, also seems to have occurred formerly to the ancients, for the following reason. *Georgius Peurbachius* [3]) having laid hand on certain writings (as he says in his treatise on trigonometry) containing the opinions of different nations, such as inhabitants of India, Egyptians, and Arabs, about the ratio of the circle's diameter to its circumference, there were also some who put it at 20,000 to 62,832, almost the same figures as above, but a little closer still to the true ratio. Now the table of sines used by those who did so had the diameter 2 with some numerical points, whose number was not only seven, as in *Regiomontanus'* table, but apparently must have been one point more, because this number of 62,832 is accurate to five figures, whereas ours is accurate only to four. This becomes evident when the limits of the ratio are taken much narrower, as was done by the famous arithmetician Mr. *Ludolf van Ceulen* [4]), to wit that when the diameter is taken

	200,000,000,000,000,000,000
the circumference is shorter than	628,318,530,717,958,647,694
but longer than	628,318,530,717,958,647,690,

so that it may apparently be concluded that before the time of *Regiomontanus* [5]) the radius in the table of sines was divided into 10,000,000 or into a number with one point more, which time may reasonably be supposed to have been the unknown Age of the Sages, since there is no evidence that this happened in known times.

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[1]) Abû Isḥâq Ibrâḥîm Ibn al-Zarqâla, usually called Abraham Arzachel, was a Jewish astronomer, who lived at Toledo in the second half of the eleventh century (*c.* 1029 to *c.* 1087).

[2]) The figure 227 instead of 223 in the Dutch text appears to be a printer's error.

[3]) See note 2 on p. 315.

[4]) On this Dutch mathematician, who lived towards the end of the sixteenth century, see the notes in Vol. II, particularly on p. 3 and p. 767.

[5]) See note 1 on page 319.

HET DERDE TEYCKEN is de Meetconst, want hoe wel de Griecken daer in seer ervaren sijn geweest, doch wort by velen bevesticht dat syse van ander gecregen hebben. Voorwaer een seer wonderbaerlicke const, vast getuychnis gevende van een seltsame wetenschap der gene, wiese oock meughen gheweest sijn, diese tot sulcken grootheyt gebrocht hebben. Hier af is ons t'meeste bescheyt ghebleven inde beginselen van *Euclides*, waer in benevens de stof der Meetconst, noch wat seer besonders, seltsaems en nuttelicx te sien en leeren is, namelick des Wijsentijts oirden in beschrijving der Wisconsten, daer af ick inde volghende vernieuwing des Wijsentijts breeder mijn gevoelen sal seggen.

VOOR VIERDE TEYCKEN schijnt datmen soude meugen houdē den handel der Damphooghde, onlancx inde Arabische spraeck weerom te voorschijn commen, en hier na verclaert int Eertclootschrifts derde bouck betuyghende datter eertijts by de * Wisconstenaers een seltsaem ondersoucking gheweest is vande weerelts ghestalt en natuerens verborghen eyghenschappen. *Mathematicos.*

HET VYFDE TEYCKEN is den wonderbaerlicken seer seltsamen handel der * Stofscheyding, by de Grieckē onbekent, die onlancx begost heeft haer weerom te vertoonen, deur welcke de menschen het wesen der stoffen tot haren grooten voordeele, anders ondersoucken en kennen, dan hemlien sonder die groote const meughelick was te begrijpen. Hier in achtmen *Hermes Trismegistus* den ervarensten gheweest te sijn daer schriftelick bescheyt af bleven is, doch onbekent wie hy was, uyt wat lant, of tot wat tijt hy leefde, hoewelmen hem voor seer oudt acht. *Alchimiæ.*

Angaende dat de Griecken met hun navolgers die *Philosophi* genoemt worden, handelen vande natuer, seggende alle stof te bestaen in vier beginselen, als eerde, water, locht, en vyer, mette vervolgen dieser uyt trecken: Seker hun neersticheyt is lovelick geweest, als gedaen hebbende watse connen, maer wachaermen t'was al van hooren seggen, met weynich bescheyt, met veel dwalinghen, en sonder kennis der oirsaken, wantse veel gehaelt wort uyt de dadelicke Stofscheyding daer sy niet af en wisten.

Angaende dat dese weerdighe Const, dese onuytputtelicke brun der wijsheyt, by velen in verachting gheroemt is, deur dien ettelicke hun daer in oeffenende bedrieghers of missers sijn, brenghende ander lieden tot schade, hun belovende gout te maken van stof gheen gout wesende: Daer wort op gheseyt, sulck misbruyck te meughen strecken tot verachting der misbruyckers, maer niet der looflicke Const.

HET SESTE TEYCKEN is de * Gheesthandel, waer in men seght dat over seer langhe tijt eenighe volcken met kennis der oirsaken hun vlietelick gheoeffent hebben, want hoewel sulcx schrickelick is, soo mercktmen nochtans wat groote wetenschappen datter uyt de * Wisconsten volghden, diemen van soodanighe grondelicke kennis voor oirsaeck houdt, en wat een seltsaem wijsheyt datter voormael byde menschen gheweest heeft, welcken tijt wy den Wijsentijt noemen. *Magia.* *Mathematicis.*

Maer hoemen de saeck na ons meyning soude meugen anleggen om weerom daer an te gheraken, dat sullen wy inde volghende VERCLARING int gemeen beschrijven: En hoemen weerom totte voorgeweten kennis der Dwaelderloopen soude meughen commen, dat sal inde beschrijving der Dwaelderloopen int besonder noch eyghentlicker gheseyt worden.

Up to this point we have spoken of the Ancients' division of the sines according to the tenth progression, but that they may moreover also have divided the arc of a quarter circle in this way, in order thus to get the ease to be set forth in the Appendix to the Heavenly Motions, may be assumed from the division of the circle into 1,600, into which mathematical instruments were formerly divided, as stated by *Ptolemy* in the 2nd chapter of his 3rd book, from which it followed that, as a quarter circle is divided according to the Egyptian manner into 90 degrees, and in mathematical instruments each degree is often subdivided into four parts, in spite of the fact that in computations the sixtieth progression was followed, here each quarter circle was thus divided into 100 degrees, and in mathematical instruments each degree was often divided into four parts, in spite of the fact that in computations the tenth progression was followed. For it does not seem likely that they, having devised the numerals according to the tenth progression and the rules for computations which are performed with regard to the tenth progression, as we shall describe more precisely in the book on miscellaneous subjects, should not have perceived the great advantage of the tenth progression in the division of a circle so frequently occurring in calculations relating to the heavenly motions.

The third sign is geometry, for though the Greeks were greatly versed in this, it is yet confirmed by many that they got it from others. It is indeed a very wonderful science, giving sure evidence of exceptional learning in those who brought it to such a high level, whoever they may have been. About this the most complete information has been preserved in the elements of *Euclid,* in which besides the subject of geometry something very peculiar, extraordinary, and useful is also to be noted and learned, *viz.* the systematic order observed by the Age of the Sages in the description of mathematics, about which I will give my views more fully in the subsequent treatise on the restoration of the Age of the Sages.

The fourth sign may apparently be considered to consist in the discussion on the height of the atmosphere, recently brought to light again by a treatise in Arabic and hereafter set forth in the third book of Geography, which states that in former times among mathematicians an exceptional inquiry into the form of the world and of nature's hidden properties took place.

The fifth sign is the marvellous and very unusual subject of alchemy, unknown among the Greeks, which recently began to make its appearance again, by means of which people are able to inquire into the nature of substances, to their great profit, in another way than was possible for them to understand without this great science. In this, *Hermes Trismegistus* [1]) is considered to have been the most skilled of those of whom written information has come down to us, but it is unknown who he was, from what country, or in what time he lived, though he is thought to be very ancient.

As to the fact that the Greeks, with their imitators, who are called *Philosophi,* treat of nature, saying that every substance consists of four elements, *viz.* earth, water, air, and fire, with the conclusions they draw therefrom, their diligence was indeed praiseworthy, for they did what they could, but alas, they had it all from hearsay, with little real knowledge, with many errors, and without

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[1]) Hermes Trismegistos is the mythical author of several treatises on alchemy, magic, and philosophy. He has many traits in common with the ancient Egyptian god Thoth, the god of the craftsmen, scribe of the gods, and inventor of writing.

VANDE VERNIEVVWING DES

VVysentijts, t'vvelck is verclaring hoet schijnt datmen de saeck mocht anlegghen, om allencx vveerom te gheraken an sulcke groote vvetenschappen alsser inden VVysentijt gevveest sijn.

ANghesien het seker is de mensch tot eenighe tijt soo seltsame groote wetenschap ghehadt te hebben als inde 6 bepaling verclaert is, en dat mijns wetens nerghens an en blijckt sijn verstant een haer vermindert te wesen, so gevet met reden vermoen, meughelick te sijn dat hy daer an weerom soude connen gheraken, by aldien hyder sulck middel als voormael toe gebruyckte. T'selve heeft my veroirsaeckt te schrijven mijn ghevoelen, van t'gene de mensch nu ghebreeckt dat hy doen hadde: Alles vervatende in vier leden, waer af de somme, die ick daer nae van elcke breeder verclaren sal, dusdanich is.

knowledge of the causes, because these are frequently deduced from practical alchemy, with which they were not acquainted.

As to the fact that this worthy science, this inexhaustible source of wisdom, has fallen into disgrace among many people because some of those who study it are cheats or evildoers, causing damage to others by promising them to make gold out of matter which is no gold, to this it is said that such abuse tends to disgrace the men committing it, but not the praiseworthy science.

The sixth sign is magic, which is said to have been diligently practised a very long time ago by certain nations with knowledge of the causes, for though this is frightening, it is yet perceived what great learning has resulted from mathematics, which is regarded as the cause of such thorough knowledge, and what extraordinary wisdom existed among men in former ages, which time we call the Age of the Sages.

Now how in our view we can set out to attain thereto again, will be described generally by us in the subsequent *Exposition.* And how one might attain to the former knowledge of the planets' motions again, will be said even more properly in particular in the description of the planets' motions [1]).

[*In connection with a conversation he had about the Age of the Sages with the jurist and humanist Hugo Grotius, Stevin received from the latter a list of references to classical authors who asserted that a very long time ago mankind possessed marvellous scientific knowledge. He includes this list in his text.*]

Of the restoration of the Age of the Sages, being an exposition of how it seems one might set out to attain gradually to such great learning again as existed in the Age of the Sages.

Since it is certain that at one time mankind possessed the exceptional, great learning that has been set forth in the 6th definition, while as far as I know there is no evidence at all that human intelligence has the least bit deteriorated, there is reason to assume that it is possible for man to attain thereto again if he uses the same means to this end as formerly. This induced me to write down my view as to what man now lacks which he then had, comprising it all in four sections, the sum of which — to be set forth in greater detail hereinafter for each of them — is as follows.

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[1]) See the note on p. 608.

I LIDT.

Ten eersten ghebreken ons seer veel dadelicke ervaringen daermen de consten een vasten gront op gheeft. Om tot sulcke ervaringhen te gheraken, soo souden hun seer veel menschen t'samen daer toe moeten begheven.

2 LIDT.

Om te gheraken tot soo grooten menichte van menschen als hier toe noodich sijn, soo souden de voorschreven ervaringhen en oeffeningen der consten ghehandelt moeten worden by een * gheslacht in sijn eyghen angeboren tael, welcke om wat besonders daer in uyt te rechten, besonderlick goet soude moeten wesen, t'welck ick sedert den Wysentijt niet en merck ghschiet te sijn, uytghenomen byde Griecken, maer dat alleenlick int stick der Meetconst, want de rest en treft niet.

Gentem.

3 LIDT.

Na dien goede talen noodich sijn, men soude om na goede talen te connen trachten, voor al moeten weten waer in talens goetheyt bestaet, want die rechte kennis nu by soo weynich menschen is, datse met d'ander wetenschappen des Wysentijts verloren schijnt.

4 LIDT.

Anghesien goede oirden in beschrijving en leering der consten, tot haer bevoordering seer behulpich is, t'waer oirboir daer op vlietich te letten, en met goede oirdeel van dies t'beste te verkiesen. Tot welck eynde ick in stof der wisconsten gheen beter en merck, dan d'oirden des Wijsentijts.

VERCLARING DES I LIDTS.

Maer om van elck deser vier leden breeder reden te geven: En ten eersten dat ons veel dadelicke ervaringhen gebreken, daermen de consten een vasten gront op geeft, ick sal tot verclaring van dien beginnen met voorbeelt des Hemelloops, tot kennis van welcke die des Wysentijts openbaerlick gherocht sijn deur een groote menichte van ervaringhen, als breeder daer af inde 6 bepaling ghesceyt is, en noch breeder ghesceyt sal worden int eerste voorstel des volghenden boucx vande *Sonloop*, voort inde boucken der Dwaelderloopen, daer wy berekende dachtafels voorbeeltsche wijse ghebruycken sullen al offe deur ervaringhen becommen waren, en bethoonen hoe t'vinden der Dwaelderloopen daer deur al een ander lichte anvanck en voortganck crijcht, danmen sedert den Wysentijt ghebruyckt heeft.

Al dit overleyt wesende, ick acht openbaer ghenouch te sijn, ghebreck van overvloet der ervaringen, oirsaeck te wesen dat de menschen met groote moeyte en hoofbreking, hun tijt overbrenghen met te soucken Hemelloopsche gedaenten die alsoo niet vindelick en sijn. T'gaet hier me, op dat ickt deur voorbeelt van Eertclootsche stof noch opentlicker verclare, als met eenen varende langs de cant vant * onbekende Zuytlandt, en siende de mont van een groote rivier daer uyt aldus beslote: Langs groote rivieren sijn vruchtbaer landen: In vruchtbaer landen langs groote rivieren verkiesen veel menschen haer wooning: Daer veel menschen woonen geraken goede Steden: Daerom an die rivier

Terra incognita Australis.

legghen

1st SECTION.

In the first place, what we lack is a large body of data obtained by practical experience, on which the sciences can be firmly founded. In order to arrive at such a body of data, a great many people would have to apply themselves jointly to this task.

2nd SECTION.

To arrive at so great a number of men as is needed for this, the aforesaid experiences and pursuit of the sciences would have to be practised by a nation in its own native language, which, if it is to accomplish something exceptional therein, would have to be exceptionally good, which I am not aware has been the case since the Age of the Sages, except among the Greeks, but this in the field of geometry alone, for to the rest it does not apply.

3rd SECTION.

Since good languages are needed, one would, to be able to strive after good languages, have to know before all what the excellence of a language consists in, for this proper knowledge is now possessed by so few men that it seems to have been lost along with the other sciences of the Age of the Sages.

4th SECTION.

Since good order in the description and teaching of the sciences is very conducive to their advance, it would be expedient to attend diligently thereto and judiciously to choose the best, for which I am not aware of any better order in the matter of mathematics than that of the Age of the Sages.

EXPOSITION OF THE 1st SECTION.

Now to speak in greater detail about each of these four sections, and in the first place that what we lack is a large body of data obtained by practical experience, on which the sciences can be firmly founded, by way of illustration I will begin with the example of astronomy, the knowledge of which was evidently attained by the people of the Age of the Sages by means of a large body of empirical data, as has been stated more fully in the 6th definition and will be stated even more fully in the first proposition of the subsequent book of the sun's motion [1]), further in the books on the planets' motions, where we shall use calculated ephemerides, for instance, as if they had been obtained by experience and show how the finding of the planets' motions thus starts and proceeds more easily than has been customary since the Age of the Sages.

All this being considered, I think it is evident enough that the lack of plenty of empirical data is the cause that people spend their time with great trouble and pondering in seeking for astronomical schemes which cannot thus be found. It is with this — if I may explain it even more clearly by a geographical example — as with a man sailing by the shores of unknown Australia, who, seeing the mouth of a large river, should draw from this the following conclusion: Along large rivers lie fertile lands. In fertile lands along large rivers big numbers of people

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[1]) From this sentence it appears that originally the treatise on the Age of the Sages was intended to precede *The Heavenly Motions*, whereas it is now the introduction to the Physical Geography.

legghen groote welvarende Steden. En of hy voort op ſulck gheſtelde(deur een gheſien deel vant heel beſluytende) ſulcke Landen en Steden in caerte teyckende, denckt eens wat ſekerheyt of ghelijckheyt die mette Landen ſoude hebben, en hoe ſulcke caerten en ſchriften ſouden overcommen mettet ghene men daer na dadelick ſage,want daer deur machmen met een verſtaen, wat ſekerheyt datter can weſen in beſluyt van eens Dwaelders heelen loop, uyt een gheſien deel ghetrocken, en hoe dat ſulcke reghelen en ſchriften ſouden connen overcommen mettet ghene wy daer na dadelick ſien.

Nu ſulcx als hier gheſeyt is vande noodighe ervaringhen in ſtof des Hemelloops,dergelijcke is oock te verſtaen van ander,als Ebbenvloet, tot wiens grondelicke kennis ons louter ervaringhen ghebreken, daer int volghende 6 bouck afgheſeyt ſal worden: S'gelijcx vant Eertcloots Stofroerſel int volgende 2 bouck beſchreven: Voort Ervaringhen der * Steroirdeelen, of voorſegginghen deur Sterren:Oock der * Stofſcheyding: En Gheneſing, waer inmen (om niet te bewijſen met, *Hippocrates* ſegt)al een ander dadelicke oeffening ſoude moeten weſen ſoo in * opſnijding, als onderſoucking vande ghedaenten der cruyden en * Gheneeſtoffen,die veel door ſtofſcheyding ghevonden worden. Maer want van elck van deſe int beſonder te ſchrijven, meer tijts ſoude behouven dan my te pas comt,en dan noodich ſchijnt daer an te beſteden, ſo ſal ickt cortheyts halven overſlaen.

Iudiciariæ Aſtrologiæ. Alchimiæ. Anatomia. Medicamentorum.

Ick houde dan voor openbaer, dat ghelijck int eerſte lidt gheſeyt is, ons ſeer veel dadelicke ervaringhen ghebreken,daermen de conſten een vaſte grondt op gheeft. Maer om nu te verclaren dat (ghelijck daer voorder ſtaet) om tot ſulcke ervarnghen te gheraken,ſeer veel menſchen t'ſamen hun daer toe ſouden moeten begheven, ick ſal weerom met voorbeelt des Hemelloops beginnen. Ten eerſten, een menſch en can niet gheduerlick by dage by nachte,Iaer uyt Iaer in, gaſlaen de Dwaelders plaetſen en alles datter noodich is: Maer een ſeer groote menichte ſulcx doende, t'ghene by d'een ghebreeckt, wort by d'ander bevonden. Ten tweeden,de ervaringhen van eenen, al warenſe in haer ſelven gewis, ſoo en verſtreckenſe anderen nochtans tot gheen ſeecker gront, om int veroirdenen der * ſpieghelinghen daer op te werck te gaen, deur diender gheen proef af en is:Maer ſeer veel verſcheyden menſchens ervaringhen, die daer nae teghen malcander overleyt ſijnde, bevonden worden ſo na t'overcommen als de ſaeck vereyſcht, daer machmen op ſteunen. Tot voorbeelt van deſen connen verſtrecken de ervaringhen onlancx ghedaen tuſſchen den Doorluchtighen Vorſt *Willem* Landtgraef van Heſſen,en den vermaerden * Gaſlagher *Tuychobrahe* in druck uytgaende: Ervaringhen voorwaer diens ghelijcke ick ſedert den Wyſentijt niet en merck ghebeurt te weſen. En ſulcke ſoudemen dan met groote menichte vinden. Ten derden,ſoo iſſer dickwils tot ſommighe plaetſen overtogen locht,inder voughen datmender in ettelicke weken geen Hemelſche lichten en ſiet: In ſulcken ghevalle canmen dan hebben d'ervaringhen van anderen ghedaen inde Landen daert clare locht was. Ten vierden ſoo ſouder tuſſchen de Gaſlaghers een eergiericheyt en twiſt te verwachten ſtaen,willende elck het ſijne voor beſt bewijſen,waer me de conſten(hoe wel de menſchen daerentuſſchẽ int ſtuck der ſeden hun dickwils miſgaen)gemeenlick gheen cleene voortganck en crijghen:Daer anders den handel by weynich menſchen beſtaende, elck ſijn vonden bewaert en verberght.

Theoriarum. Obſervatorem.

Angaende ſommighe achten de ſtof te weerdich te ſijn om vande gemeente ghehandelt te worden, en alleen den Vorſten toeſtaet: Mijn ghevoelen is daer anders, want de Vorſten des Eertrijcx ſijn weynich,en onder die weynighe iſſer

 luttel

choose to live. Where there live many people, good cities arise. Therefore on this river there are large and prosperous cities. And if further on this hypothesis (drawing a conclusion about the whole from the part he had seen) he were to draw a map of such lands and cities, just think what accuracy or similarity to the lands this would have and how such maps and descriptions would agree with what would be actually seen afterwards, for from this it may at the same time be understood what degree of certainty there may be in a conclusion about the whole motion of a planet drawn from the sight of a part, and how such rules and descriptions might agree with what we actually see afterwards.

The same statement that has here been made about the necessary empirical data in astronomical matters also applies to other subjects, such as ebb and flow, for the thorough knowledge of which we merely lack empirical data, about which something will be said in the subsequent 6th book. Likewise about the motion of the earth's matter, described in the subsequent 2nd book. Further empirical data concerning astrological judgments or predictions by means of the stars. Also empirical data of alchemy, and medical science, in which (to avoid proving things by means of the statement: *Hippocrates* says) there would have to be much more practical work, both in anatomy and in the examination of the properties of herbs and medicines, which are often found by means of alchemy. But because it would take more time to write about each of these subjects in particular than suits me and seems necessary to spend on it, I will, for the sake of brevity, omit this.

I therefore regard it as evident that, as has been said in the first section, what we lack is a large body of empirical data, on which the sciences can be firmly founded. Now to explain that (as is further said there), in order to arrive at such a body of data, a great many people would have to apply themselves jointly to this ask, I will again begin with the example of astronomy. In the first place, one man cannot continually, by night and by day, year in and year out, observe the positions of the planets and all that is necessary; but when a large number of people do this, what is lacking in the observations of one man will be found in those of another. Secondly, the data obtained by one man, even if they were exact in themselves, yet do not serve others as a sure basis on which to proceed in framing the theories, because they have not been checked. But when the data obtained by a great many different people, having been compared with each other, are found to agree as closely as the matter requires, one can rely thereon. As an example of this may be taken the data recently obtained on the one hand by the illustrious Prince William, Landgrave of Hesse [1]), and on the other hand by the famous observer Tycho Brahe, whose work was then being published: data indeed which I am not aware have been equalled since the Age of the Sages. And such data would then be found very plentifully. Thirdly, the sky will often be overcast in some places, so that for some weeks no heavenly bodies are seen; in such a case one may then rely on the data obtained by others in the countries where the sky was clear. Fourthly, ambition and rivalry would be apt to arise among observers, each wishing to prove his own work best, owing to which the sciences (although men meanwhile often misbehave in moral respects) usually make no inconsiderable progress, whilst on the other hand, if the branch of science is

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[1]) Wilhelm IV von Hessen (Kassel 1532 - *ibid.* 1592) was a diligent observer up to 1567, when he succeeded his father; later he was able to attract excellent collaborators to Kassel. He gave special care to the determination of time.

luttel dieder een natuerlicke gheneghentheyt toe hebben. Siet eens tot voorbeelt van desen t'gene den Coninck *Alfonsus* weervoer, die tottet maken der tafels op sijn naem uytgaende, over de vier hondert duysent ducaten oncosten dede: Seker sijn yver tot sulcken const was lovelick, maer wat isser eyntlick uytgherecht? niet besonders, want sijn Wisconstnaers hebben sonder nieuwe ervaringhen, te werck ghegaen op *Ptolemeus* bloote stelling, waer uyt met die groote schat niet dan wat onsekers ghemaeckt en conde worden. Maer dese const byde ghemeente ghehandelt wesende, men can alles met meerder sekerheyt en kennis der oirsaken voor niet crijghen.

Nu sulcke voorbeelt als ick hier tot verclaring des voornemens ghetrocken heb uyt Hemelloopsche stof, derghelijcke soudemen oock meughen bybrengen uyt ander consten: Maer achtende hier me ghenouch te blijcken dat om in elck veel ervaringhen te crijgen, noodich is dat seer veel menschen t'samen hun daer toe souden moeten begheven, ick sal die om cortheydts wille overslaen, en voortvaren.

VERCLARING DES 2 LIDTS.

Angaende het tweede lidt, te weten dat om te gheraken tot soo grooten menichte van menschen als hier toe noodich sijn, dese oeffening der consten soude moeten ghehandelt worden by een gheslacht in sijn eyghen angheboren tael, welcke om wat besonderlicx daer in uyt te rechten, besonderlick goet soude moeten wesen, hier op segh ick aldus: Soude een ghemeente haer in een const oeffenen, sy soude moeten de tael verstaen daerse in gheschreven is, t'welck haer eyghen tael moest wesen, want hoe wel ettelicke ouders hun kinders t'Latijn doen leeren, waer in men de vrye consten nu meest handelt, de selve sijn weynich int ansien vande gemeente. Ten anderen leertmen de Ionckheyt t'Latijn, om hun eyntlick te begheven totte Rechten, Godheyt, of ghenesing: Isser onder de sulcke ymant die hem daer na gantschelick totte Wisconsten schickt, dat gheburt seer selden, en ghemeenelick teghen hun ouders danck, ghelijck onder anderen den vermaerden Gaslager *Tuychobrahe* hem schrijft gheburt te sijn: Daer isser onder die oock een grooten deel, welcke hun tijt voort deurbrenghen in oeffening der Latijnsche spraeck, leerende veersen der * Dichters van buyten, om op alle dingen die in gemeene t'saemspraeck voorvallen, een Latijns veers te connen vervoughen: Soucken voort bloemkens van woorden en spreucken om in haer brieven en schriften te pas te brenghen: En hoe wel sulcke opghetoeyde stijl an sommighe mishaecht, soo isser nochtans veel ander dieder hun niet me versaen en connen. In somme men vintter onder soodanige weynich die hun volcommelick totte wisconsten begheven, en daerom ist noodich ghelijck wy gheseyt hebben dese stof ghehandelt te worden inde ghemeentens aenghebo-ren tael. Doch moetse daer benevens noch goet sijn, connende alles uytbeelden dat totte saeck noodich is. Maer want dit stuck der talen een vande voornaemste punten is, die my doen wanhopen van weerom tot een wysetijt te meughen gheraken, om dat als vooren gheseyt is, metten onderganck van veel wetenschappen des selfden wysentijts, oock t'ondergegaen schijnt s'menschen kennis of oirdeel van goetheyt der talen, en dattet swaricheyt sal hebben hun dat te doen verstaen, soo moet ick na mijn vermeughen daer af breeder mijn ghevoelen verclaren. Tis openbaer dat de Francoysen (op dat wy met voorbeelt der Fransche tael beginnen) de vrye consten in haer spraeck meer beschrijven als an-

Poëtarum.

der

practised by few people, each of them will keep his findings to himself and conceal them.

Concerning the fact that some people consider the matter to be too sublime to be studied by ordinary people and to be fit only for princes, I hold a different view, for the princes of the earth are few, and among those few there are few who have a natural inclination for it. Just see, for instance, how King Alphonsus fared, who spent more than four hundred thousand ducats on the compilation of the tables published under his name [1]). His zeal with respect to this science was indeed laudable, but what has been ultimately achieved? Nothing much, for his mathematicians proceeded without new-found empirical data on *Ptolemy*'s mere hypothesis, from which with that large sum only something uncertain could be accomplished. Now when this science is studied by ordinary people, everything can be obtained for nothing with greater certainty and knowledge of the causes.

Now examples similar to the one I have here taken from astronomy, to explain my intention, might also be adduced from other sciences. But since I consider it is now evident enough that to obtain a large number of empirical data in each of them it is necessary that a great many people should apply themselves jointly to it, I will omit them, for brevity's sake, and continue.

EXPOSITION OF THE 2nd SECTION.

As to the second section, to wit that to arrive at so great a number of men as is needed for this, the aforesaid pursuit of the sciences would have to be practised by a nation in its own native language, which, if it is to accomplish something exceptional therein, would have to be exceptionally good, about this I say as follows: if ordinary people were to study a science, they would have to understand the language in which it is written, which should be their own language, for though some parents have their children taught Latin, in which the liberal arts are now usually treated, these are few in comparison with the people at large. On the other hand young people are taught Latin in order that they may ultimately study law, divinity or medicine. It rarely happens that among these there is anyone who thereafter devotes himself wholly to mathematics, and then it is against his parents' wishes, as *e.g.* the famous observer Tycho Brahe reports it happened to him. There are also among them a great many who further spend their time in practising the Latin language, learning verses of the poets by heart in order that they may apply a Latin verse to anything occurring in an ordinary conversation. They further collect flowers of speech and aphorisms in order to quote them in their letters and writings. And although such an ornate style displeases some people, there are nevertheless many others who cannot get enough of it. Briefly, there are found few among them who devote themselves wholly to mathematics and for this reason it is necessary, as we have said, that this subject should be discussed in the native language of the community. But in addition this language also has to be good and to be able to render everything that is required for the matter. Now because this question of the language is one of the chief points which make me despair we shall ever reach an Age of the Sages again, because — as has been said before — along with the decline of many sciences of the said Age of the Sages men's knowledge or appreciation of the excellence of a language also seems to have declined, and it will be difficult to

[1]) See note 2 on p. 315.

der volckẽ,t'welck wel oirſaeck is datter hemlien in haer gemeente meer menſchen daer in oeffenẽ, dant ſoudẽ by aldienſer niet af en handeldẽ,maer wantſe daer in ſeer veel Griecksche en Arabiſche conſtwoordẽ ſtellẽ, de groote voortganck met kennis der oirſakẽ,om weerom tot een Wijſentijt te geraken,en can daer uyt niet volgen: Want alſmẽ de conſtwoorden niet grondelick en verſtaet, als by voorbeelt inde Franſche tael te ſeggẽ van *Proſtapherefe*, *Parallaxe*, *Nadir*, *Almincantarat*, en veel dergelijcke, haer beteyckening geduerlick t'onthouden valt de gemeente laſtich,de oeffening moeylick,verdrietich,en de ſaeck vã ſlappe voortganck,ſy en connẽ niet verbeteren gebreckige conſtwoorden die dickwils beter bepaling vereyſſchen,noch de dwalingen mercken dieder uyt volgẽ, dan verblijden hun te connen ſpreken woordẽ die ander haer lantſlieden niet en verſtaen,en datmen hun met verwondering voor Hoochgeleerden hout.

make them understand this, I have to set forth my view of this more fully to the best of my ability. It is known that the French (to start with the example of the French language) describe the liberal arts more frequently in their own language than do other nations, which is probably the cause that more of the community study them than would be the case if they did not discuss them, but great progress use in their studies very many Greek and Arabic technical terms, the great progress involving knowledge of the causes, necessary to reach an Age of the Sages again, cannot result therefrom. For when the technical terms are not thoroughly understood, as when, for instance, in the French language the words Prostaphérèse, Parallaxe, Nadir, Almucantarat and the like are used, it is difficult for ordinary people to remember their meaning constantly, the study of the sciences is difficult and annoying, and the progress of the matter is poor; they cannot correct defective technical terms, which often call for a better definition, nor note the errors resulting therefrom, but they are glad that they can use words which others of their countrymen do not understand and that they are looked upon with admiration as very learned men.

[In order to show how the use of good technical terms affects the pursuit of science, Stevin points to the word "proportion", which failed to be understood by students of musical theory, thus leading to confused and unpractical systems of tuning. If, however, the Dutch word "evenredigheid" ("equi-ratio") had been used, equal temperament would automatically have been adopted.

The admixture of foreign words with a language, such as it takes place in French, is an impoverishment rather than an enrichment. Contrary to the general view therefore Stevin asserts that the French language is neither rich nor suitable for science. There are indeed charming French poems, which Stevin himself has read with great pleasure. But this merely proves that France counts many great poets; the language itself is extremely defective, because it contains so large a number of foreign words which ordinary people do not understand. — The same applies to Italian and Spanish, though to a less extent.

In order to assess the value of a language, Stevin wants to ascertain to what extent it is able to serve as a vehicle for science, in particular for mathematics. French, Italian, Spanish or Latin cannot express the mathematical sciences unless by using Greek words. Greek on the contrary is a very suitable language, because compound words are formed very easily in that language. Even better is Dutch, which is able to forge even shorter and plainer compounds of this kind from mostly monosyllabic basic words.

To prove this, Stevin gives lists of more than two thousand monosyllabic Dutch words, whilst on the other hand he finds only small numbers of monosyllabic Latin or Greek words. After this he shows by means of four instances how the meaning of Dutch compounds is always unambiguous and immediately clear to anyone. He who gains an insight into the character of the Dutch language cannot but marvel at its excellence. In historical times there has been no nation clever enough to grasp the great importance of such a language structure and to produce such a language. This language accordingly might have arisen in the Age of the Sages. If the Dutch should properly appreciate again their language (which is spoken in its purest form in North Holland), they might help to bring about the speedy restoration of the Age of the Sages.

Those who do not know what the language of science is, think that the acquisition of learning makes our lives gloomy and distracts us from important practical matters. Thus there are many people who think that Prince Maurice by his devotion to the sciences is distracted from the business of government. But to this it has to be answered that the study of sciences in the systematic formulation and style of the Age of the Sages makes things easier and aids us in accomplishing many great things.

The modes of expression of scientific style are next dicussed by Stevin in five chapters.

1. *The style of Euclid, in which a rigorous distinction is made between proposition and problem. Each of them is subdivided into the supposition, the property to be proved, the construction, and the proof (datum, quaesitum, constructio, demonstratio). Such a subdivision presents great advantages. One can first fully study the theoretical proposition, which one has to understand thoroughly; afterwards one can then give one's undivided attention to the practical rules (example: with multiplication, division, extraction of roots). Ptolemy did not make use of this mathematical style, which undoubtedly dates back to the*

Age of the Sages. Peurbach, Regiomontanus, and Copernicus on the other hand did make use of it, just like Stevin himself in his Mathematical Memoirs.

2. *The propositions should be preceded by definitions. Stevin relates how in certain circumstances he had to get to know more about earthwork, wattling, carpentry, brick-laying, forging, etc. For this, the workers in these trades were his best teachers. He began by asking them for the meaning of the words he did not understand, wrote down the definitions of these words, and learned them by heart. It appeared that he could then talk quite easily with the workers and that he thus quickly improved his knowledge.*
   In Stevin's „Arithmétique" all the definitions are given at the beginning; another time he would prefer to open each chapter with those definitions which pertain to that chapter.
3. *Dichotomy.*
   By this, Stevin means the classification of the subject under discussion into two (or more) possible cases; in the discussion of the spherical triangle, for instance, the following cases are to be distinguished:
   the side AC is less than 90 degrees;
   the side AC is equal to 90 degrees;
   the side AC is more than 90 degrees.
4. *Anaphora: the same concepts should invariably be referred to by the same terms* [1]*).*
5. *The separate discussion of theory and practical application. In a final chapter Stevin shows that theory and practice are both necessary.*]

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[1]) In rhetoric, this word designates a figure consisting in the repetition of a word or a phrase at the beginning of two or more successive sentences.

VANT MENGHEN DER *SPIEGHELING EN DAET.

Theoriæ & praxis.

WAntter int stick des oirdens noch een verschil valt vant menghen der spiegheling en daet, soo moet ick daeraf mijn ghevoelen seggen, eerst haer beteyckening verclarende voor de ghene diet onbekent mocht sijn. Spiegheling is een verdochten handel sonder natuerlicke stof, ghelijck onder anderen sijn de Spieghelinghen des Spieghelaers *Euclides*, handelende * deur stelling van grootheden en ghetalen, maer elck ghescheyden van natuerlicke stof. Daet is een handel die wesentlick met natuerlicke stof gheschiet, als lant en wallen meten, de menichte der roen of voeten tellen dieder in sijn, en dierghelijcke. T'besluyt vande voorstellen der Spiegheling is volcommen, maer der daet onvolcommen: Als by voorbeelt de Spiegheling vint en bewijst dat den helft des uytbrengs vande * hanghende en gront eens wisconstich driehoucx, volcommelick gheeft het inhoudt des plats sonder eenich gebreck of overschot: Maer een wesentlick driehouck van lant of ander glatter stof dadelick ghemeten sijnde, t'besluyt is daer af onvolcommen, eensdeels om dat wy gheen langde soo nau meten en connen, dattet gheen duysentste deel der dickte eens haers en schilt, of al waert by gevalle heel effen, tis onbewijselick. Ten anderen om dat gheen natuerlicke linien soo heel recht, noch natuerlicke vlacken soo heel plat en sijn, als de wisconstighe bepalinghen vereysschen, of al waren sy soo heel recht en plat, ten is niet bewijselick. De eyghenschap en t'eynde der Spiegeling is datse verstreckt tot seker gront vande manier der wercking inde daet, alwaermen deur nauwer en moeyelicker toesicht de volcommenheyt der Spiegheling so na mach commen, als de saecks einde tot Smenschen ghebruyck vereyscht.

Per Hypotesin.

Perpendicularis.

Hier uyt is te verstaen, dat wanneer sommighe de * Wisconsten van onvolcommenheyt beschuldighen, deur dien veel dadelicke werckinghen niet heel effen uyt en commen, datter kennis gebreeckt des onderscheyts tusschen Spiegeling en Daet, tusschen Wisconstighen en tuychwerckelicken handel: Want de Daet of * tuychwerckelicken handel om de boveschreven redenen altijt onvolcommen moet wesen.

Mathematicas artes.

Mechanicam.

Dese twee deelen Spiegeling en daet sijn so verscheydē, dat menich mensch hem t'eenemael tottet een begheeft, sonder van t'ander kennis te hebben, ghelijck menich leeraer met sijn toehoorders inde * ghemeen scholen ghebeurt, die hun gheduerlick in Spieghelingen oeffenen, als in *Euclides* * beginselen der Meetconst, sonder dadelick te meten landen, wallen, of vaten, of yet anders te doen daer de Daet in bestaet: En weerom verkeert so vintmen dadelicke Lantmeters, welcke alle reghels diese besighen gelooven, of toestaen waer te wesen, sonder inde Spiegeling t'ondersoucken de oirsaken en bewijs: Ia sommighen en weten niet datter sulcke oirsaken en bewijs af sijn.

Universitatibus.

Elementis.

Angaen-

ON THE COMBINATION OF THEORY AND PRACTICE.

Because, in the matter of orderly exposition, opinions differ with regard to the combination of theory and practice, I feel bound to give my view about this, first explaining their meaning for those who are not acquainted with it. Theory is a fictitious operation without natural material, such as *e.g.* the theories of the theoretician *Euclid,* which operate by the assumption of quantities and numbers, but each of them without connection with natural material. Practice is an operation which essentially takes place with natural material, such as the measurement of land and ramparts, counting the number of rods or feet contained therein, and the like. The conclusion of theoretical propositions is perfect, but that of practical propositions is imperfect. Thus, for instance, the theory finds and proves that half the product of the height and the base of a mathematical triangle perfectly gives the area of the plane surface, without any deficiency or surplus; but when a real triangle of land or smoother material is measured actually, the conclusion is imperfect, in the first place because we cannot measure any length so exactly that it does not differ by one thousandth part of a hair's breadth, or even if it is quite exact, it cannot be proved. In the second place because no natural lines are quite so straight nor natural plane surfaces quite so flat as the mathematical definitions require, or even if they are quite straight and flat, it cannot be proved. The property and the end of theory is that it furnishes a sure foundation for the method of practical operation, in which by closer and more painstaking care one may get as near to the perfection of the theory as the purpose of the matter requires for the benefit of man.

From this it is to be understood that if some people accuse the mathematical sciences of imperfection, because many practical operations do not produce results which are quite exact, they lack knowledge of the difference between theory and practice, between mathematical and mechanical operations, for practice or mechanical operation on the above account must always be imperfect.

These two sections, theory and practice, are so different that many people apply themselves altogether to the one, without being acquainted with the other, as is the case with many lecturers and their audience in the universities, where they constantly study theories, *e.g. Euclid*'s elements of geometry, without actually measuring lands, ramparts or vessels or doing anything else in which practice consists. And, conversely, practical surveyors are to be found who take on trust all the rules they apply or regard them as true, without examining the causes and the proof in the theory; nay, some do not even know that such causes and proofs exist.

Angaende ettelicke segghen de Spiegheling sonder Daet onnut te wesen, het schijnt datmen de saeck met beter onderscheyt soude meughen insien. Om hier af mijn ghevoelen te verclaren, ick segh by voorbeelt aldus: Datmen t'werck eens aerbeyders die boomen int bosch afhout, soude voor onnut achten, om dat hy der self gheen huysen, schepen, molens, sluysen, tonnen, kisten, beelden, en dierghelijcke me en maeckt, dat en waer openbaerlick niet wel ghesëyt, want hoewel sulcke wetenschap in een mensch looflick is, nochtans ghemerckt hy met boomen af te houwen, an veel ander stof levert, om elck hem in sijn ambacht te oeffenen, soo en is sijn aerbeyt niet te verachten: Maer dat hy die boomen afhieuwe om te laten verrotten, sonder nut daer af te verwachten, dat waer dwaselick ghedaen: En alsoo ist oock mette Spieghelaers in vrye consten, sy connen den Doenders stof leveren en voorderlick sijn, sonder self Doenders te wesen: Als den Spieghelaer *Euclides*, die wy niet en bevinden Doender gheweest te hebben, heeft nochtans voorstellen beschreven den dadelicken Boumeesters, Landtmeters, en ander doenders seer voorderlick: Den Spieghelaer *Ptolemeus* en meer ander die gheen dadelicke Stierlien en waren, hebben nochtans reghelen beschreven den dadelicken Stierlien op groote Zeevaerden, en ander hun daer in oeffenende seer nut: Ia sulcx dat de dadelicke Stierlien self sich voor meesters achten, als sy verstaen de reghelen door sulcke Spieghelaers beschreven, hoewelse nochtans gheen dadelicke Stierlien en waren. Daerom een Spieghelaers Spieghelinghen die ander Doenders te sta commen, en sijn niet onnut al en is hy self gheen Doender.

Tot hier toe deur ettelicke omstandighen verclaert hebbende wat Spiegeling en Daet beteyckent, soo is te weten dat d'oude Wisconstenaers met oock ettelicke nieuwe, van yder dier twee deelen onvermengt int besonder handelen, welcke oirden wy daert te pas comt oock volghen: Doch wantter by ettelicke een ander ghevoelen is, die Spiegheling en daet met malcander menghen, om teen mette ander t'samen te leeren, so moet ick om breeder reden mijns doens te verclaren, daer af mijn ghevoelen segghen.

Voor al soo ist te weten dat der Menschen natuerlicke gheneghentheden angaende de Daet seer verscheyden sijn, waer toe noch helpen de oirsaken die desen anders ontmoeten en dringhen als dien: Den eenen heeft natuerlicke lust met eenighe dringhende oirsaken totte Stercktebou, en dinghen de crijch angaende: Den anderen tot Landtmeten: De derde tot Wijnschroon: De vierde tot saken de groote Zeevaerden belanghende: De vijfde totten * Huysbou: De seste totte Spiegheling alleen: De sevende tot ettelicke van dese, of tot altemael, met noch oneyndelicke ander. Nu also een der voornaemste eynden vande beschrijving der vrye consten streckt, om daer deur te crijghen veel menschen, die ten ghemeenen oirboire met lichticheyt gheraken ter kennis van t'ghene daer sy hun toe begheven, soo willen wy eens overlegghen, of dat inde leering gheschien can door vermenging des daets mette Spiegheling, tot welcken eynde ick aldus segh: Alsmen onder spiegheligh voorstellen ettelicke des daets vermengt, en dat van een afcomst ghetrocken uyt de oneyndelicke menichte diemen daer af beschrijft, de selve dadelicke voorstellen en sullen misschien niet sijn van die afcomst daer den leerlinck na tracht: Als by voorbeelt, datmen inde Meetconst tusschen de spiegheligh voorstellen eenighe beschrijft des daets, waer in voorbeelden commen neem ick der meting deur het * schuyfcruys van ongherakelicke veynsters en pylaren van ghestichten; Maer t'can ghebeuren dat den leerlinck tot sulcke afcomst van Meetdaet gheen lust en sal hebben, denckende misschien dattet inde Daet luttel ghebruycx heeft, oock gheen ghenouchsaem seker-

Architecturam.

Radium.

As regards the assertion of some people that theory without practice is useless, it would seem that this matter should be considered more critically. To set forth my view about this, I say, for instance, as follows: it would evidently not be right to regard the work of a labourer, who cuts down trees in the wood, as useless because he does not personally make houses, ships, mills, canal-locks, barrels, chests, sculptures, and the like therewith, for although such knowledge is praiseworthy in a man, yet considering that by cutting down trees he supplies many others with material with which each of them can pursue his trade, his work is not to be despised. But if he were to cut down those trees to let them rot, without expecting any benefit of them, that would be acting foolishly. And thus it is also with the theoreticians in the liberal arts: they are able to furnish the practicians with material and to be of use to them, without themselves being practicians. Thus the theoretician *Euclid,* whom we do not find to have been a practician, nevertheless described propositions which are of great use to practical architects, surveyors, and other practicians. The theoretician *Ptolemy* and several others, who were no practical navigators, nevertheless described rules which are of great use to practical navigators during long voyages and to others pursuing this art, even to the extent that practical navigators consider themselves masters when they are conversant with the rules described by such theoreticians, although nevertheless the latter were no practical navigators. Accordingly the theories of a theoretician which are of use to others who are practicians are not useless, even though he himself is no practician.

Having so far explained by means of some ample considerations what is the meaning of theory and practice, I would say that it is to be noted that the ancient mathematicians as well as some modern ones treat of each of these two sections in particular without combining them, an order which we also follow where this is proper. But because a different view is held by some people, who combine theory and practice, in order to teach the two things together, I have to give my views about this, so as to explain the reason of my conduct more fully.

Before all it is to be noted that people's natural inclinations towards practice vary widely, a fact which is also promoted by the causes, which occur to and impel some people in a different way from others. One man has a natural liking, with some causes impelling him thereto, for the art of fortification and things concerned with war, another for surveying, a third for displacing barrels of wine, a fourth for matters concerned with great voyages, a fifth for architecture, a sixth for theory alone, a seventh for some or all of them, with an infinite number of others as well. Now since one of the main ends of the description of the liberal arts is thus to get many people who, for the benefit of all, easily acquire the knowledge of the subject to which they apply themselves, we will consider for a moment whether in instruction this can take place by a combination of practice and theory, to which end I say as follows: If among theoretical propositions one included some practical ones, and that of a species chosen from the infinite number described, these practical propositions may not be of the species the pupil is aiming at; thus, for instance, if in geometry among the theoretical propositions one describes some practical ones, in which there occur instances — I assume — of measurements, by means of the cross-staff, of inaccessible windows and columns of buildings. But it may be that the pupil has no liking for this species of practical measurement, thinking perhaps that it is little used in practice nor produces kerheyt, om op ſulcke gevonden maten van pylaren in ander ghebou te werck tegaen, of ander invallen die hem meughen te voor commen: Boven dien en ſal hyder niet vinden * d'afcomſt daer hy na tracht: Sulcx dat by aldien hy al wil verſtaen watter int bouck is, ſal moeten leeren dat hy niet en begheert te weten. Maer de Spiegheling als een oirdentlick geketent werck alleen beſchreven ſijnde, en den leerlinck die verſtaende, ſy verſtreckt hem tot ghemeene gront, om innerlick te begrijpen alſulcken deel der Daet als hy uyt verſcheyden beſchreven Daden int licht uytgaende, na ſijn behaghen verkieſen ſal.

Speciem.

Voor beſluyt, ick heb mijn ghevoelen verclaert hoet ſchijnt datmen de ſaeck an mocht legghen, om metter tijt weerom te gheraken tot ſulcke groote wetenſchappen alſſer inden Wyſentijt gheweeſt ſijn, ghelijck t'voornemen was. En met opſicht van ſulcken gront, ſal ick de volghende handelinghen beſchrijven.

EERSTE BOVCK DES
EERTCLOOTSCHRIFTS
EYNDE.

sufficient certainty to proceed for another building on the basis of the measures of columns thus found, or for other reasons that may occur to him, while moreover he will not there find the species which he is aiming at, so that if he wants to understand all that the book contains, he will have to learn what he does not wish to know. But if the theory alone is described as a systematic chain of reasoning and if the pupil understands it, it will serve him as a general basis for grasping mentally any part of practice which he chooses at his own pleasure from different descriptions of practice that are published.

To conclude, I have set forth my view as to how it seems one might set out to attain gradually to such great learning again as existed in the Age of the Sages, as I intended to do. And with respect to this basis I will write the following treatises.

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